OVERMUGGED MOCK PAPER 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ELEMENTARY MATHEMATICS   **4048/01**

Specimen Paper Marking Scheme

Date: **1 September 2021**   **Duration: 2 hours**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Write your name on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

At the end of the exam, fasten all your work securely together.
The number of marks is given in brackets [   ] at the end of each question or part question
The total number of marks for this paper is **80**.

Setter: Ong Kai Wen

This question paper consists of <u>22</u> printed pages including the cover page

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1.   Write the following numbers in order of size, starting with the **largest**

$$\left(\frac{6}{7} - \frac{5}{8}\right), \qquad 0.2, \qquad 0.2\dot{3}, \qquad 0.\dot{2}0\dot{3}$$

<u>Solution</u>

$$\left(\frac{6}{7} - \frac{5}{8}\right) = 0.232142\ldots$$

$$0.2 = 0.200000\ldots$$

$$0.2\dot{3} = 0.233333\ldots$$

$$0.\dot{2}0\dot{3} = 0.203203\ldots$$

Answer ....$0.2\dot{3}, \qquad \left(\frac{6}{7} - \frac{5}{8}\right), \qquad 0.\dot{2}0\dot{3}, \qquad 0.2$...................................... [**1**]

[S4 HSS P1/2020 PRELIM QN 1]

[**Total**: **1 mark**]

___

2. **(a)** Factorise the following completely

$$x^2 + 9y^2 - 6xy - 1$$

Solution

$$x^2 + 9y^2 - 6xy - 1 = (x^2 - 6xy + 9y^2) - 1$$
$$= (x - 3y)^2 - 1$$
$$= (x - 3y + 1)(x - 3y - 1)$$

Answer $(x - 3y + 1)(x - 3y - 1)$ [2]

(b) Express the following as a single fraction

$$\frac{2}{3(x-4)^2} + \frac{1}{4-x}$$

Solution

$$\frac{2}{3(x-4)^2} + \frac{1}{4-x} = \frac{2}{3(x-4)^2} - \frac{1}{x-4}$$
$$= \frac{2 - 3(x-4)}{3(x-4)^2}$$
$$= \frac{14 - 3x}{3(x-4)^2}$$

Answer $\dfrac{14 - 3x}{3(x-4)^2}$ [2]

[S4 ANDSS P1/2020 PRELIM Qn 2]

[**Total**: **4 marks**]

3. Show that the following is divisible by **5** for all positive integer values

$$3^{3x+2} - 9^{\frac{3}{2}x} + (27)^{x+1}$$

<u>Solution</u>

$$3^{3x+2} - 9^{\frac{3}{2}x} + 27^{x+1} = 3^{3x+2} - 3^{3x} + 3^{3x+3}$$
$$= 9(3^{3x}) - 3^{3x} + 27(3^{3x})$$
$$= 35(3^{3x})$$
$$= (5)(7)(3^{3x})$$

From the above simplification, we can see that 5 is a factor of the expression. Hence, the following is indeed divisible by 5 for all positive integer values

Answer .... $(5)(7)(3^{3x})$ .... [2]

[S4 AHS P1/2020 PRELIM Qn 4]

[**Total**: **2 marks**]

_____

4. Mrs Huang bought some chicken floss buns and hotdog buns for an outing. The ratio of the number of chicken floss buns to the number of hotdog buns bought was **11**:**7**. At the end of the outing, the number of each type of buns left was **4**. The ratio of the number of chicken floss buns to the number of hotdog buns consumed was **8**:**5**. Calculate the total number of buns that Mrs Huang bought

<u>Solution</u>

Let x be the total number of buns Mrs Huang bought

$$\frac{\left(\frac{11}{18}x - 4\right)}{\left(\frac{7}{18}x - 4\right)} = \frac{8}{5}$$

$$5\left(\frac{11}{18}x - 4\right) = 8\left(\frac{7}{18}x - 4\right)$$

$$\frac{55}{18}x - 20 = \frac{56}{18}x - 32$$

$$\frac{1}{18}x = 12$$

$$x = 216$$

Answer .... 216 .... [4]

[S4 BGSS P1/2020 PRELIM Qn 21]

[**Total**: **4 marks**]

_____

5. A farm rears chickens and sheep. Each chicken has **2** legs and each sheep has **4** legs. The total number of legs is **100** while the total number of heads is **35**. Find the number of chickens and sheep in the farm

Note: You are not allowed to use "Guess-and-Check" to solve this question

<u>Solution</u>

Let the number of chickens be C and the number of sheep be S

$2C + 4S = 100 \dots \dots \dots (1)$
$C + S = 35 \dots \dots \dots (2)$

From Equation (2),
$C = 35 - S \dots \dots \dots (3)$

Substitute Equation (3) into Equation (1),
$2(35 - S) + 4S = 100$
$2S = 30$
$S = 15$
$\therefore C = 20$

*Answer* .... Chicken $= 20$, Sheep $= 15$ .... [3]

[S4 BPGHS P1/2020 PRELIM Qn 6]

[Total: 3 marks]

_____

6. On the diagram below, mark and label a point to represent element x given the following

$$\{x\} \subset (A \cap B) \cup C$$
$$x \notin A$$
$$B = \{w, y, z\}$$

Answer:
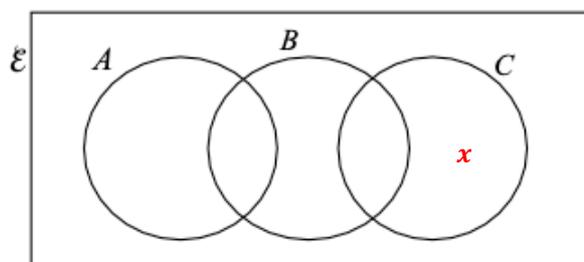
[S4 CHIJ CHIJS P1/2020 PRELIM Qn 6(b)]

[Total: 1 mark]

7.
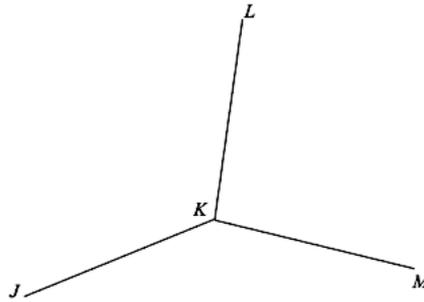
In the diagram, JK and KM are two sides of a regular decagon. JK and KL are two sides of a regular hexagon

(a) Calculate the interior angle of the regular decagon

<u>Solution</u>

$$\text{Interior angle} = \frac{(10-2)(180°)}{10}$$
$$= 144°$$

Answer ....<u>**144°**</u>................... [**1**]

(b) Explain if MK and KL are two sides of a regular polygon. Show your working clearly

<u>Solution</u>

$$\text{Interior angle}_{\text{hexagon}} = \frac{(6-2)(180°)}{6}$$
$$= 120°$$
$$\angle LKM = 360° - 144° - 120°$$
$$= 96° \text{ (angles about a point)}$$

$$\therefore \frac{(n-2)(180°)}{n} = 96°$$
$$96n = 180n - 360$$
$$n = \frac{360}{84}$$
$$= 4.285\ldots (\notin \mathbb{Z}^+)$$

**Since the number of sides is not an integer, MK and KL are not two sides of a regular polygon.**

Answer ....<u>**Not a regular polygon**</u>............... [**3**]

[S4 HIHS P1/2020 PRELIM Qn 20]

[**Total: 4 marks**]

8.
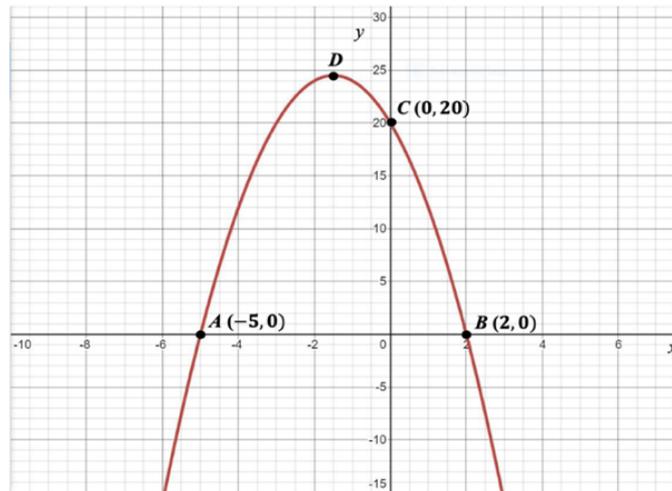
The graph of the following curve is drawn below. It cuts the axes at points A, B and C

$$y = -2(x^2 + 3x - 10)$$

(a) Find the coordinates of the maximum point D

Solution

$$\text{Midpoint} = \left(\frac{-5 + 2}{2}, 0\right)$$

$$= \left(-1\frac{1}{2}, 0\right)$$

$$\therefore y = -2\left(\left(-1\frac{1}{2}\right)^2 + 3\left(-1\frac{1}{2}\right) - 10\right)$$

$$= 24\frac{1}{2}$$

Answer .... $\left(-1\frac{1}{2}, 24\frac{1}{2}\right)$ .... [1]

(b) Write down the equation of the line of symmetry of the graph

Answer .... $x = -1\frac{1}{2}$ .... [1]

(c) Use the graph to solve the equation

$$x^2 + 3x - 10 = 4$$

Solution

$x^2 + 3x - 10 = 4$

$-2(x^2 + 3x - 10) = -8$

$y = -8$

Need to sketch the line $y = -8$ onto the graph
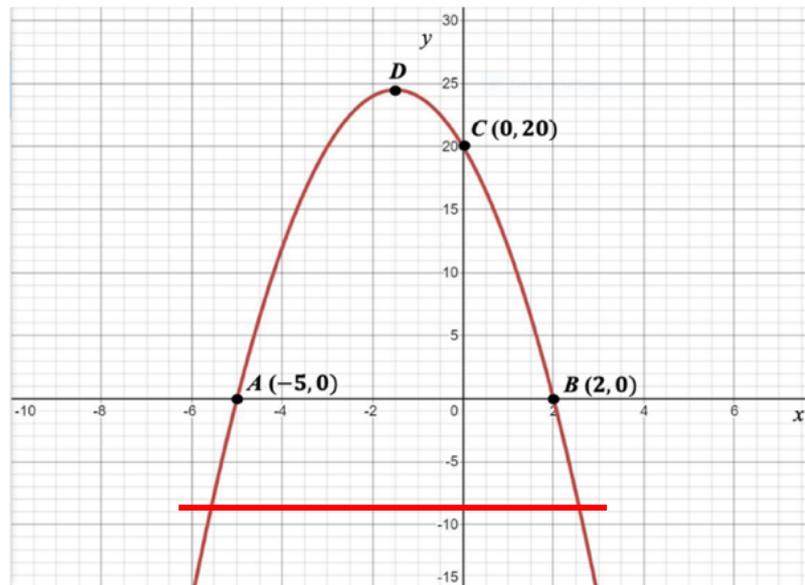
$x = -5.5$ **or** $x = 2.5$ **(accepted range:** $x \pm 0.25$**)**

$$x = -5\frac{1}{2} \text{ or } x = 2\frac{1}{2}$$
Answer .... [2]

(d) Explain why the equation does not have solutions for some values of k

$$-2(x^2 + 3x - 10) = k$$

Answer For $-2(x^2 + 3x - 10) = k$ to have no solutions, the horizontal line $y = k$
does not have any intersection points with the curve, and lies directly above
the curve
.... [1]

[S4 HYSS P1/2020 PRELIM Qn 9]

[Total: 5 marks]

9.
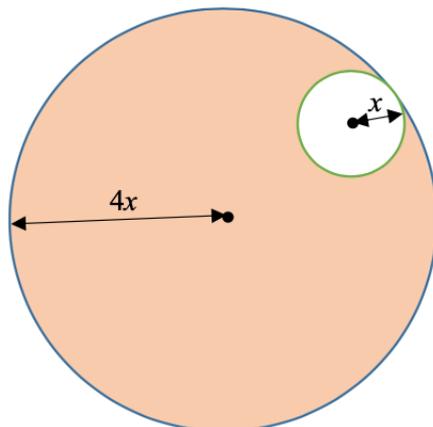
The diagram shows two circles with radii x and $4x$. A point is chosen, at random, inside the larger circle. Find, in the simplest fractional form, the probability that this point is in the shaded area

Solution

$$\text{P(shaded area)} = \frac{\pi(4x)^2 - \pi(x)^2}{\pi(4x)^2}$$

$$= \frac{15\pi x^2}{16\pi x^2}$$

$$= \frac{15}{16}$$

Answer .... $\dfrac{15}{16}$ [2]

[S4 MSS P1/2020 PRELIM Qn 7]

[**Total: 2 marks**]

10.
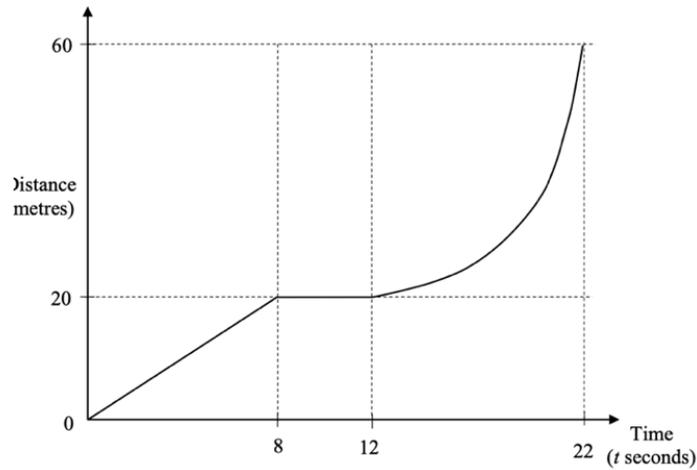
The diagram above shows the distance-time graph of the first **22** seconds of a journey

(a) Find the speed during the first **8** seconds

Solution

$$\text{Speed} = \frac{20}{8}$$

$$= 2\frac{1}{2} \text{ m/s}$$

Answer .... $2\frac{1}{2}$ **m/s** [1]

(b) What is the acceleration when $t = 4$

Answer .... **0 m/s²** [1]

(c) It is given that the speed increases at a uniform rate between $t = 12$ and $t = 22$, and the speed is **8 m/s** when $t = 22$. Sketch the speed-time graph for the first **22** seconds in the axes below

[2]
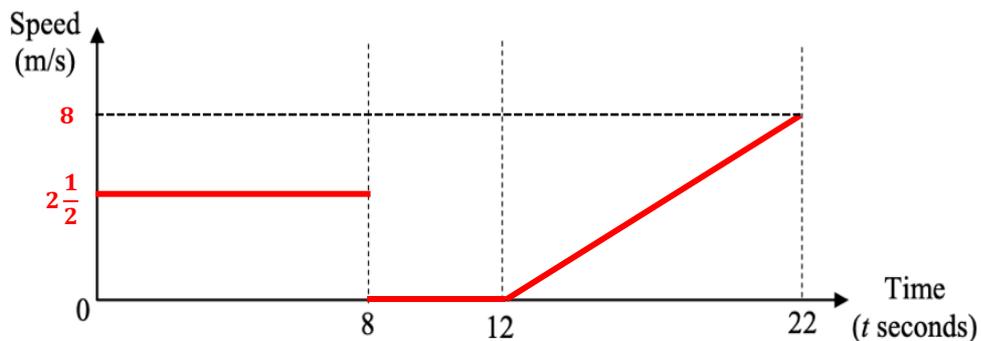
(d) Change $8\ \text{m/s}$ into kilometres per hour

<u>Solution</u>

$$8\ \text{m/s} = \frac{\left(\frac{8}{1000}\right)}{\left(\frac{1}{3600}\right)}$$

$$= 28.8\ \text{km/h}$$

Answer ....<u>28.8 km/h</u>........................ [1]

[S4 MGS P1/2020 PRELIM Qn 15]

[**Total**: **5 marks**]

11. Two bottles of hand sanitisers are geometrically similar. The height of the smaller bottle and larger bottle are **10 cm** and **19 cm** respectively. The price of the smaller and larger bottle are $**4. 90** and $**15. 10** respectively. Explain which bottle gives the better value for money

<u>Solution</u>

To find which bottle gives the better value, we first need to find the volume of each bottle

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$= \frac{10^3}{19^3}$$

$$= \frac{1000}{6859}$$

$$\text{Unit price for smaller bottle} = \frac{\$4.90}{1000}$$

$$= \$0.0049/\text{cm}^3$$

$$\text{Unit price for larger bottle} = \frac{\$15.10}{6859}$$

$$= \$0.0022/\text{cm}^3$$

As the unit price of the smaller bottle is smaller than the unit price of the larger bottle, it will be better value for money to buy the larger bottle

Answer ....**Larger bottle**................... [3]

[S4 NHHS P1/2020 PRELIM Qn 4]

[**Total**: **3 marks**]

12. (a) It takes **6** people to lay an area of tiles in **8** days. How long will it take if there are **16** people to do the same job?

Solution

6 people to lay an area in 8 days

1 person to lay an area in $(8 \times 6) = 48$ days

16 people to lay an area in $\left(\dfrac{48}{16}\right) = 3$ days

Answer ....**3 days**................... [**1**]

(b) The intensity, I, of a light source on an object is inversely proportional to the square of distance, d, between the light source and the object. If the distance between the light source and the object is decreased to **40**% of the original distance, calculate the percentage increase in the intensity of the light source on the object

Solution

$I \propto \dfrac{1}{d^2}$

$I = \dfrac{k}{d^2}$

New $d = 0.6d$

$\therefore I_{new} = \dfrac{k}{(0.4d)^2}$

$= 6\dfrac{1}{4}\left(\dfrac{k}{d^2}\right)$

$= 6\dfrac{1}{4}I_{original}$

$\% \text{ increase} = \dfrac{\left(6\dfrac{1}{4}I_{original} - I_{original}\right)}{I_{original}} \times 100\%$

$= 525\%$

Answer ....**525%**................... [**2**]

[S4 NGHS P1/2020 PRELIM Qn 10]

[**Total**: **3 marks**]

13.
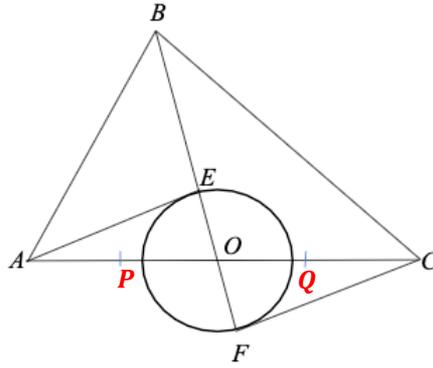
In the diagram, $OA = OC$, AE and FC are tangents to the circle with centre O. OE and OF are radii of a circle. BF is a straight line passing through point E and O

(a) Show that triangle OAE and triangle OCF are congruent. Give a reason for each statement you make

[3]

<u>Solution</u>

$OA = OC$ **(given)**

$OE = OF$ **(radius)**

$\angle AOE = \angle COF$ **(vertically opposite angles)**

By the SAS congruency test, $\triangle OAE \equiv \triangle OCF$

OR

$OA = OC$ **(given)**

$OE = OF$ **(radius)**

$\angle AEO = \angle CFO = 90°$ **(radius is perpendicular to tangent)**

By the RHS congruency test, $\triangle OAE \equiv \triangle OCF$

OR

$\angle AOE = \angle COF$ **(vertically opposite angles)**

$\angle AEO = \angle CFO = 90°$ **(radius is perpendicular to tangent)**

$OA = OC$ **(given)**

**By the AAS congruency test, $\triangle OAE \equiv \triangle OCF$**

(b) Points, P and Q lie on the circumference of the circle such that triangle APE is congruent to triangle CQF. On the diagram, mark the positions for the points, P and Q

[1]

[S4 PCSS P1/2020 PRELIM Qn 14]

[Total: 4 marks]

14.
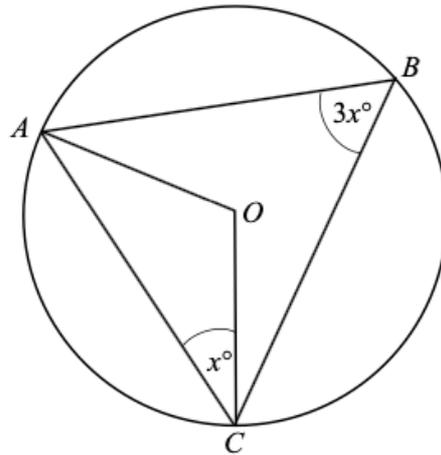
In the diagram, A, B and C are points on a circle, centre O. $\angle ACO = x°$ and $\angle ABC = 3x°$

Find x

Solution

$\angle AOC = 2(3x°)$

$\qquad = 6x°$ (angle at centre $= 2 \times$ angle at circumference)

$OA = OC$ (radius of circle)

$6x° + x° + x° = 180°$
$x° = 22.5$
$x = 22.5$

Answer ....22.5................... [3]

[S4 QSS P1/2020 PRELIM Qn 16]

[Total: 3 marks]

15.
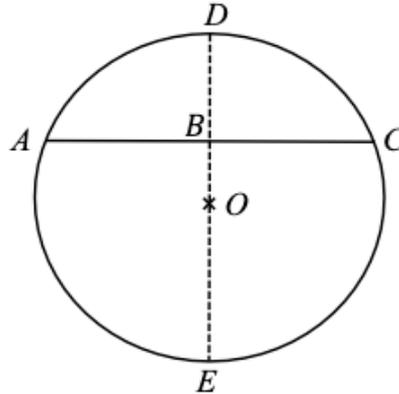
The diagram shows a circle, centre O, radius **16 cm**. B is the mid-point of the chord AC, DE is a diameter and $BE = 25$ **cm**

(a) Calculate $\angle AOC$ is in radians

<u>Solution</u>

$$\cos \angle BOC = \frac{BO}{OC}$$

$$\angle BOC = \cos^{-1}\left(\frac{25 - 16}{16}\right)$$

$$\angle AOC = 2\cos^{-1}\left(\frac{9}{16}\right)$$

$$= 1.946779\ldots$$

$$= 1.95 \text{ radians } (3.\,\text{s.\,f.})$$

Answer ....<u>**1.95 radians**</u>................ [2]

(b) Hence, find the area of the segment $ABCE$

<u>Solution</u>

$$\text{Area of segment } ABCE = \text{Area of circle} - \text{Area of leaf } ABD$$

$$= \text{Area of circle} - (\text{Area of sector } AOC - \text{Area of triangle } AOC)$$

$$= \pi(16)^2 - \left\{\frac{1}{2}(16)^2\left[2\cos^{-1}\left(\frac{9}{16}\right)\right] - \frac{1}{2}(16)^2\sin\left[2\cos^{-1}\left(\frac{9}{16}\right)\right]\right\}$$

$$= 674.11891\ldots$$

$$= 674 \text{ cm}^2 \ (3.\,\text{s.\,f.})$$

Answer ....<u>**674 cm²**</u>................ [3]

[S4 SST P1/2020 PRELIM Qn 19]

[Total: 5 marks]

16.

Huge Increase in students studying in library
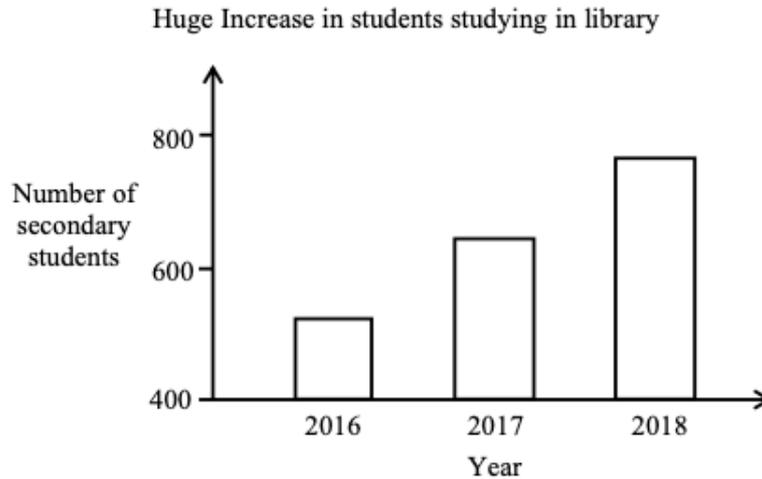
The graph below shows the number of secondary school students studying in the library between **2016** and **2018**

(a) State one way in which the graph is misleading and explain why it is misleading

Answer ....**Vertical axis does not start from 0 and seems to suggest that the number of**..........

..........**students studying in the library in 2018 is three times of 2016**..........

....

.... **[1]**

(b) Do you agree that the percentage of secondary school students studying in the library has been increasing between years **2016** to **2018**

Answer ....**Disagree. The total enrolment of each year is not given and hence we are unable**..........

..........**to calculate the percentage of students studying in the library each year. Thus, it**..........

..........**is not right to draw such conclusions**..........

.... **[1]**

[S4 SMSS P1/2020 PRELIM Qn 5]

[**Total**: **2 marks**]

17. (a) Solve the inequalities

$$-2 < \frac{2(7 + 5x)}{8} < 10$$

Solution

$$-2 < \frac{2(7 + 5x)}{8} < 10$$

$-16 < 14 + 10x$ **and** $14 + 10x < 80$

$-30 < 10x$ $\quad\quad\quad\quad 10x < 66$

$-3 < x$ $\quad\quad\quad\quad\quad x < 6\frac{3}{5}$

$-3 < x < 6\frac{3}{5}$

Answer .... [2]
$$-3 < x < 6\frac{3}{5}$$

(b) Write down all the prime numbers that satisfy the above inequality

Answer .... $2, 3, 5$ .... [1]

[S4 CHIJ SNGS P1/2020 PRELIM Qn 5]

[Total: 3 marks]

_____

18. Mr Smith plans to invest his money in unit trust with a bank. His target is to earn an interest of $\$10,000$ after **5** years. The bank pays **3%** compound interest per annum compounded yearly. Calculate, to the nearest hundred dollars, the minimum amount of money Mr Smith has to invest

Solution

Let the amount Mr Simth plan to invest be P

$$P\left(1 + \frac{3}{100}\right)^5 - P = 10\,000$$

$$P\left[\left(1 + \frac{3}{100}\right)^5 - 1\right] = 10\,000$$

$$P = \frac{10\,000}{\left(1 + \frac{3}{100}\right)^5 - 1}$$

$= 62\,784.85713\ldots$

$= \$62\,800 \text{ (nearest hundred dollars)}$

Answer .... $\$62\,800$ .... [3]

[S4 TKSS P1/2020 PRELIM Qn 18]

[Total: 3 marks]

19.
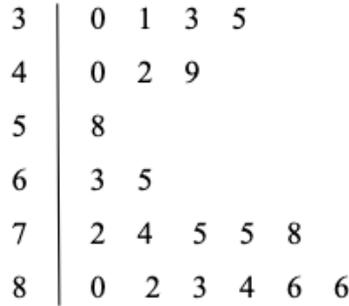
```
3 | 0  1  3  5
4 | 0  2  9
5 | 8
6 | 3  5
7 | 2  4  5  5  8
8 | 0  2  3  4  6  6
Key  3 | 0  means 30 marks
```

The stem-and-leaf diagram shows the marks of **21** students who took a class test. The test was marked out of **100**

(a) Find the
  (i) median mark

<div align="right">

Answer ....**72 marks**.................. [**1**]

</div>

  (ii) interquartile range

  Solution

$$\text{Interquartile range} = 81 - 41$$
$$= 40$$

<div align="right">

Answer ....**40 marks**.................. [**1**]

</div>

(b) A student said that the test was easy. Do you agree? Justify your answer

Answer **The test is easy as the median mark is 72 marks** ....

....

....

.... [**2**]

[S4 TSS P1/2020 PRELIM Qn 21]

<div align="right">

[**Total**: **4 marks**]

</div>

20.
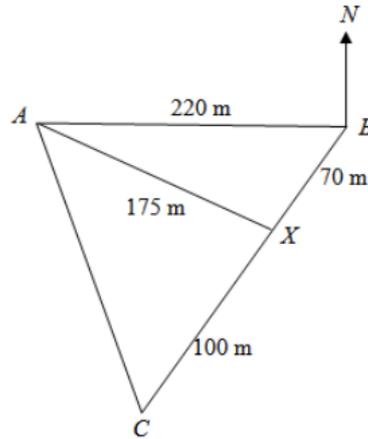
A, B and C are three points on a horizontal field. A is **220 m** due west of B. X is a point on BC such that $BX = 70$ **m**, $CX = 100$ **m** and $AX = 175$ **m**

(a) Calculate the bearing of C from B

<u>Solution</u>

$$\cos B = \frac{220^2 + 70^2 - 175^2}{2(220)(70)}$$

$$B = \cos^{-1}\left(\frac{220^2 + 70^2 - 175^2}{2(220)(70)}\right)$$

$$\text{Bearing of } C \text{ from } B = 360° - 90° - \cos^{-1}\left(\frac{220^2 + 70^2 - 175^2}{2(220)(70)}\right)$$

$$= 227.409°$$

$$= 227.4° \, (1.\,\text{d.}\,\text{p.})$$

Answer ....**227.4°**................................. [3]

(b) Calculate the shortest distance from X to AB

<u>Solution</u>

Let the point where the shortest distance cuts AB be M

$$\sin \angle MBX = \frac{MX}{XB}$$

$$MX = 70 \sin\left[\cos^{-1}\left(\frac{220^2 + 70^2 - 175^2}{2(220)(70)}\right)\right]$$

$$= 47.37338\ldots$$

$$= 47.4 \, \text{m} \, (3.\,\text{s.}\,\text{f.})$$

Answer ....**47.4 m**................................. [3]

[S4 YISS P1/2020 PRELIM Qn 22]

[Total: 6 marks]

21. The line **AB** is drawn below

    (a) Construct triangle **ABC** where ∠**ABC** = **80°** and **BC** = **6 cm**

                                                                                                      [1]

    (b) Construct the bisector of ∠**BAC**

                                                                                                             [1]

    (c) Construct the perpendicular bisector of **BC**

                                                                                                               [1]

    (d) Mark clearly a possible point which is outside the triangle, equidistant from **B** and **C**, and nearer to **AB** than **AC**. Label this point **P**

                                                                                                               [1]

Solution
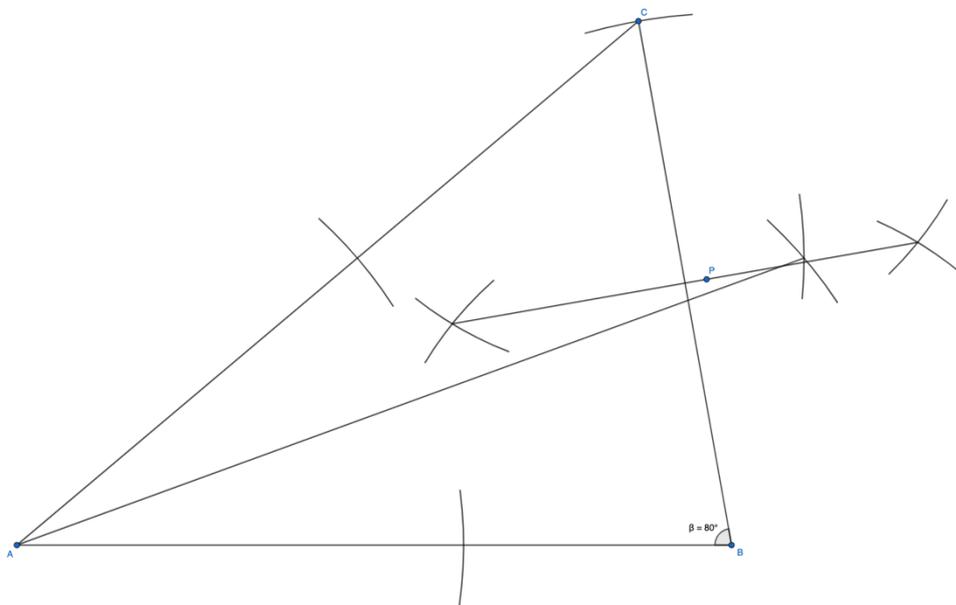
[S4 ZHSS P1/2020 PRELIM Qn 19]

[Total: 4 marks]

22.
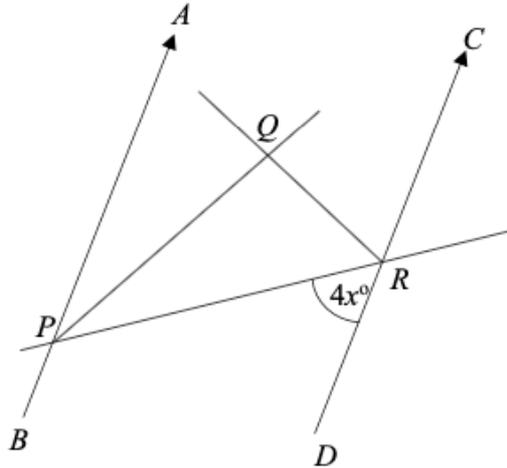
In the figure, AB is parallel to CD. PQ and QR bisects $\angle APR$ and $\angle CRP$ respectively.

Given that $\angle DRP = 4x°$, show that PQ is perpendicular to QR

[**3**]

<u>Solution</u>

$$\angle QRP = \frac{180° - 4x°}{2}$$
$$= 90° - 2x° \text{ (supplementary angles)}$$

$\angle APR = 4x° \text{ (alternate angles, } AB \text{ parallel to } CD\text{)}$

$\text{Since } QP \text{ bisects } \angle APR, \angle QPR = 2x°$

$\angle PQR + \angle QRP + \angle QPR = 180° \text{ (angles in a triangle)}$
$\angle PQR + 90° - 2x° + 2x° = 180°$
$\angle PQR = 90°$

$\text{Hence, } PQ \text{ is perpendicular to } QR \text{ (shown)}$

[S4 AHS P1/2020 PRELIM Qn 19]

[**Total: 3 marks**]

23.
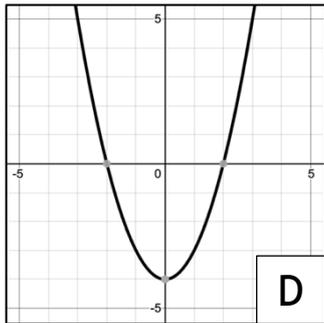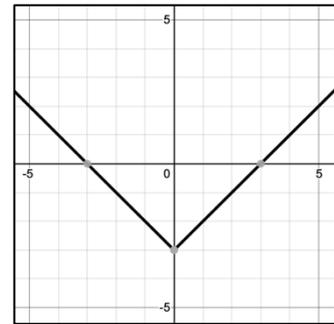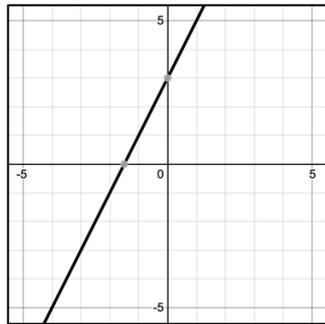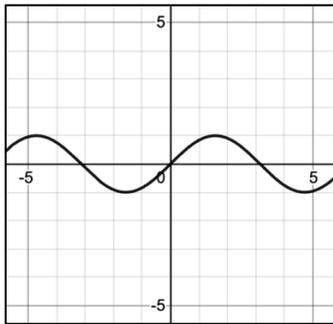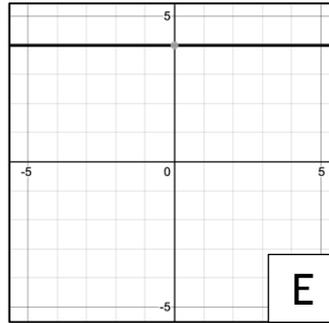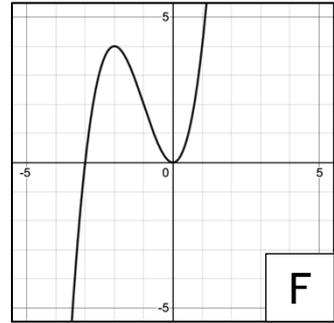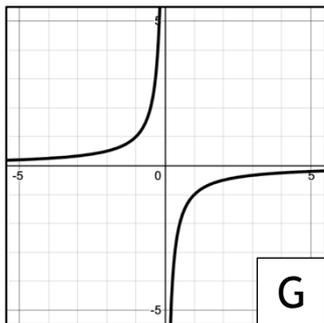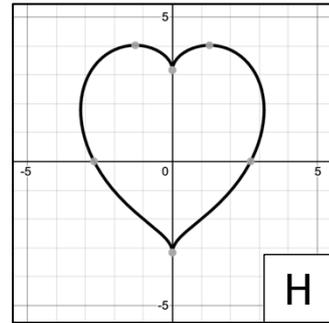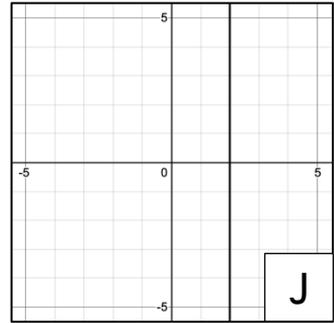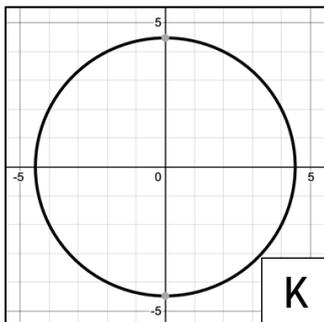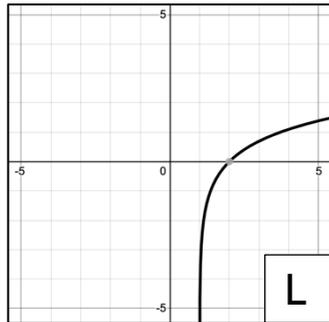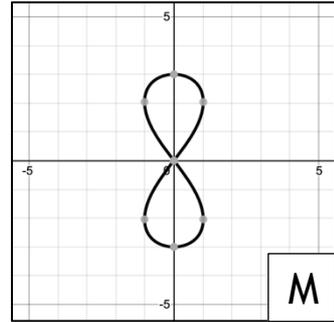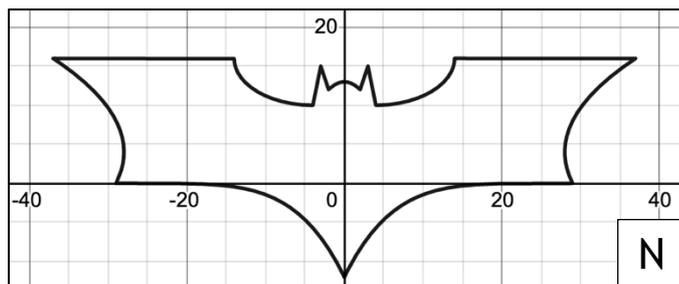
Each of the **13** plots on the previous page, labelled A to N, show graphs of varying equations. Answer the following questions. For some of the questions, the number of graphs required to get the full credit of the question is indicated at the end of each sub-part. No justification is required.

(a) Which of the following graphs passes through origin? **(3 graphs)**

[**1**]

<u>Solution</u>

Graphs A, F, M

(b) Which of the following graphs have only **1** line of symmetry? **(4 graphs)**

[**1**]

<u>Solution</u>

Graphs C, D, H, N

(c) Which of the following graphs are fully enclosed and when extended will not tend towards infinity? **(4 graphs)**

[Hint: Think about what does the phrase "fully enclosed" mean]

[**1**]

<u>Solution</u>

Graphs H, K, M, N

(d) Which of the following graph(s) will correspond to the line of symmetry of a quadratic curve of the form $ax^2 + bx + c = 0$

[**1**]

<u>Solution</u>

Graph J

(e) <u>**TWO**</u> of the curves have its general-form equation labelled incorrectly. Take α to be an arbitrary constant. Identify the error and relabel it with the correct equation in the blanks provided. Use the arbitrary constant α if necessary

Graph	General-Form Equation
B	$y = mx + c$
D	$y = ax^2 + bx + c$
E	$x = \alpha$
F	$y = ax^3 + bx^2 + cx + d$
G	$y = \dfrac{\alpha}{x^2}$

Graph .... *E* .... Is labelled incorrectly. The correct equation should be .... $y = \alpha$ ....

Graph ...... *G* ...... Is labelled incorrectly. The correct equation should be .... $y = -\dfrac{\alpha}{x}$ ....

[**2**]

[**Total: 6 marks**]

End of Paper ☺