Name: _____ Material: E-Math Mock Paper 2

OVERMUGGED MOCK PAPER 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ELEMENTARY MATHEMATICS **4048/02**

Specimen Paper MARKING SCHEME

Date: **1 September 2021** **Duration: 2 hours 30 minutes**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Write your name on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

At the end of the exam, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question
The total number of marks for this paper is 100.

Setter: Ong Kai Wen

This question paper consists of 22 printed pages including the cover page

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone } = \pi rl$$

$$\text{Surface area of a sphere } = 4\pi r^2$$

$$\text{Volume of a cone } = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere } = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. (a) Given that

$$p = 4q(r - 2)$$

(i) Evaluate p when $q = 2$ and $r = -3$

Solution

$$\begin{aligned} p &= 4(2)(-3 - 2) \\ &= -40 \end{aligned}$$

Answer $p = -40$ **[1]**

(ii) Express r in terms of p and q

Solution

$$\begin{aligned} p &= 4q(r - 2) \\ r - 2 &= \frac{p}{4q} \\ r &= \frac{p}{4q} + 2 \end{aligned}$$

Answer $r = \frac{p}{4q} + 2$ **[1]**

(b) Simplify

$$\frac{9 - 6x}{3 - 2x + 3y - 2xy}$$

Solution

$$\begin{aligned} \frac{9 - 6x}{3 - 2x + 3y - 2xy} &= \frac{3(3 - 2x)}{3(1 + y) - 2x(1 + y)} \\ &= \frac{3(3 - 2x)}{(3 - 2x)(1 + y)} \\ &= \frac{3}{1 + y} \end{aligned}$$

Answer $\frac{3}{1 + y}$ **[3]**

(c) (i) Express the following in the form $(x-h)^2+q$

$$7-4x+x^2$$

Solution

$$x^2-4x+7=(x-2)^2-(2)^2+7$$
$$=(x-2)^2+3$$

Answer $(x-2)^2+3$ [2]

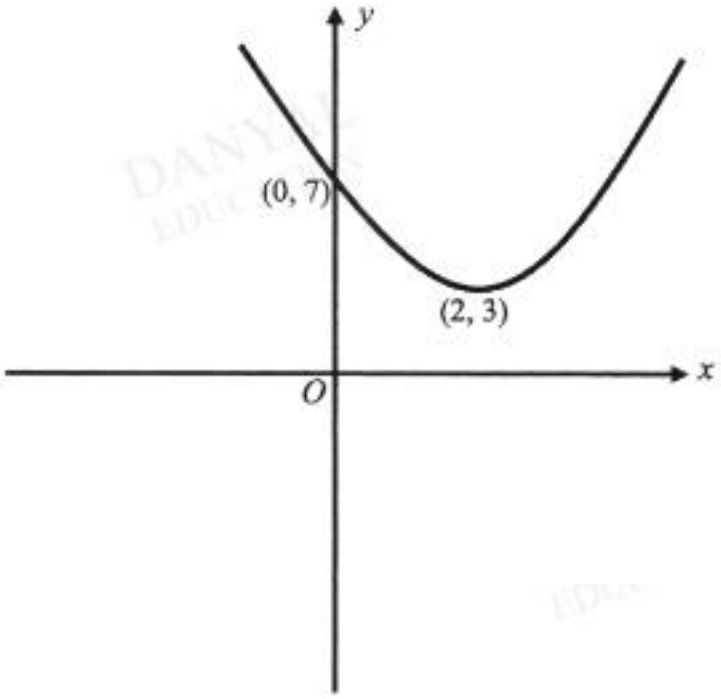
(ii) Sketch the graph of

$$y=7-4x+x^2$$

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the graph

[2]

Answer



(iii) Write the equation of the line of symmetry of the graph of

$$y=7-4x+x^2$$

Answer $x=2$ [1]

(d) Solve the equation

$$\frac{x}{2-x}+\frac{5}{3x-1}=\frac{5+10x}{(3x-1)(2-x)}$$

Solution

$$\frac{x}{2-x}+\frac{5}{3x-1}=\frac{5+10x}{(3x-1)(2-x)}$$
$$x(3x-1)+5(2-x)=5+10x$$
$$3x^2-x+10-5x-5-10x=0$$
$$3x^2-16x+5=0$$
$$(3x-1)(x-5)=0$$
$$x=\frac{1}{3}\ (\text{rej}) \quad \text{or} \quad x=5$$

Answer $x=5$ ..... [3]

[S4 ZHSS P2/2020 PRELIM Qn 1]

[Total: 13 marks]

2. The variables x and y are connected by the equation

$$y = 2x + \frac{30}{x} - 16$$

Some corresponding values of x and y are given in the table below

x	1.75	2	2.5	3	4	5	6	7	8
y	4.64	p	1	0	−0.5	0	1	2.29	3.75

(a) Calculate the value of p

Solution

$$p = 2(2) + \frac{30}{(2)} - 16$$
$$= 3$$

Answer ..... 3 ..... [1]

(b) In the grid given on the next page, draw the graph, for $1.75 \leq x \leq 8$, of

$$y = 2x + \frac{30}{x} - 16$$

Use a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leq x \leq 8$
Use a scale of 2 cm to represent 0.5 unit, draw a vertical y-axis for $-0.5 \leq y \leq 5$
On your axes, plot the points given in the table and join them with a smooth curve

[3]

Graph is drawn at the back of this question

(c) By drawing a tangent, find the gradient of the curve at $(6, 1)$

Solution

$$\text{Gradient} = \frac{3.33 - 0}{8 - 5.14}$$
$$= 1.64335\ldots$$
$$= 1.64$$

Answer ..... 1.64 ..... [2]

(d) Use your graph to find

(i) the range of values of x for which $y < 2.5$

Answer $2.09 < x < 7.15$ ..... [1]

(ii) the solutions to the equation

$$2x + \frac{30}{x} = 18$$

Solution

$$2x + \frac{30}{x} = 18$$
$$2x + \frac{30}{x} - 16 = 2$$
$$y = 2$$

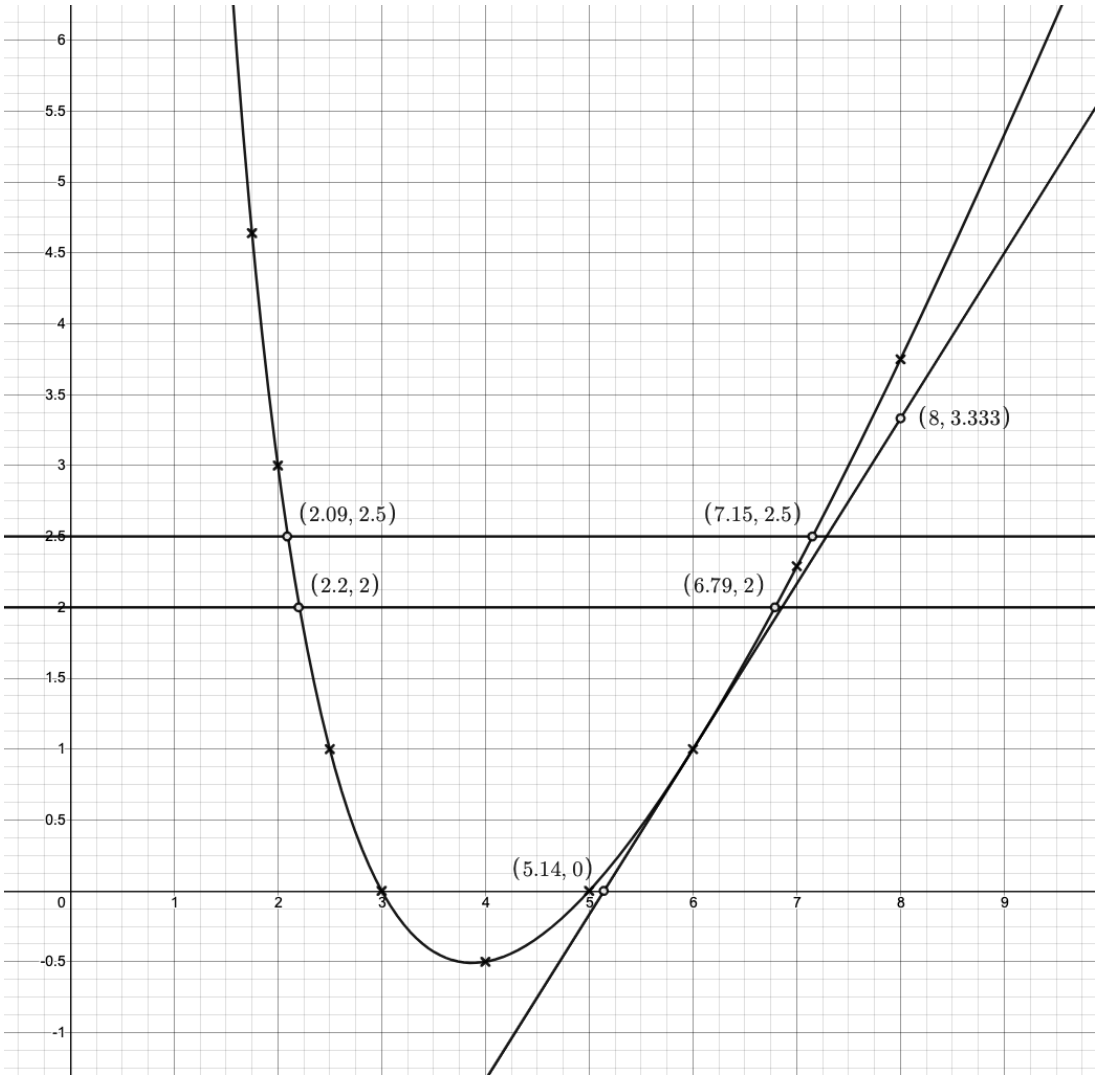
Draw the line $y = 2$

Answer $x = 2.2$ or $x = 6.79$ ..... [2]

[S4 HIHS P2/2020 PRELIM Qn 10]

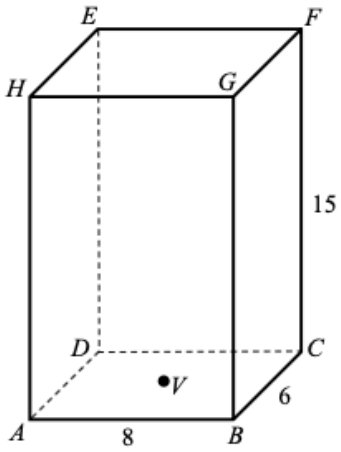
[**Total: 9 marks**]

Graph for Question 2



The solutions to this question is based on my drawings of the question, the solutions may vary dependent on your graph

3.



In the diagram, $\boldsymbol{ABCDEFGH}$ is a cuboid with dimensions $\mathbf{8\ cm}$ by $\mathbf{6\ cm}$ by $\mathbf{15\ cm}$. $\boldsymbol{V}$ is the centre of the rectangular base

(a) Show that $\boldsymbol{EV = 15.8\ cm}$, correct to $\mathbf{3}$ significant figures

[**2**]

Answer

Solution

$$\boldsymbol{DV = \frac{1}{2}\sqrt{6^2 + 8^2}}$$
$$\boldsymbol{= 5\ cm}$$

$$\boldsymbol{EV = \sqrt{15^2 + 5^2}}$$
$$\boldsymbol{= \sqrt{250}}$$
$$\boldsymbol{= 15.81138\ldots}$$
$$\boldsymbol{= 15.8\ cm\ (3.s.f.)\ (shown)}$$

(b) Calculate $\angle ACE$

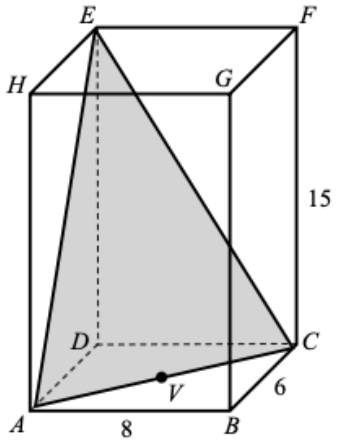
Solution

$$EC = \sqrt{15^2 + 8^2}$$
$$= 17 \text{ cm}$$

$$\cos \angle ACE = \frac{17^2 + 5^2 - 250}{2(17)(5)}$$
$$\angle ACE = \cos^{-1}\left(\frac{32}{85}\right)$$
$$= 67.885 \ldots$$
$$= 67.9° \ (1.\text{d.p.})$$

Answer $67.9°$ [2]

(c) A pyramid $EDAC$ is cut out from the cuboid



(i) Find the total surface area of the pyramid

Solution

$$\text{Total surface area} = \frac{1}{2}(15)(6) + \frac{1}{2}(15)(8) + \frac{1}{2}(8)(6) + \frac{1}{2}(17)(10)\sin\left[\cos^{-1}\left(\frac{32}{85}\right)\right]$$
$$= 207.74642 \ldots$$
$$= 208 \text{ cm}^2 \ (3.\text{s.f.})$$

Answer 208 cm^2 [2]

(ii) Another pyramid is to be made with volume half of pyramid $\boldsymbol{EDAC}$. Given that the two pyramids are geometrically similar, find the vertical height of the smaller pyramid

Solution

$$\frac{V_{\text{small}}}{V_{\text{large}}} = \left(\frac{h_{\text{small}}}{h_{\text{large}}}\right)^3$$

$$\frac{h_{\text{small}}}{h_{\text{large}}} = \sqrt[3]{\frac{1}{2}}$$

$$h_{\text{small}} = \sqrt[3]{\frac{1}{2}} \times 15$$

$$= 11.905507\ldots$$

$$= 11.9 \text{ cm } (3.\text{s.f.})$$

Answer 11.9 cm ..... [2]

[S4 YISS P2/2020 PRELIM Qn 8]

[**Total**: **8 marks**]

4. A bird flew $\mathbf{36\ km}$ at a constant speed of $\boldsymbol{x}\ \mathbf{km/h}$ from $\boldsymbol{A}$ to $\boldsymbol{B}$ (outward journey) when there was no wind. When the bird flew back from $\boldsymbol{B}$ to $\boldsymbol{A}$ (return journey) against the wind direction, its speed was decreased by $\mathbf{2\ km/h}$

(a) Write down, in terms of $\boldsymbol{x}$, the time taken for the bird to fly from $\boldsymbol{A}$ to $\boldsymbol{B}$

Answer $\dfrac{\mathbf{36}}{\boldsymbol{x}}\ \mathbf{hrs}$ **[1]**

(b) Write down, in terms of $\boldsymbol{x}$, the time taken for the bird to fly back from $\boldsymbol{B}$ to $\boldsymbol{A}$

Answer $\dfrac{\mathbf{36}}{\boldsymbol{x-2}}\ \mathbf{hrs}$ **[1]**

(c) Given that the return journey took the bird $\mathbf{75\ minutes}$ longer than the outward journey, form an equation in $\boldsymbol{x}$ and show that it simplifies to

$$\mathbf{5}\boldsymbol{x}^{\mathbf{2}}\mathbf{-10}\boldsymbol{x}\mathbf{-288=0}$$

[2]

Answer

Solution

$$\frac{\mathbf{36}}{\boldsymbol{x-2}}-\frac{\mathbf{36}}{\boldsymbol{x}}=\frac{\mathbf{75}}{\mathbf{60}}$$

$$\frac{\mathbf{36}\boldsymbol{x}-\mathbf{36}\boldsymbol{x}+\mathbf{72}}{\boldsymbol{x}(\boldsymbol{x}-\mathbf{2})}=\frac{\mathbf{5}}{\mathbf{4}}$$

$$\mathbf{72(4)}=\mathbf{5}\boldsymbol{x}(\boldsymbol{x}-\mathbf{2})$$

$$\mathbf{5}\boldsymbol{x}^{\mathbf{2}}\mathbf{-10}\boldsymbol{x}\mathbf{-288=0\ (shown)}$$

(d) Solve the equation $5x^2 - 10x - 288 = 0$, giving your answers to 2 decimal places

Solution

$$5x^2 - 10x - 288 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(-288)}}{2(5)}$$

$$= \frac{10 \pm \sqrt{5860}}{10}$$

$x = 8.655063\ldots$ or $x = -6.655063\ldots$

$x = 8.66\ (2.\text{d.p.})$ $x = -6.66\ (2.\text{d.p.})$

Answer $x = 8.66$ or $x = -6.66$ [3]

(e) Find the time taken by the bird on its return journey, giving your answer to the nearest minute

Solution

$$\text{Time taken} = \frac{36}{\left(\frac{10 + \sqrt{5860}}{10}\right) - 2}$$

$$= 5.40941\ldots \text{hrs}$$

$$= 324.56488\ldots \text{min}$$

$$= 325 \text{ min (nearest minute)}$$

Answer 325 minutes [2]

[S4 NHHS P2/2020 PRELIM Qn 9]

[**Total**: **9 marks**]

5. The first three terms in a sequence of numbers, T_1, T_2, T_3,... are given below

$$T_1 = 3 + 9 = 12 = 3 \times 2^2$$
$$T_2 = 3 + 9 + 15 = 27 = 3 \times 3^2$$
$$T_3 = 3 + 9 + 15 + 21 = 48 = 3 \times 4^2$$

(a) (i) Find T_4

$T_4 = 3 + 9 + 15 + 21 + 27 = 75 = 3 \times 5^2$

Answer ..... [1]

(ii) Find an expression, in terms of n for T_n

$T_n = 3(n+1)^2$

Answer ..... [1]

(iii) Find the value of p and the value of q given that

$$T_p = 3 + 9 + 15 + \cdots + q = 363$$

Solution

$363 = 3(n+1)^2$

$(n+1)^2 = 121$

$n + 1 = \pm 11 \ (\text{rej} - 11)$

$\therefore p = 10, \quad q = 63$

Answer $p = 10, \quad q = 63$ ..... [2]

(iv) Explain why T_n is an odd number when n is even

Solution

When n is even, $(n+1)$ is odd. The product of 3 odd numbers is always an odd number

Answer product of 3 odd numbers is odd ..... [1]

(b) The nth term of a different sequence is given by

$$P_n = 3n - 6$$

Siti is asked to find the difference between T_{50} and P_{50} and her answer is 7660. Without any calculation of values, is Siti correct? Justify your answer

Answer $T_n - P_n$ is a multiple of 3. Since 7660 is not divisible by 3, Siti's answer is wrong .....

..... [2]

[S4 NGHS P2/2020 PRELIM Qn 6]

[**Total**: **7 marks**]

6. **(a)** Find the matrix $\boldsymbol{A}$ such that

$$4\boldsymbol{A} + \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 6 & 8 \end{pmatrix}$$

Solution

$$4\boldsymbol{A} = \begin{pmatrix} 4 & 3 \\ 6 & 8 \end{pmatrix} - \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 8 & 4 \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} \frac{1}{2} & 1\frac{1}{2} \\ 2 & 1 \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} \frac{1}{2} & 1\frac{1}{2} \\ 2 & 1 \end{pmatrix}$$

Answer ..... [2]

(b) Given that $\boldsymbol{D} = \begin{pmatrix} 3 & -4 \\ 0 & 2 \end{pmatrix}$, $\boldsymbol{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{F} = \begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix}$, evaluate the following

$$2\boldsymbol{D} + \boldsymbol{E} - 2\boldsymbol{F}$$

Solution

$$\begin{aligned} 2\boldsymbol{D} + \boldsymbol{E} - 2\boldsymbol{F} &= 2\begin{pmatrix} 3 & -4 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2\begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -8 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -4 & -2 \\ 2 & 8 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -8 \\ 0 & 5 \end{pmatrix} - \begin{pmatrix} -4 & -2 \\ 2 & 8 \end{pmatrix} \\ &= \begin{pmatrix} 12 & -6 \\ -2 & -3 \end{pmatrix} \end{aligned}$$

Answer $\begin{pmatrix} 12 & -6 \\ -2 & -3 \end{pmatrix}$ ..... [2]

(c) Two cafés sell coffee from different countries. The sale and price of the coffee at the two cafés are shown in the table below

	Ethiopia ($8)	Myanmar ($6)	Colombia ($7)
Café A	**18**	**9**	**10**
Café B	**15**	**7**	**24**

(i) Represent the information above by a $\mathbf{2 \times 3}$ matrix $\boldsymbol{C}$

Answer $\begin{pmatrix} 18 & 9 & 10 \\ 15 & 7 & 24 \end{pmatrix}$ ..... [1]

(ii) Evaluate the matrix $\boldsymbol{S}$ where

$$\boldsymbol{S} = \boldsymbol{C}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Solution

$$\boldsymbol{S} = \begin{pmatrix} 18 & 9 & 10 \\ 15 & 7 & 24 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 18+9+10 \\ 15+7+24 \end{pmatrix}$$
$$= \begin{pmatrix} 37 \\ 46 \end{pmatrix}$$

Answer $\begin{pmatrix} 37 \\ 46 \end{pmatrix}$ ..... [1]

(iii) Explain what the elements of $\boldsymbol{S}$ represent

Answer It represents the total cups of coffee sold at café A and B respectively

..... [1]

(iv) Represent the price of the coffee using matrix $\boldsymbol{P}$

Answer $\begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$ ..... [1]

(v) Using the matrix $\boldsymbol{C}$ and $\boldsymbol{P}$, find the total sales $\boldsymbol{T}$ by each café

Solution

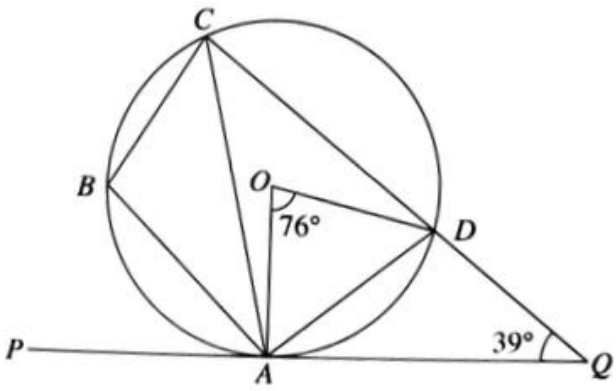
$$\boldsymbol{T} = \begin{pmatrix} 18 & 9 & 10 \\ 15 & 7 & 24 \end{pmatrix}\begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$$
$$= \begin{pmatrix} 268 \\ 330 \end{pmatrix}$$

Answer $\begin{pmatrix} 268 \\ 330 \end{pmatrix}$ ..... [2]

[S4 TSS P2/2020 PRELIM Qn 10]

[**Total**: **10 marks**]

7.



A, B, C and D are points on the circle with centre O. PAQ is a tangent to the circle at A and meets the line CD extended at point Q. Given that $\angle AOD = 76°$ and $\angle DQA = 39°$, find, giving your reasons for each answer

(a) $\angle DAQ$

Solution

$$\angle OAD = \frac{180° - 76°}{2}$$
$$= 52° \text{ (angles in an isosceles triangle)}$$

$$\angle DAQ = 90° - 52°$$
$$= 38° \text{ (radius is perpendicular to tangent)}$$

Answer ..... $38°$ ..... [2]

(b) $\angle ABC$

Solution

$$\angle CDA = 38° + 39°$$
$$= 77° \text{ (exterior angle = sum of interior opposite angles)}$$

$$\angle ABC = 180° - 77°$$
$$= 103° \text{ (angles in opposite segments)}$$

Answer ..... $103°$ ..... [2]

(c) $\angle OAC$

Solution

$$\angle ACD = \frac{76°}{2}$$
$$= 38° \text{ (angle at centre } = 2 \times \text{angle at circumference)}$$

$$\angle CAD = 180° - 38° - 39°$$
$$= 103° \text{ (angles in a triangle)}$$

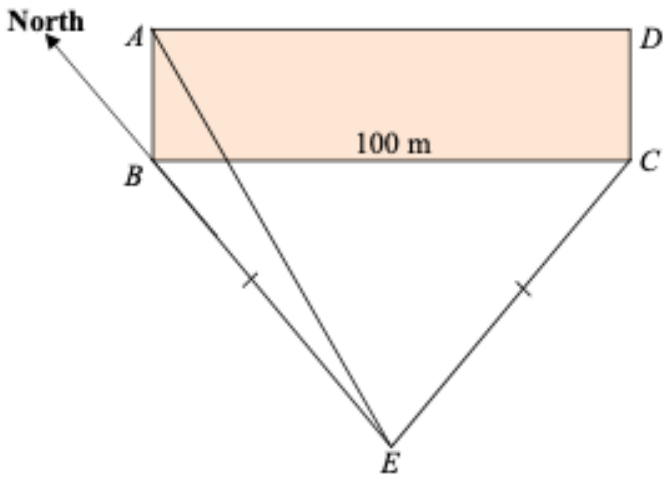
$$\angle OAC = \angle CAD - \angle OAD$$
$$= 103° - 90°$$
$$= 13°$$

Answer ..... $13°$ [3]

[S4 HSS P2/2020 PRELIM Qn 2]

[**Total**: **7 marks**]

8.



In the diagram, $ABCD$ represents a vertical cliff face. The bottom of the cliff, BC is 100 m and is at sea level. A boat is in the sea at E. B is due North of E and the bearing of C from E is $070°$. E is known to be equidistant from B and C

(a) Find the bearing of C from B

Solution

$$\angle EBC = \frac{180° - 70°}{2}$$
$$= 55° \text{ (angles in an isosceles triangle)}$$

$$\text{Bearing of } C \text{ from } B = 125°$$

Answer $125°$ [2]

(b) Show that $BE = 87.17$ m

[2]

Answer

Solution

$$\frac{\sin 70°}{100} = \frac{\sin 55°}{BE}$$
$$BE = \frac{100 \sin 55°}{\sin 70°}$$
$$= 87.17 \text{ m (shown)}$$

(c) Find the area of ΔBCE, leaving your answer to the nearest square metre

Solution

$$\text{Area of } \Delta BCE = \frac{1}{2}\left(\frac{100\sin 55°}{\sin 70°}\right)^2 \sin 70$$
$$= 3570.3700\ldots$$
$$= 3750\ \text{m}^2\ (3.\text{s.f.})$$

Answer $3570\ \text{m}^2$ [2]

(d) It is given that the angle of depression of E from A is $20.2°$. Find the height of the cliff BA, leaving your answers in 3 significant figures

Solution

$$\tan \angle BEA = \frac{AB}{BE}$$
$$\tan 20.2° = \frac{AB}{\left(\frac{100\sin 55°}{\sin 70°}\right)}$$
$$AB = \left(\frac{100\sin 55°}{\sin 70°}\right)\tan 20.2°$$
$$= 32.073176\ldots$$
$$= 32.1\ \text{m}\ (3.\text{s.f.})$$

Answer $32.1\ \text{m}$ [2]

(e) Find the length of CP given that P is a point on BC such that

$$\frac{\text{Area of } \Delta BEP}{\text{Area of } \Delta CEP} = \frac{3}{2}$$

Solution

$$\frac{\text{Area of } \Delta BEP}{\text{Area of } \Delta CEP} = \frac{3}{2}$$
$$\frac{BP}{CP} = \frac{3}{2}$$
$$CP = 40\ \text{m}$$

Answer $40\ \text{m}$ [2]

[S4 BPGHS P2/2020 PRELIM Qn 6]

[Total: 10 marks]

9. Mr Tan is looking to purchase a new car to drive to and from work on weekdays and for leisure on weekends. He estimates the total distance travelled is about $\mathbf{1500\ km}$ per month. The average cost of petrol is $\$\mathbf{2.20}$ per litre. To buy a new car, he must pay a down payment of $\mathbf{30}\%$ of the selling price of the car before he can take a car loan for the remaining amount from a bank. He has to pay back the loan by monthly instalment. A $\mathbf{5}$-year car loan simple interest rate offered by most banks is $\mathbf{2.28}\%$ per annum. Mr Tan shortlisted $\mathbf{3}$ cars with all the relevant cost as shown in the table

	Car $\boldsymbol{A}$	Car $\boldsymbol{B}$	Car $\boldsymbol{C}$
Selling Price	$\$\mathbf{87\ 999}$	$\$\mathbf{108\ 999}$	$\$\mathbf{107\ 888}$
Fuel Consumption (km per litre)	$\mathbf{17.2}$	$\mathbf{14.9}$	$\mathbf{17.8}$
Car Insurance per year	$\$\mathbf{1\ 200}$	$\$\mathbf{1\ 500}$	$\$\mathbf{1\ 500}$
Engine Capacity (in cubic cm)	$\mathbf{1\ 598}$	$\mathbf{1\ 499}$	$\mathbf{1\ 197}$
Monthly Car Maintenance	$\$\mathbf{200}$	$\$\mathbf{200}$	$\$\mathbf{200}$
Monthly Car Park Charges	$\$\mathbf{150}$	$\$\mathbf{150}$	$\$\mathbf{150}$

He also finds out that the annual road tax of the car is determined by the engine capacity of the car is as follows:

$$\textbf{Annual Road Tax} = [\$\mathbf{500} + \mathbf{0.75}(\textbf{Engine Capacity} - \mathbf{1000})] \times \mathbf{0.782}$$

(a) After reviewing the information, Mr Tan decides to buy Car $\boldsymbol{A}$

(i) Calculate the down payment

Solution

$$\textbf{Car } \boldsymbol{A} \textbf{ downpayment} = \$\mathbf{87\ 999} \times \mathbf{30}\%$$
$$= \$\mathbf{26\ 399.70}$$

Answer $\$\mathbf{26,399.70}$ ..... [**1**]

(ii) Calculate the total monthly expenditure, including monthly instalment payment and all the other monthly cost

Solution

We shall first calculate the monthly car instalment

$$\textbf{Loan amount} = \$87\,999 - \$26\,399.70$$
$$= \$61\,599.30$$
$$\textbf{Loan Interest after 5 years} = (\$61\,599.30)(2.28\%)(5)$$
$$= \$7\,022.32$$
$$\textbf{Monthly car instalment} = \frac{\$61\,599.30 + \$7\,022.32}{5(12)}$$
$$= \$1\,143.69$$

Next, we shall calculate the expenditure on the car

$$\textbf{Amount of petrol needed} = \frac{1500\text{ km}}{17.2\text{ km}/l}$$
$$= 89.209\text{ litres}$$
$$\textbf{Monthly petrol consumption} = \$2.20 \times (89.209)$$
$$= \$191.86 \ldots\ldots\ldots (1)$$
$$\textbf{Monthly car insurance} = \frac{\$1200}{12}$$
$$= \$100 \ldots\ldots\ldots (2)$$
$$\textbf{Monthly road tax} = \frac{[500 + 0.75(1598 - 1000)] \times 0.782}{12}$$
$$= \$61.81 \ldots\ldots\ldots (3)$$
$$\textbf{Monthly Maintenance cost} = \$200 \ldots\ldots\ldots (4)$$
$$\textbf{Monthly Car Park charges} = \$150 \ldots\ldots\ldots (5)$$

$$\textbf{Total monthly cost} = \$1\,143.69 + \$191.86 + \$100 + \$61.81 + \$200 + \$150$$
$$= \$1\,847.36$$

Answer $\$1847.36$ [5]

(b) An alternative to buying a car is to rent an electric car that is easily accessible from his house and workplace. Subscription per month is $\$\mathbf{15}$ and it cost $\mathbf{33}$ cents per minute of use. Mr Tan estimates that the average daily travel time to and from work is about $\mathbf{1\ hour}$ and $\mathbf{20}$ minutes. On weekends, he needs to use the car for about $\mathbf{5\ hours}$ for leisure activities. Calculate his monthly expenditure to rent a car

Solution

$$\textbf{Monthly cost of renting a car}$$
$$= \$15 + (80\ \text{min})(20\ \text{working days}) \times \$0.33 + (60\ \text{min})(4\ \text{weekends}) \times \$0.33$$
$$= \$924$$

There is a variety of answers here, depending on the number of working days and weekends students use in their calculations

Answer Between $\$939$ and $\$1064.40$ [3]

(c) Do you think Mr Tan should buy or rent a car? Justify your answer

Answer Renting is better as there is no need to pay down payment and monthly instalment

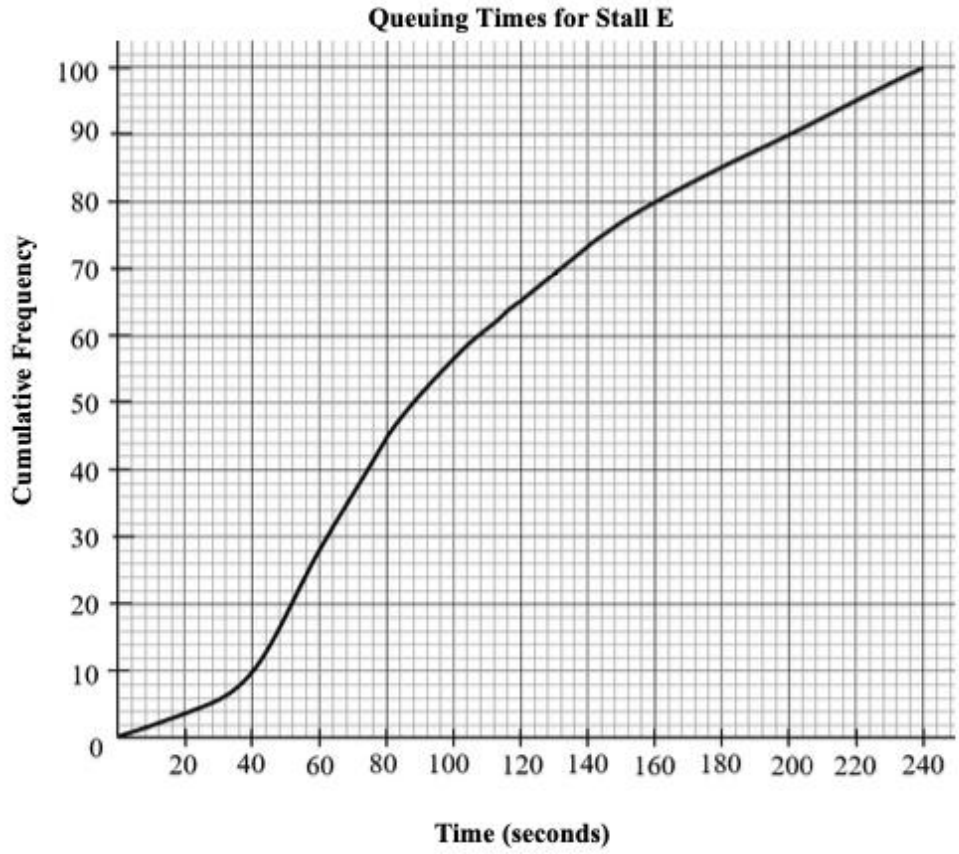
..... [1]

[S4 AHS P2/2020 PRELIM Qn 9]

[**Total: 10 marks**]

10. Nadirah observes that the queue at one of the school's canteen stall, Stall $\boldsymbol{E}$, is always long. She decides to do a project to improve the situation

She finds information about the times, in seconds, spent by $\mathbf{100}$ students in the queue for Stall $\boldsymbol{E}$. The cumulative frequency curve shows the distribution of the queuing time



(a) Complete the grouped frequency table for the queuing times for Stall $\boldsymbol{E}$

Time (sec)	$0 \le t < 40$	$40 \le t < 80$	$80 \le t < 120$	$120 \le t < 160$	$160 \le t < 200$	$200 \le t < 240$
Frequency	10	35	20	15	10	10

[1]

(b) Calculate an estimate of the mean queuing time of the **100** students

Solution

$$\bar{X} = \frac{10(20) + 35(60) + 20(100) + 15(140) + 10(180) + 220(10)}{100}$$
$$= \frac{10400}{100}$$
$$= 104 \text{ seconds}$$

Answer ..... **104 s** ..... [1]

(c) Calculate an estimate of the standard deviation

Solution

$$S.D. = \sqrt{\frac{10(20)^2 + 35(60)^2 + 20(100)^2 + 15(140)^2 + 10(180)^2 + 220(10)^2}{100} - (104)^2}$$
$$= \sqrt{\frac{1432000}{100} - 104^2}$$
$$= 59.194\ldots$$
$$= 59.2 \text{ seconds}$$

Answer ..... **59.2 s** ..... [2]

(d) A student claims that **75**% of students queuing at Stall ***E*** had to wait at least **144** seconds. Is this claim true? Explain your answer

Answer ..... **Incorrect as 25% of students had to wait at least 144 seconds** .....

.....

.....

..... [1]

(e) A few weeks later, Nadirah recorded the queuing time of another **100** students. She observes that the longest queuing time is now **200** seconds and the median queuing time is smaller. State two possible ways the cumulative frequency curve for this set of data differs from the given curve

Answer ..... **The curve is less wide and the median is shifted to the left** .....

.....

.....

..... [2]

[S4 CHIJ SNGS P2/2020 PRELIM Qn 9(a)]

[**Total**: **7 marks**]

11. The most popular ride at the Outer Space amusement park is the Ferris wheel. The diagram below shows a model of the Ferris wheel at the park

Each day, the Ferris wheel rides run from $\mathbf{10:30\ am}$ to $\mathbf{8:30\ pm}$ with an hour break for maintenance work at $\mathbf{3\ pm}$

The Ferris wheel has $\mathbf{16}$ passenger cabins. There are seats for $\mathbf{3}$ passengers in each cabin. Due to recent rules on social distancing, it is mandatory to have an empty seat in between two passengers in each cabin

Before each ride, passengers take about $\mathbf{5}$ minutes to be seated and undergo safety checks. At the end of each ride, passengers take about $\mathbf{2}$ minutes to disembark

The table below shows the ticket prices for one ride on the Ferris wheel for weekdays and weekends. Tickets must be used on the same day of purchase

Weekday Ticket	$\mathbf{\$18}$
Weekend Ticket	$\mathbf{\$23}$

(a) Given that the Ferris wheel makes 1 revolution every 2 minutes and makes 4 revolutions per ride, calculate the total number of rides each cabin on the Ferris wheel makes in one day

Solution

$$\textbf{Time taken for 1 ride} = \textbf{4 revolutions} \times \textbf{2 minutes}$$
$$= \textbf{8 minutes}$$

$$\textbf{Number of rides per day} = \frac{\mathbf{60}}{\mathbf{8+2+5}} \times \mathbf{9}$$
$$= \mathbf{36\ rides}$$

Answer ..... $\mathbf{36}$ ..... **[2]**

(b) Calculate the maximum number of passengers that the Ferris wheel can take in one day

Solution

$$\text{Maximum number of passengers} = 2 \text{ passengers} \times 16 \text{ cabins} \times 36 \text{ rides}$$
$$= 1152 \text{ passengers}$$

Answer 1152 ..... [1]

(c) If all the seats are taken up for every ride, calculate the total amount of money that the park can collect from the sales of tickets for Ferris wheel rides on a weekday

Solution

$$\text{Total collected} = 1152 \times \$18$$
$$= \$20\,736$$

Answer $20 736 ..... [1]

(d) To commemorate its tenth birthday, the amusement park is planning to give out free tickets for its Ferris wheel rides.

Propose a sensible number of tickets to be given free such that the amount collected from the sales of tickets sales for the day covers its operating cost of $\$\mathbf{10\,000}$. It is also estimated that $\mathbf{40}\%$ to $\mathbf{60}\%$ of the total possible seats will be taken for each ride

Explain your proposal clearly and state any assumptions made

[**6**]

Solution

We shall split this solution into 2 sections: Weekday tickets and Weekend tickets

Weekday tickets

$$\text{Max number of tickets to cover operating cost} = \frac{\$10\,000}{18}$$
$$= 555.6$$
$$\approx 556 \text{ (nearest whole)}$$

Assuming 40% occupancy,

$$\text{Number of free tickets} = \left(\frac{40}{100} \times 1152\right) - 556$$
$$= -96 \text{ (not possible)}$$

Assuming 50% occupancy,

$$\text{Number of free tickets} = \left(\frac{50}{100} \times 1152\right) - 556$$
$$= 20$$

Assuming 60% occupancy,

$$\text{Number of free tickets} = \left(\frac{60}{100} \times 1152\right) - 556$$
$$= 135$$

Weekday tickets

$$\text{Max number of tickets to cover operating cost} = \frac{\$10\,000}{23}$$
$$= 434.8$$
$$\approx 435 \text{ (nearest whole)}$$

Assuming 40% occupancy,

$$\text{Number of free tickets} = \left(\frac{40}{100} \times 1152\right) - 435$$
$$= 26$$

Assuming 50% occupancy,

$$\text{Number of free tickets} = \left(\frac{50}{100} \times 1152\right) - 435$$
$$= 141$$

Assuming 60% occupancy,

$$\text{Number of free tickets} = \left(\frac{60}{100} \times 1152\right) - 435$$
$$= 256$$

Each possible scenario has been calculated above as shown

[S4 SST P2/2020 PRELIM Qn 12]

[Total: 10 marks]

End of Paper ☺