Name: _____ Material: E-Math Mock Paper 2

OVERMUGGED MOCK PAPER 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ELEMENTARY MATHEMATICS **4048/02**

Specimen Paper

Date: **1 September 2021** **Duration: 2 hours 30 minutes**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Write your name on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value of $\mathbf{3.142}$, unless the question requires the answer in terms of π.

At the end of the exam, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question
The total number of marks for this paper is $\mathbf{100}$.

Setter: Ong Kai Wen

This question paper consists of 22 printed pages including the cover page

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone } = \pi rl$$

$$\text{Surface area of a sphere } = 4\pi r^2$$

$$\text{Volume of a cone } = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere } = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1. (a) Given that

$$\boldsymbol{p = 4q(r - 2)}$$

(i) Evaluate $\boldsymbol{p}$ when $\boldsymbol{q = 2}$ and $\boldsymbol{r = -3}$

Answer ..... [**1**]

(ii) Express $\boldsymbol{r}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$

Answer ..... [**1**]

(b) Simplify

$$\frac{\boldsymbol{9 - 6x}}{\boldsymbol{3 - 2x + 3y - 2xy}}$$

Answer ..... [**3**]

(c) (i) Express the following in the form $(x-h)^2+q$

$$7-4x+x^2$$

Answer ..... [2]

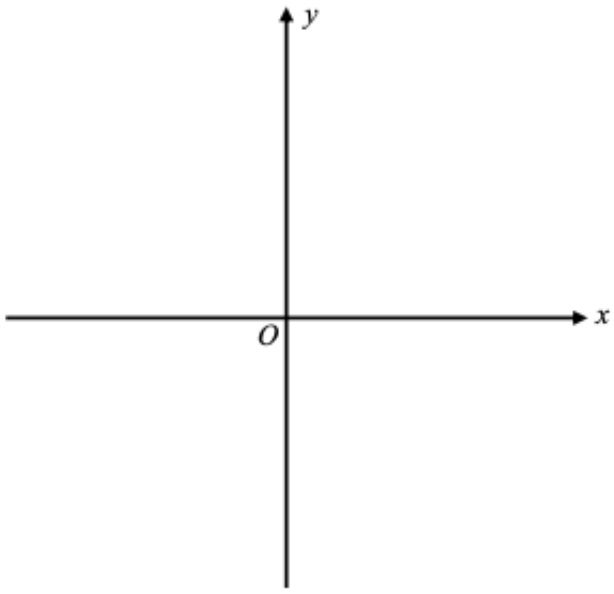
(ii) Sketch the graph of

$$y=7-4x+x^2$$

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point on the graph

[2]

Answer



(iii) Write the equation of the line of symmetry of the graph of

$$y=7-4x+x^2$$

Answer ..... [1]

(d) Solve the equation

$$\frac{x}{2-x}+\frac{5}{3x-1}=\frac{5+10x}{(3x-1)(2-x)}$$

Answer **.....** [**3**]

[S4 ZHSS P2/2020 PRELIM Qn 1]

[**Total**: **13 marks**]

2. The variables $\boldsymbol{x}$ and $\boldsymbol{y}$ are connected by the equation

$$\boldsymbol{y = 2x + \frac{30}{x} - 16}$$

Some corresponding values of $\boldsymbol{x}$ and $\boldsymbol{y}$ are given in the table below

$\boldsymbol{x}$	$\mathbf{1.75}$	$\mathbf{2}$	$\mathbf{2.5}$	$\mathbf{3}$	$\mathbf{4}$	$\mathbf{5}$	$\mathbf{6}$	$\mathbf{7}$	$\mathbf{8}$
$\boldsymbol{y}$	$\mathbf{4.64}$	$\boldsymbol{p}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{-0.5}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{2.29}$	$\mathbf{3.75}$

(a) Calculate the value of $\boldsymbol{p}$

Answer ..... [**1**]

(b) In the grid given on the next page, draw the graph, for $\mathbf{1.75 \leq x \leq 8}$, of

$$\boldsymbol{y = 2x + \frac{30}{x} - 16}$$

Use a scale of $\mathbf{2\ cm}$ to represent $\mathbf{1\ unit}$, draw a horizontal $\boldsymbol{x}$-axis for $\mathbf{0 \leq x \leq 8}$
Use a scale of $\mathbf{2\ cm}$ to represent $\mathbf{0.5\ unit}$, draw a vertical $\boldsymbol{y}$-axis for $\mathbf{-0.5 \leq y \leq 5}$
On your axes, plot the points given in the table and join them with a smooth curve

[**3**]

(c) By drawing a tangent, find the gradient of the curve at $(\mathbf{6}, \mathbf{1})$

Answer ..... [**2**]

(d) Use your graph to find

(i) the range of values of x for which $y < 2.5$

Answer ..... [1]

(ii) the solutions to the equation

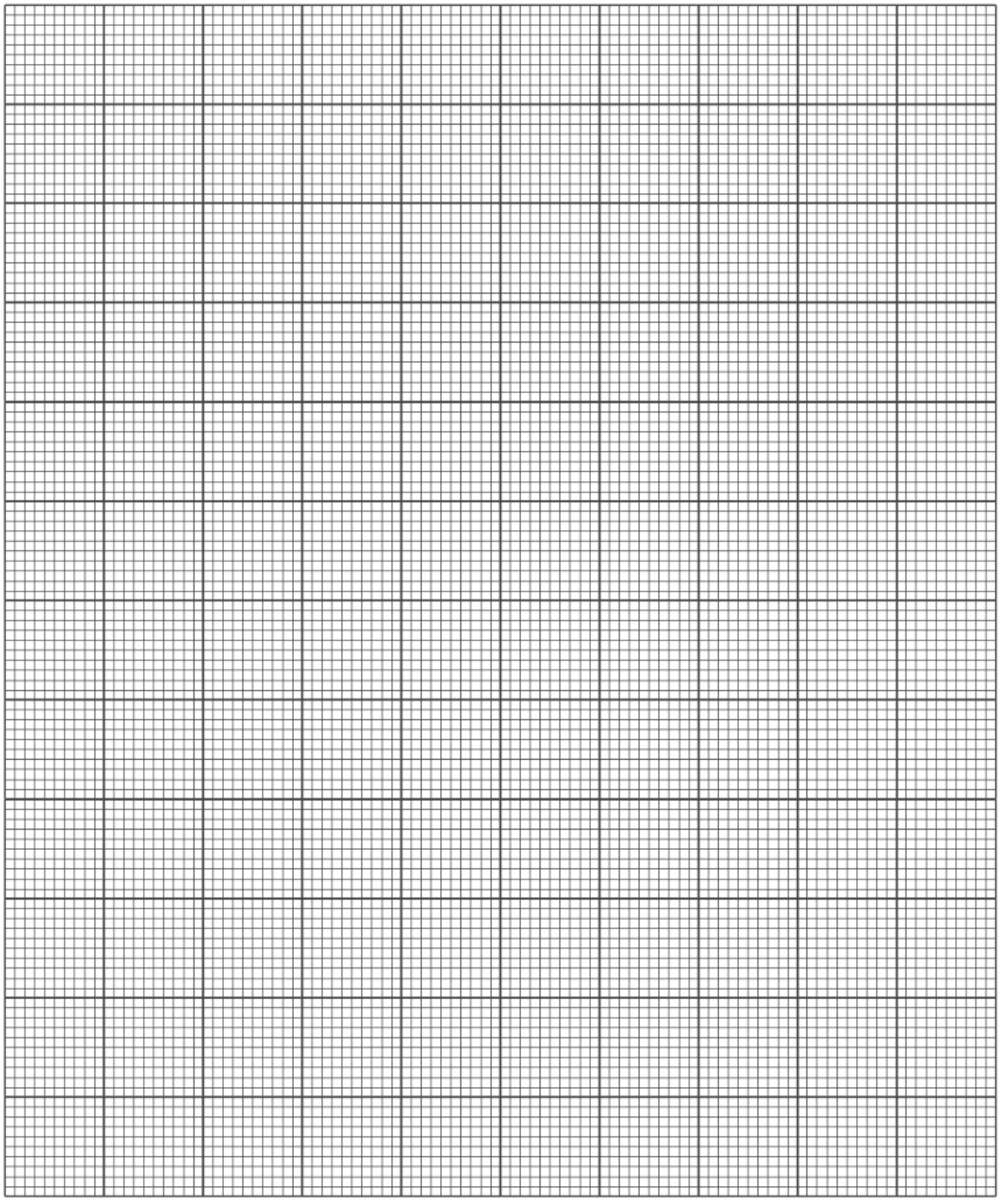
$$2x + \frac{30}{x} = 18$$

Answer ..... [2]

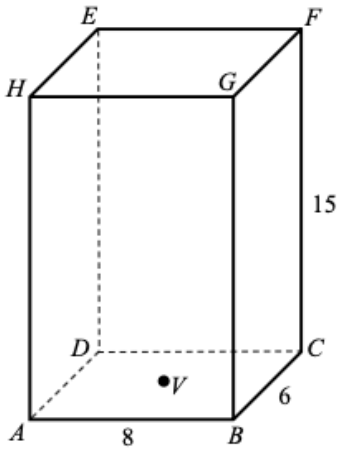
[S4 HIHS P2/2020 PRELIM Qn 10] (9)

[**Total**: **9 marks**]

Graph for Question 2



3.



In the diagram, $\boldsymbol{ABCDEFGH}$ is a cuboid with dimensions $\mathbf{8\ cm}$ by $\mathbf{6\ cm}$ by $\mathbf{15\ cm}$. $\boldsymbol{V}$ is the centre of the rectangular base

(a) Show that $\boldsymbol{EV = 15.8\ cm}$, correct to **3** significant figures

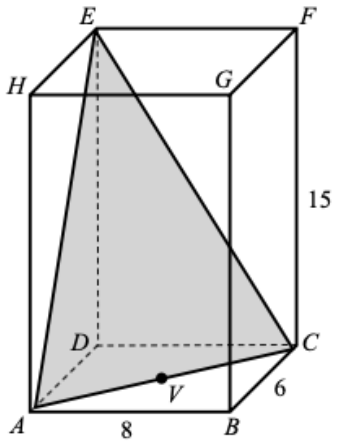
[**2**]

Answer

(b) Calculate $\angle \boldsymbol{ACE}$

Answer ..... [**2**]

(c) A pyramid $EDAC$ is cut out from the cuboid



(i) Find the total surface area of the pyramid

Answer ..... [**2**]

(ii) Another pyramid is to be made with volume half of pyramid $\boldsymbol{EDAC}$. Given that the two pyramids are geometrically similar, find the vertical height of the smaller pyramid

Answer ..... [**2**]

[S4 YISS P2/2020 PRELIM Qn 8]

[**Total**: **8 marks**]

4. A bird flew $\mathbf{36\ km}$ at a constant speed of $\boldsymbol{x}\ \mathbf{km/h}$ from $\boldsymbol{A}$ to $\boldsymbol{B}$ (outward journey) when there was no wind. When the bird flew back from $\boldsymbol{B}$ to $\boldsymbol{A}$ (return journey) against the wind direction, its speed was decreased by $\mathbf{2\ km/h}$

(a) Write down, in terms of $\boldsymbol{x}$, the time taken for the bird to fly from $\boldsymbol{A}$ to $\boldsymbol{B}$

Answer ..... [**1**]

(b) Write down, in terms of $\boldsymbol{x}$, the time taken for the bird to fly back from $\boldsymbol{B}$ to $\boldsymbol{A}$

Answer ..... [**1**]

(c) Given that the return journey took the bird $\mathbf{75\ minutes}$ longer than the outward journey, form an equation in $\boldsymbol{x}$ and show that it simplifies to

$$\mathbf{5}\boldsymbol{x}^{\mathbf{2}} - \mathbf{10}\boldsymbol{x} - \mathbf{288} = \mathbf{0}$$

[**2**]

Answer

(d) Solve the equation $\mathbf{5}\boldsymbol{x}^{\mathbf{2}} - \mathbf{10}\boldsymbol{x} - \mathbf{288} = \mathbf{0}$, giving your answers to $\mathbf{2}$ decimal places

Answer ..... [**3**]

(e) Find the time taken by the bird on its return journey, giving your answer to the nearest minute

Answer ..... [**2**]

[S4 NHHS P2/2020 PRELIM Qn 9]

[**Total**: **9 marks**]

5. The first three terms in a sequence of numbers, $\boldsymbol{T_1}$, $\boldsymbol{T_2}$, $\boldsymbol{T_3}$,… are given below

$$\boldsymbol{T_1 = 3 + 9 = 12 = 3 \times 2^2}$$
$$\boldsymbol{T_2 = 3 + 9 + 15 = 27 = 3 \times 3^2}$$
$$\boldsymbol{T_3 = 3 + 9 + 15 + 21 = 48 = 3 \times 4^2}$$

(a) (i) Find $\boldsymbol{T_4}$

Answer ..... [**1**]

(ii) Find an expression, in terms of $\boldsymbol{n}$ for $\boldsymbol{T_n}$

Answer ..... [**1**]

(iii) Find the value of $\boldsymbol{p}$ and the value of $\boldsymbol{q}$ given that

$$\boldsymbol{T_p = 3 + 9 + 15 + \cdots + q = 363}$$

Answer ..... [**2**]

(iv) Explain why $\boldsymbol{T_n}$ is an odd number when $\boldsymbol{n}$ is even

Answer ..... [**1**]

(b) The $\boldsymbol{n}$**th** term of a different sequence is given by

$$\boldsymbol{P_n = 3n - 6}$$

Siti is asked to find the difference between $\boldsymbol{T_{50}}$ and $\boldsymbol{P_{50}}$ and her answer is $\boldsymbol{7660}$. Without any calculation of values, is Siti correct? Justify your answer

Answer .....

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.....

..... [**2**]

[S4 NGHS P2/2020 PRELIM Qn 6]

[**Total**: **7 marks**]

6. **(a)** Find the matrix $\boldsymbol{A}$ such that

$$4\boldsymbol{A} + \begin{pmatrix} 2 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 6 & 8 \end{pmatrix}$$

Answer ..... [2]

(b) Given that $\boldsymbol{D} = \begin{pmatrix} 3 & -4 \\ 0 & 2 \end{pmatrix}$, $\boldsymbol{E} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\boldsymbol{F} = \begin{pmatrix} -2 & -1 \\ 1 & 4 \end{pmatrix}$, evaluate the following

$$2\boldsymbol{D} + \boldsymbol{E} - 2\boldsymbol{F}$$

Answer ..... [**2**]

(c) Two cafés sell coffee from different countries. The sale and price of the coffee at the two cafés are shown in the table below

	Ethiopia ($8)	Myanmar ($6)	Colombia ($7)
Café A	**18**	**9**	**10**
Café B	**15**	**7**	**24**

(i) Represent the information above by a $\mathbf{2} \times \mathbf{3}$ matrix $\boldsymbol{C}$

Answer ..... [**1**]

(ii) Evaluate the matrix $\boldsymbol{S}$ where

$$\boldsymbol{S} = \boldsymbol{C}\begin{pmatrix}\mathbf{1}\\\mathbf{1}\\\mathbf{1}\end{pmatrix}$$

Answer ..... [**1**]

(iii) Explain what the elements of $\boldsymbol{S}$ represent

Answer .....

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..... [**1**]

(iv) Represent the price of the coffee using matrix $\boldsymbol{P}$

Answer ..... [**1**]

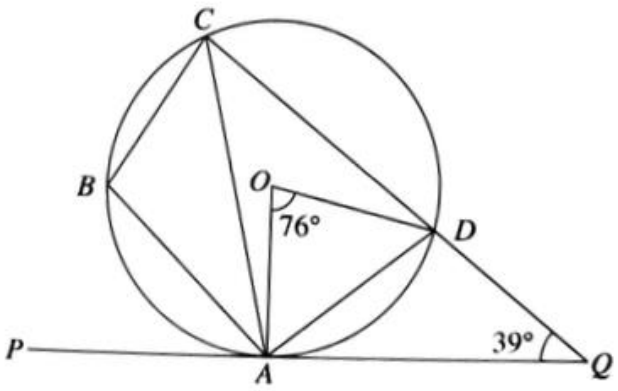
(v) Using the matrix $\boldsymbol{C}$ and $\boldsymbol{P}$, find the total sales $\boldsymbol{T}$ by each café

Answer ..... [**2**]

[S4 TSS P2/2020 PRELIM Qn 10]

[**Total**: **10 marks**]

7.



$\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are points on the circle with centre $\boldsymbol{O}$. $\boldsymbol{PAQ}$ is a tangent to the circle at $\boldsymbol{A}$ and meets the line $\boldsymbol{CD}$ extended at point $\boldsymbol{Q}$. Given that $\angle \boldsymbol{AOD} = \mathbf{76}°$ and $\angle \boldsymbol{DQA} = \mathbf{39}°$, find, giving your reasons for each answer

(a) $\angle \boldsymbol{DAQ}$

Answer ..... [**2**]

(b) $\angle \boldsymbol{ABC}$

Answer ..... [**2**]

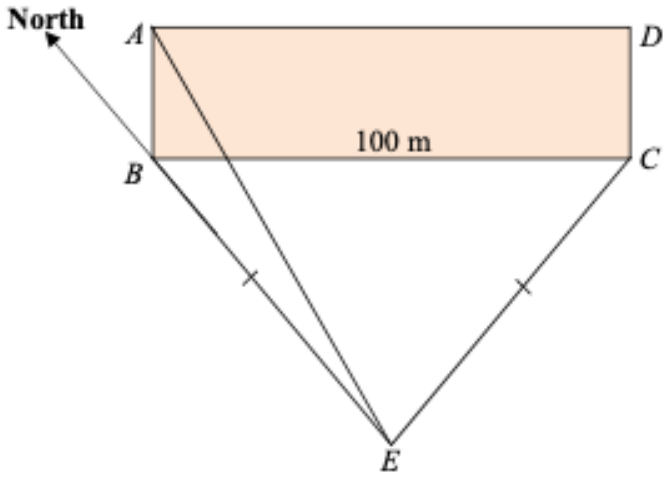
(c) $\angle \boldsymbol{OAC}$

Answer ..... [**3**]

[S4 HSS P2/2020 PRELIM Qn 2]

[**Total**: **7 marks**]

8.



In the diagram, $\boldsymbol{ABCD}$ represents a vertical cliff face. The bottom of the cliff, $\boldsymbol{BC}$ is $\mathbf{100\ m}$ and is at sea level. A boat is in the sea at $\boldsymbol{E}$. $\boldsymbol{B}$ is due North of $\boldsymbol{E}$ and the bearing of $\boldsymbol{C}$ from $\boldsymbol{E}$ is $\mathbf{070°}$. $\boldsymbol{E}$ is known to be equidistant from $\boldsymbol{B}$ and $\boldsymbol{C}$

(a) Find the bearing of $\boldsymbol{C}$ from $\boldsymbol{B}$

Answer ..... [**2**]

(b) Show that $\boldsymbol{BE = 87.17\ m}$

[**2**]

Answer

(c) Find the area of $\Delta \boldsymbol{BCE}$, leaving your answer to the nearest square metre

Answer ..... [**2**]

(d) It is given that the angle of depression of $\boldsymbol{E}$ from $\boldsymbol{A}$ is $\mathbf{20.2°}$. Find the height of the cliff $\boldsymbol{BA}$, leaving your answers in **3** significant figures

Answer ..... [**2**]

(e) Find the length of $\boldsymbol{CP}$ given that $\boldsymbol{P}$ is a point on $\boldsymbol{BC}$ such that

$$\frac{\textbf{Area of } \Delta \boldsymbol{BEP}}{\textbf{Area of } \Delta \boldsymbol{CEP}} = \frac{\mathbf{3}}{\mathbf{2}}$$

Answer ..... [**2**]

[S4 BPGHS P2/2020 PRELIM Qn 6]

[**Total**: **10 marks**]

9. Mr Tan is looking to purchase a new car to drive to and from work on weekdays and for leisure on weekends. He estimates the total distance travelled is about $\mathbf{1500\ km}$ per month. The average cost of petrol is $\$\mathbf{2.20}$ per litre. To buy a new car, he must pay a down payment of $\mathbf{30}\%$ of the selling price of the car before he can take a car loan for the remaining amount from a bank. He has to pay back the loan by monthly instalment. A $\mathbf{5}$-year car loan simple interest rate offered by most banks is $\mathbf{2.28}\%$ per annum. Mr Tan shortlisted $\mathbf{3}$ cars with all the relevant cost as shown in the table

	Car $\boldsymbol{A}$	Car $\boldsymbol{B}$	Car $\boldsymbol{C}$
Selling Price	$\mathbf{\$87\ 999}$	$\mathbf{\$108\ 999}$	$\mathbf{\$107\ 888}$
Fuel Consumption (km per litre)	$\mathbf{17.2}$	$\mathbf{14.9}$	$\mathbf{17.8}$
Car Insurance per year	$\mathbf{\$1\ 200}$	$\mathbf{\$1\ 500}$	$\mathbf{\$1\ 500}$
Engine Capacity (in cubic cm)	$\mathbf{1\ 598}$	$\mathbf{1\ 499}$	$\mathbf{1\ 197}$
Monthly Car Maintenance	$\mathbf{\$200}$	$\mathbf{\$200}$	$\mathbf{\$200}$
Monthly Car Park Charges	$\mathbf{\$150}$	$\mathbf{\$150}$	$\mathbf{\$150}$

He also finds out that the annual road tax of the car is determined by the engine capacity of the car is as follows:

$$\textbf{Annual Road Tax} = [\$\mathbf{500} + \mathbf{0.75}(\textbf{Engine Capacity} - \mathbf{1000})] \times \mathbf{0.782}$$

(a) After reviewing the information, Mr Tan decides to buy Car $\boldsymbol{A}$

(i) Calculate the down payment

Answer ..... [$\mathbf{1}$]

(ii) Calculate the total monthly expenditure, including monthly instalment payment and all the other monthly cost

Answer ..... [$\mathbf{5}$]

(b) An alternative to buying a car is to rent an electric car that is easily accessible from his house and workplace. Subscription per month is $\$\mathbf{15}$ and it cost $\mathbf{33}$ cents per minute of use. Mr Tan estimates that the average daily travel time to and from work is about $\mathbf{1\ hour}$ and $\mathbf{20}$ minutes. On weekends, he needs to use the car for about $\mathbf{5\ hours}$ for leisure activities. Calculate his monthly expenditure to rent a car

Answer ..... [**3**]

(c) Do you think Mr Tan should buy or rent a car? Justify your answer

Answer .....

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.....

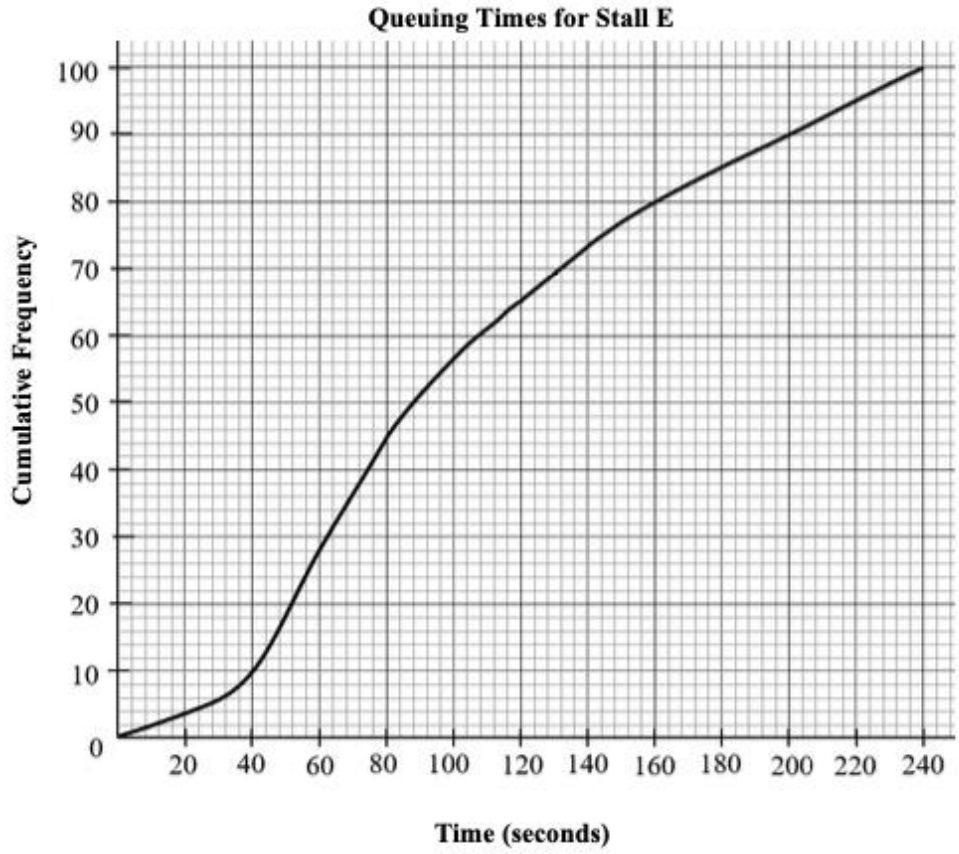
..... [**1**]

[S4 AHS P2/2020 PRELIM Qn 9]

[**Total**: **10 marks**]

10. Nadirah observes that the queue at one of the school's canteen stall, Stall $\boldsymbol{E}$, is always long. She decides to do a project to improve the situation

She finds information about the times, in seconds, spent by $\mathbf{100}$ students in the queue for Stall $\boldsymbol{E}$. The cumulative frequency curve shows the distribution of the queuing time



(a) Complete the grouped frequency table for the queuing times for Stall $\boldsymbol{E}$

Time (sec)	$0 \leq t < 40$	$40 \leq t < 80$	$80 \leq t < 120$	$120 \leq t < 160$	$160 \leq t < 200$	$200 \leq t < 240$
Frequency	10	35	20			10

[**1**]

(b) Calculate an estimate of the mean queuing time of the $\mathbf{100}$ students

Answer ..... [$\mathbf{1}$]

(c) Calculate an estimate of the standard deviation

Answer ..... [$\mathbf{2}$]

(d) A student claims that $\mathbf{75}\%$ of students queuing at Stall $\boldsymbol{E}$ had to wait at least $\mathbf{144}$ seconds. Is this claim true? Explain your answer

Answer .....
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.....
..... [$\mathbf{1}$]

(e) A few weeks later, Nadirah recorded the queuing time of another $\mathbf{100}$ students. She observes that the longest queuing time is now $\mathbf{200}$ seconds and the median queuing time is smaller. State two possible ways the cumulative frequency curve for this set of data differs from the given curve

Answer .....
.....
.....
..... [$\mathbf{2}$]

[S4 CHIJ SNGS P2/2020 PRELIM Qn 9(a)]

[**Total**: **7 marks**]

11. The most popular ride at the Outer Space amusement park is the Ferris wheel. The diagram below shows a model of the Ferris wheel at the park

Each day, the Ferris wheel rides run from $\mathbf{10:30\ am}$ to $\mathbf{8:30\ pm}$ with an hour break for maintenance work at $\mathbf{3\ pm}$

The Ferris wheel has $\mathbf{16}$ passenger cabins. There are seats for $\mathbf{3}$ passengers in each cabin. Due to recent rules on social distancing, it is mandatory to have an empty seat in between two passengers in each cabin

Before each ride, passengers take about $\mathbf{5}$ minutes to be seated and undergo safety checks. At the end of each ride, passengers take about $\mathbf{2}$ minutes to disembark

The table below shows the ticket prices for one ride on the Ferris wheel for weekdays and weekends. Tickets must be used on the same day of purchase

Weekday Ticket	$\mathbf{\$18}$
Weekend Ticket	$\mathbf{\$23}$

(a) Given that the Ferris wheel makes 1 revolution every 2 minutes and makes 4 revolutions per ride, calculate the total number of rides each cabin on the Ferris wheel makes in one day

Answer ..... [**2**]

(b) Calculate the maximum number of passengers that the Ferris wheel can take in one day

Answer ..... [**1**]

(c) If all the seats are taken up for every ride, calculate the total amount of money that the park can collect from the sales of tickets for Ferris wheel rides on a weekday

Answer ..... [**1**]

(d) To commemorate its tenth birthday, the amusement park is planning to give out free tickets for its Ferris wheel rides.

Propose a sensible number of tickets to be given free such that the amount collected from the sales of tickets sales for the day covers its operating cost of $\$\mathbf{10\,000}$. It is also estimated that $\mathbf{40}\%$ to $\mathbf{60}\%$ of the total possible seats will be taken for each ride

Explain your proposal clearly and state any assumptions made

Answer .....

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..... [**6**]

[S4 SST P2/2020 PRELIM Qn 12]

[**Total**: **10 marks**]

End of Paper ☺