SECONDARY 2 MATHEMATICS
Quadratic Equations, Quadratic Identities, Direct & Inverse Proportion

Name: ____________________________ Date: _________________________

Solving quadratic equations

Quadratic equations, expressed in the form $ax^2 + bx + c = 0$, can be solved using factorisation. Not all quadratic equations can be solved this way though, we will learn other methods in Secondary 3 ☺

- RHS of the equation should always be **zero**
- After factorisation, equate each bracket to zero
    - Anything applied by zero will always be zero. Hence, either of the brackets will be zero
    - Ie: $a \times b = 0$ implies $a = 0$ or $b = 0$

Worked Example 1

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$$x + 5 = 0 \text{ or } x - 2 = 0$$

$$x = -5 \text{ or } x = 2$$

x	5	$5x$
x	-2	$-2x$
x^2	3	$3x$

Reminders:

- Remember to include ALL workings, including the table for the “box method” or “cross method”. You will not be given full marks without this working step.
- You can make use of the calculator to help you factorise / check for the answers, but ensure that all workings are written down.
- DO NOT rely on a calculator when practicing and you will need to get used to presenting all workings. It will not come to you magically during exams.

Worked Example 2

(a) Factorise the following completely.

$$x^2 - 25x + 144$$

(b) Hence, solve the following equation.

$$y^4 - 25y^2 + 144 = 0$$

Worked Example 3

Solve the following equation

$$\frac{2x}{2x-3} + 1 = \frac{1}{2-3x}$$

Tips:

- Combine the fraction on the LHS before simplifying
- Always look out for **lowest common denominator**, don't be too quick to cross multiply, sometimes you'll be at a dead end
- Simplify as you go along, it will make the question more manageable

Practice on your own ☺

Solve the following equations.

a) $(p-2)(p+5)=7p-10$ $\quad p=0,4$

b) $4(1+k)(1-k)=6k+(2k-1)(k+1)$ $\quad k=\frac{1}{2},-\frac{5}{3}$

c) $9(x-4)^2=(2x+1)^2$ $\quad x=\frac{11}{5},13$

d) $3(2-y)(2y-3)=(y-2)^2$ $\quad y=\frac{11}{7},2$

e) $\frac{x-7}{11}=\frac{11}{x-7}$ $\quad x=-4,18$

<u>Worked Example 4</u>

Given that $9x^2 - 6xy + y^2 = 0$, find the value of $\frac{x}{y}$.

Tip:

- Look out for identities if possible. Here, we have $a^2 - 2ab + b^2 = (a - b)^2$
- You don't need to solve for x and y individually to get $\frac{x}{y}$

<u>Worked Example 5</u>

Given that $16x^2 - 40xy = -25y^2$, find the value of $\frac{2x}{5y}$.

Worked Example 6

One of the solutions of the equation $2x^2 + 5x = -p$ is $x = \frac{1}{2}$.

(i) Find the value of p.

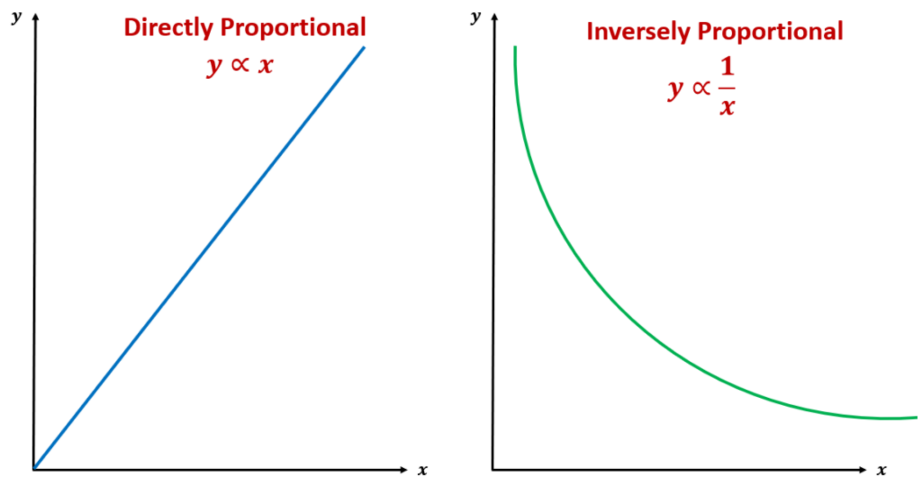
(ii) Hence, find the other solution of the equation.

Direct Proportion

- relationship between two quantities, when one quantity is multiplied by a constant to get the other
- also known as a linear relationship
- formula: $y = kx$
- when x increases, y increases at a rate directly proportional to x.
- Similarly, when x decreases, y also decreases proportionally

Inverse Proportion

- relationship between two quantities – when one quantity increases, the other decreases such that their product remains constant.
- formula: $yx = k$ or $y = \frac{k}{x}$
- when x increases, y increases at a rate inversely proportional to x, such that the product stays the same.
- similarly, when x decreases, y increases such that the product stays the same



Steps to solving proportion questions:

1. Write the appropriate equation, relating the variables given.
   - Eg: x directly proportional to $y^3 \Rightarrow$ we get $y^3 = kx$
   - Eg: T inversely proportional to square root of $F \Rightarrow$ we get $T = \frac{k}{\sqrt{F}}$
2. Substitute the values of the variables given in the question, to solve for k.
3. Using the value of k from step 2, **restate your equation.**
4. Using a new value given in the question, find the value of the other variable.

Worked Example 1

The marks of a student's exam is directly proportional to the square of number of hours spent studying. Given that Tommy spend a total of 12 hours studying, and obtained a score of 85 marks, find the number of hours spent by Jonathan studying, if he obtained a score of 60 marks.

[Solution]

Step 1: Let m and h represent the number of marks a student gets and the number of hours they spent studying.

$$m = kh^2$$

Step 2: Given that $m = 85$ when $h = 12$

$$85 = k(12)^2$$

$$k = \frac{85}{144}$$

Step 3:

$$m = \frac{85}{144}h^2$$

Step 4: Hence, when $m = 60$

$$60 = \frac{85}{144}h^2$$

$$h^2 = 60 \div \frac{85}{144} = \frac{1728}{17}$$

$$h = \pm\sqrt{\frac{1728}{17}} = 10.1 \ (\because h > 0)$$

Hence, Jonathan spends 10 hours studying.

Worked Example 2

It is given that $y + 3$ is directly proportional to x. The sum of values of y when $x = 4$ and $x = 7$ is 49. Find the value of y when $x = 9$.

$y = 42$

[2023/RVHS/EOY]

Worked Example 3

In a factory, 15 machines produce 200 bottles in 8 hours. Find the amount of time needed for 9 machines to produce 600 bottles.

40 hours

[2023/RVHS/EOY]

Worked Example 4

R is directly proportional to L and inversely proportional to the square of d.

When $L = 16$ and $d = 0.4, R = 600$.

(i) Express R in terms of L and d.

(ii) Calculate the value of R when $L = 50$ and $d = 2$.

(ii) $R = 75$

[2023/MGS/EOY]

Worked Example 5

It is given that the price P of a luxury item is inversely proportional to the square root of the demanded quantity, q. When demanded quantity was 12, the price of a luxury bag was found to be $3000.

(a) Calculate the price of the luxury bag when the demanded quantity is 20, giving your answer to the nearest dollar.

A luxury watch experienced a 300% increase in demanded quantity in 2024.

(b) (i) Find an expression for the new price in terms of P.

(ii) Calculate the percentage decrease in the price.

[2023/MGS/EOY]