

Chapter 2: Equations and Inequalities

CHAPTER ANALYSIS

1) Discriminant $\left(b^{2}-4 a c\right)$

- $b^{2}-4 a c>0: 2$ real and distinct roots
- $b^{2}-4 a c<0$ : No real roots
- $b^{2}-4 a c=0: 2$ real and equal roots (1 root)

2) Making use of discriminant and real roots to identify the relationship between a line and curve

- Line is a tangent to the curve
- Line does not intersect the curve
- Line intersects the curve at 2 points

3) Solving quadratic inequalities

## QUADRATICEQUATIONS ANDITS ROOTS

A quadratic equation is usually expressed in the form of $a x^{2}+b x+c=0$.

Methods to solve quadratic equations and obtain its roots (solutions):

1) Factorisation


Solving the equation gives $\boldsymbol{x}=\boldsymbol{\alpha}$ or $\boldsymbol{x}=\boldsymbol{\beta}$
2) Quadratic formula

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \text { where, } a \neq 0
$$

(This is the reason why the discriminant matters!
Can you guess why?)

## IMPORTANT

Students are not allowed to assume the discriminant signature as this is a show question. Students are supposed to reach the discriminant signature on their own, hence, proving the claim of the question

## Example

Show that the following has real and distinct roots for all real values of $x$

$$
(p+1) x^{2}+(4 p+3) x+2 p-1=0
$$

[S4 CWSS P1/2009 PRELIM Qn 9(b)]

## Solution:

To show that the function has real and distinct roots

$$
\begin{aligned}
& \text { WTS: } b^{2}-4 a c>0 \\
& D=(4 p+3)^{2}-4(p+1)(2 p-1) \\
&=16 p^{2}+24 p+9-4\left(2 p^{2}+p-1\right) \\
&=16 p^{2}+24 p+9-8 p^{2}-4 p+4 \\
&=8 p^{2}+20 p+13 \\
&=8\left(p^{2}+\frac{5}{2} p+\frac{13}{8}\right) \\
&=8\left[\left(p+\frac{5}{4}\right)^{2}-\left(\frac{5}{4}\right)^{2}+\frac{13}{8}\right] \\
&=8\left(p+\frac{5}{4}\right)^{2}+\frac{1}{2}
\end{aligned}
$$

$$
\text { Since }\left(p+\frac{5}{4}\right)^{2}>0
$$

$$
\begin{gathered}
8\left(p+\frac{5}{4}\right)^{2}>0 \\
8\left(p+\frac{5}{4}\right)^{2}+\frac{1}{2}>0
\end{gathered}
$$

[^0]
## Discriminant of a quadratic equation

 and its roots$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad \text { Discriminant }=b^{2}-4 a c
$$

| Nature of Roots | Discriminant | Graphical Representation |
| :---: | :---: | :---: |
| 2 real and distinct roots | $b^{2}-4 a c>0$ |  |
| 2 equal roots | $b^{2}-4 a c=0$ |  |
| No real roots | $b^{2}-4 a c<0$ |  |

Sub the values of discriminant (positive, negative or zero) into the quadratic formula above and find out how many solutions are there (equivalent to number of roots). Note: You cannot square root a negative number, hence there is no solution.

## IMPORTANT

Students are not allowed to assume the discriminant signature as this is a show question. Students are supposed to reach the discriminant signature on their own, hence, proving the claim of the question

## Example

Show that the line meets the curve at 2 distinct points for all real values of $k$

$$
\begin{gathered}
y=5-k \\
y=x^{2}-k x
\end{gathered}
$$

[S3 SQSS P1/2011 MYE Qn 10(b) (MODIFIED)]
Solution:

$$
\begin{gathered}
y=5-k \ldots \ldots(1) \\
y=x^{2}-k x \ldots \ldots .(2)
\end{gathered}
$$

Take Equation (1) = Equation (2)

$$
\begin{aligned}
5-k & =x^{2}-k x \\
x^{2}-k x+(k-5) & =0
\end{aligned}
$$

To show that the line meets the curve

$$
\begin{aligned}
& \text { WTS: } b^{2}-4 a c>0 \\
& \begin{aligned}
D & =(-k)^{2}-4(1)(k-5) \\
& =k^{2}-4 k+20 \\
& =(k-2)^{2}-(2)^{2}+20 \\
& =(k-2)^{2}+16
\end{aligned}
\end{aligned}
$$

Since $k$ can take any real value,

$$
(k-2)^{2}>0
$$

$$
(k-2)^{2}+16>0
$$

Hence, the line will meet the curve at 2 distinct points for all real values of $k$ (shown)

Relationship between line and curve
Step 1: Solve the equations simultaneously by substituting the equation of oVERPGGED the line into the equation of the curve to eliminate one of the variables (make $x$ the only unknown)
Step 2: Make one side equals to 0 and find the discriminant


## SOLVING QUADRATIC INEQUALITIES

Step 1: Expand the equation and make one side equals to zero.

Step 2: Factorise the quadratic equation into this form:

$$
a(x-\alpha)(x-\beta)=0
$$

Step 3: Sketch the quadratic graph
(*Make use of the coefficient of $x^{2}$ to identify the shape of graph*)


## SOLVING QUADRATIC INEQUALITIES

Step 4: Label the $\boldsymbol{x}$-intercepts of the graph

Step 5: Shade the region between the curve and the $\boldsymbol{x}$ axis and you will obtain your answer.

If you need a question example for better understanding, check out this video on Overmugged's TikTok channel: https://vt.tiktok.com/ZSFj43eEF/


## CONDITIONS FOR QUADRATIC EQUATION TO BE ALWAYS POSITIVE OR ALWAYS NEGATIVE

- ALWAYS POSITIVE

1) Coefficient of $x^{2}$, also known as $a>0$
2) Discriminant $b^{2}-4 a c<0$

- ALWAYS NEGATIVE

1) Coefficient of $x^{2}$, also known as $a<0$
2) Discriminant $b^{2}-4 a c<0$

Discriminant must always be negative as there should not be any intersections between the curve and the $x$-axis for it to always be above or below the $x$-axis.

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07

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$$
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$$



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[^0]:    Hence, the function has real and distinct roots (shown)

