Chapter 1: Quadratic Equations

Chapter Analysis

1. **Solving Quadratic Equations using:**

   - Completing the Square

   - Quadratic Formula

   - Graphical Method

2. **Solving Fractional Equations by reducing to Quadratic Equations**

3. **Solving Real-World Problems involving Quadratic Equations**

4. **Sketching the graph of Quadratic Functions**

Solving Quadratic Equation by Completing the Square

Example

$$(x-2)^2 - 9 = 0$$
$$(x-2)^2 = 9$$
$$(x-2) = 3 \quad or \quad (x-2) = -3$$
$$x = 5 \quad or \quad x = -1$$

Whenever you apply a square root, there will always be a positive and negative answer

Tip: If you end up with an equation looking like this: $(x-2)^2 + 9 = 0$, there is no solution as $(x-2)^2$ can never be negative.

Steps

In terms of the general case:

$$f(x) = ax^2 + bx + c$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c$$
$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 + \left[c - a\left(\frac{b}{2a}\right)^2\right]$$
$$= a(x - h)^2 + k$$

Real example:

$$f(x) = x^2 + 9x + 12$$
$$= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 12$$
$$= \left(x + 4\frac{1}{2}\right)^2 - 8\frac{1}{4}$$

- **Steps:**

  1. Rearrange the quadratic equation $f(x) = ax^2 + bx + c = 0$ in the form of

  $$f(x) = a(x - h)^2 + k = 0$$

  2. Make $(x - h)^2$ the subject of the equation

  $$f(x) = (x - h)^2 = \frac{k}{a}$$

  3. Take the square roots of both sides of the equation and solve for the values of x

  $$x = h + \sqrt{\frac{k}{a}} \qquad x = h - \sqrt{\frac{k}{a}}$$

Solving Quadratic Equation by Quadratic Formula

- Method works for <u>ANY</u> quadratic equation
- Steps:
    1. The roots of the quadratic equation $f(x) = ax^2 + bx + c = 0$ can be obtained by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$$

Tip 1: When the question said to express your answer in 3.s.f or 2d.p, you should use the quadratic formula to get the solutions.
Tip 2: If you get a negative value for $b^2 - 4ac$, there is no solution!

Solving Quadratic Equation by Graphical Method

- Using the graphical method, the solutions of the quadratic equation are the points of intersection between the quadratic curve and the x-axis.
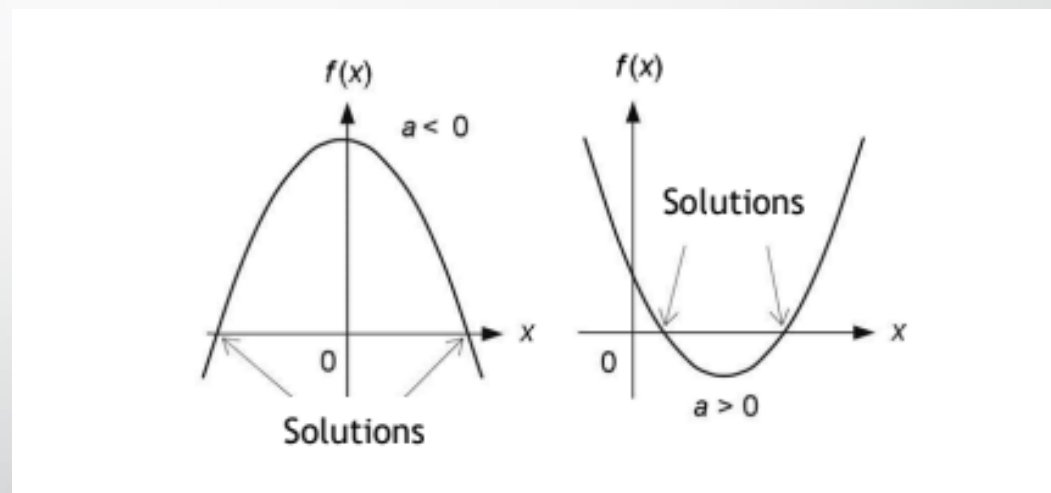
Solving Fractional Equations by reducing to Quadratic Equations

Step 1: Identify the lowest common denominator between the two fractions.

Step 2: Make the denominators of the fractions to be the same.

Step 3: Convert the fractional equations into a quadratic equation.

$$\frac{2x}{2x-3} + 1 = \frac{1}{2-3x}$$

$$\frac{1}{2-3x} - \frac{2x}{2x-3} = 1$$

$$\frac{(2x-3) - 2x(2-3x)}{(2-3x)(2x-3)} = 1$$

$$2x - 3 - 4x + 6x^2 = 4x - 6 - 6x^2 + 9x$$

$$12x^2 - 15x + 3 = 0$$

$$3(4x^2 - 5x + 1) = 0$$

$$3(4x - 1)(x - 1) = 0$$

$$x = \frac{1}{4} \quad \text{or} \quad x = 1$$

	-1	$-x$
$4x$		
x	-1	$-4x$
$4x^2$	1	$-5x$

Common Mistake Made

Example of an __incorrect__ cancellation:

$$\frac{2(x-5)(x+5) - x^2 + 10x}{(x-1)(x+5)}$$

o Performing a cancellation like this is __unacceptable__ as the $(x+5)$ term only appears for the $2(x-5)(x+5)$ and $(x+5)(x-1)$ term. The $-x^2 + 10x$ expression does not contain $(x+5)$ term, hence we cannot cancel the $(x+5)$ term like this

Solving Real-World Problems involving Quadratic Equations

Example:

Worked Example A10.1.3

Nancy is planning a holiday to the United States. On 1 March 2017, she exchanged S$3000 into US dollars (US$) at Kumar's Money Exchange at a rate of US$1 = S$$x$

(a) Find an expression, in terms of x, for the amount of US$ she will received from Kumar's Money Exchange

(b) On 15 March 2017, she decided to exchange another S$2100 into US$ at Lee's Money Exchange at a rate of US$1 = S$$(x - 0.1)$. Find an expression, in terms of x, for the amount of US$ she received from Lee's Money Exchange

(c) Given that Nancy received a total of US$3500 from the two Money Exchanges, form an equation in x and show that it simplifies to the following equation

$$70x^2 - 109x + 6 = 0$$

(d) Solve the equation $70x^2 - 109x + 6 = 0$, giving your answer to 4 decimal places

(e) Suggest a reason as to why one of the answers has to be rejected

(f) Hence, find the exchange rate between S$ and US$ offered by Lee's Money Exchange

(g) Is it better for Nancy to change her currency on 1 March or 15 March? Justify your answer with appropriate workings

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Things to take note of:

1) Do not spend too much time trying to solve part (c). Focus on the next few parts instead. If you are unable to obtain the equation shown, you can still solve part (d) as the question has provided you with the correct quadratic equation to use!

2) In part (d), the question has said to give your answer to 4 d.p, hence use QUADRATIC FORMULA.

3) You don't have to reject any solutions in part (d) as its merely asking you to solve the equation. You should only reject when you are applying the value with the context of the question.

Sketching the graph of Quadratic Functions

- To find the x-intercept, sub $y = 0$.
- To find the y-intercept, sub $x = 0$.
- To find the turning point, use either Complete the square (Refer to page 3) or finding the line of symmetry (we can compute the line of symmetry by finding the midpoint: $\frac{x_1 + x_2}{2}$, where x_1 and x_2 are the x-intercepts.)

- To determine the shape of the graph, we look at the coefficient of x^2.
  - If the coefficient of x^2 is more than 0, it is a happy face.
  - If the coefficient of x^2 is less than 0, it is a sad face.
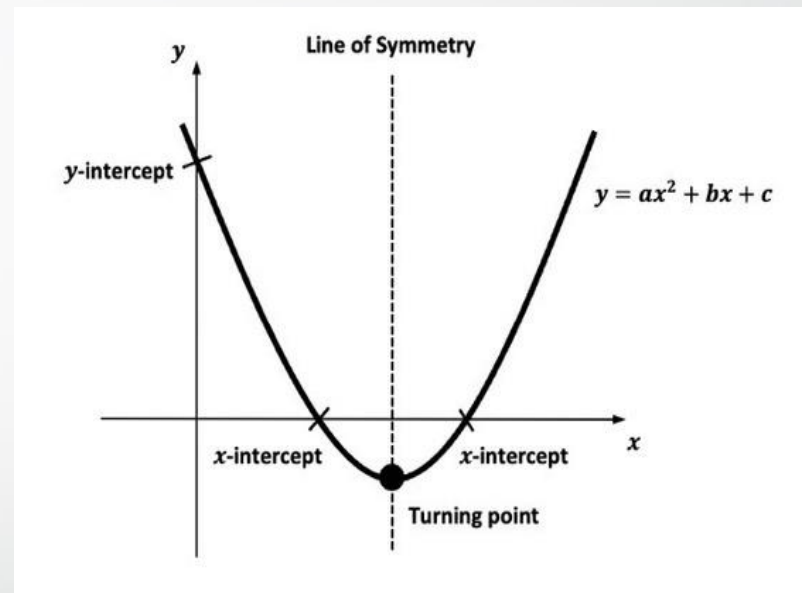
Happy Face
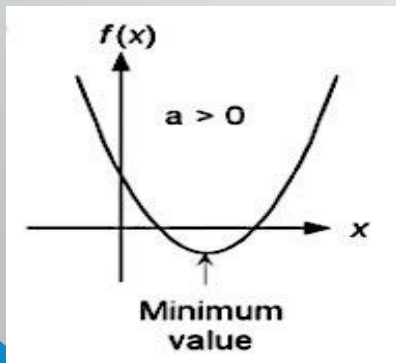
Sad Face
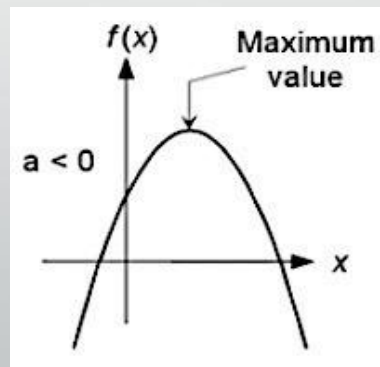
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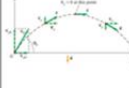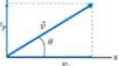
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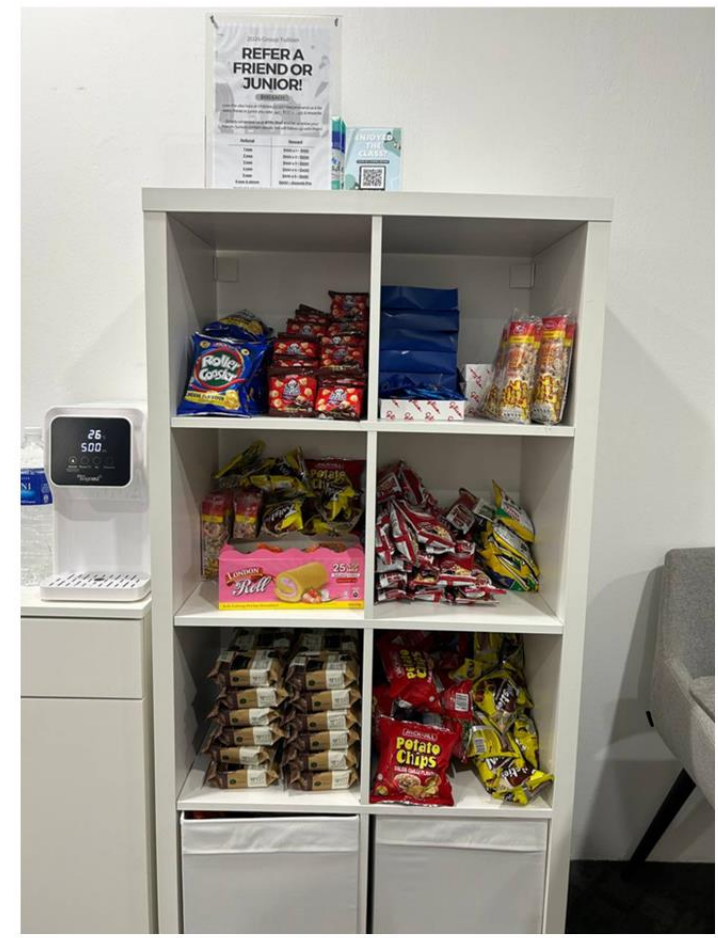
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