



CHAPTER 4: POLYNOMIALS

$$b)(a^2 - ab + b^2)$$

$$b)(a^2 + ab + b^2)$$

CHAPTER ANALYSIS

- Multiplication and Division of Polynomials
- Use of remainder and factor theorems, including factorizing polynomials and solving cubic equations
- Use of:
 - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
 - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$



WHAT IS A POLYNOMIAL?

- If the question states that the degree of the polynomial is n , the highest power of the polynomial will be n .

Example: The degree of the polynomial is 4.

Equation will be:

$$Ax^4 + Bx^3 + Cx^2 + Dx + E$$

A function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

- n / all powers must be a positive integer

Common Mistake

Some students have difficulty identifying which functions are polynomials

The following are **NOT** polynomials

$$x^2 - 2x + 3x^{\frac{1}{2}}$$

The power of $3x^{\frac{1}{2}}$ is not a **positive integer**

$$x^2 - \frac{2}{x} - \frac{5}{x^3}$$

The powers of $\frac{2}{x}$ and $\frac{5}{x^3}$ are not **positive integers**

Why?

Although at face value it does look like positive integers, the variable x is in the denominator

By indices law:

$$\frac{2}{x} = 2x^{-1} \quad \frac{5}{x^3} = 5x^{-3}$$



MULTIPLICATION AND DIVISION OF POLYNOMIALS

- Division of polynomials involve using long division, so do recap on those!!

Steps in Long Division:

- 1) Divide: Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.
- 2) Multiply: Multiply the entire divisor by the first term of the quotient
- 3) Subtract: Subtract the result from the dividend to get a remainder
- 4) Repeat: Repeat the process with the remainder as the new dividend until the remainder is zero or of a lower degree than the divisor.

Division Algorithm

Suppose that $f(x)$ and $g(x)$ are 2 polynomials where $g(x) \neq 0$. We can find 2 polynomials $q(x)$ and $r(x)$ such that

$$f(x) = g(x) \cdot q(x) + r(x)$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Polynomial	Terminology
$f(x)$	Dividend
$g(x)$	Divisor
$q(x)$	Quotient
$r(x)$	Remainder

The degree of $g(x)$ is always less than the degree of $f(x)$

$$\begin{array}{r} q(x) \text{ [Quotient]} \\ g(x) \text{ [Divisor]} \overline{) f(x) \text{ [Dividend]}} \\ \hline r(x) \text{ [Remainder]} \end{array}$$



REMAINDER AND FACTOR THEOREM

- If we are dividing by $(x - b)$, we will have to substitute in $x = b$ and write $f(b)$.

*Tip: Make the bracket $(x - b)$ to be equal to 0.

- The main idea of Remainder Theorem and Factor Theorem are the same. The only difference is that for Factor Theorem, the remainder is always equal to 0.
- As it is a factor, it will be exactly divisible, therefore leaving no remainder.

The Remainder Theorem states that when a polynomial $f(x)$ (of degree ≥ 1) is divided by $(ax \mp b)$ where $a \neq 0$, the remainder R is given by

$$R = f\left(\pm \frac{b}{a}\right)$$

Reasoning and Methodology:

$$f(x) = (ax \mp b) \cdot q(x) + R$$

$q(x)$ (Quotient) is a polynomial that is usually not given in questions. We want to get rid of $q(x)$ and force the whole term to be 0 so that the only thing remaining is $f(x) = R$. To force $(ax \mp b) \cdot q(x)$ to be 0, we will find values of x that will cause $(ax \mp b) = 0$

$$(ax \mp b) = 0 \Rightarrow x = \pm \frac{b}{a}$$



FACTORISATION OF POLYNOMIALS

- To determine factors of a polynomial, you can use the calculator to check.

*For Casio fx-96 SG PLUS, key in: **MODE,3,4** and key in the coefficients and constant, respectively. The calculator will solve the equation and you will be able to derive the factors from there.

*For Casio fx-97 SG PLUS, key in: **MENU,5,2,SELECT DEGREE** and key in the coefficients and constant, respectively. The calculator will solve the equation and you will be able to derive the factors from there.

Example

When I solve the equation using the calculator, if one of the solutions is $x = 1$, the factor will be $(x - 1)$.

- Determine one of the factors of $f(x)$ using “trial and error”

- if more factors can be determined using “trial and error”, then perform this method for the other factors as well
- Although the method of “trial and error” is most typically **frowned upon**, there is a smart way to perform this “trial and error” method
 - * Key into the calculator the polynomial to check for all the factors
 - * Choose one of the factors that gives a beautiful solution (\mathbb{Z} or \mathbb{Q}) and present that factor as the “trialed” factor

Presentation of solutions

Follow the following template to show the smart “trial and error”.

Let the factor be α and

$$f(x) = x^3 - 2x^2 - 5x + 6$$

- Claim that x is the factor

$$\text{Let } x = \alpha$$

- Show that the substitution results in 0

$$f(\alpha) = (\alpha)^3 - 2(\alpha)^2 - 5(\alpha) + 6 = 0$$

- Write a concluding statement

Hence, by the factor theorem, $(x - \alpha)$ is a factor



FACTORISATION OF POLYNOMIALS

- Once we have derived one of the factors of the polynomials,

Step 1: You will “let $(x - 1)$ be a factor” and use Factor Theorem to prove that remainder is equals to 0.

Step 2: Find the remaining factors using either of the TWO methods below.

Method 1: Apply Long Division, using $f(x) \div \text{factor found in step 1}$.

Method 2: Comparison Method.

$f(x) = (x - 1)(Ax^2 + Bx + C)$, whereby $f(x)$ is the polynomial you have.

TIP: Always start by comparing the highest power and the constant. You will be able to derive A and C.

Step 3: Factorise the quadratic equation found in Step 2 into two brackets.

Step 3 is applicable regardless of the method used in Step 2.



SOLVING OF POLYNOMIALS

- Once we have derived all the factors of the polynomials, express it in the form of 3 brackets.

Example:

$$f(x) = 2x^3 - 11x^2 - x + 30 = (2x + 3)(x - 5)(x - 2)$$

$$\text{Since } f(x) = 0, (2x + 3)(x - 5)(x - 2) = 0.$$

Therefore, the solutions are : $x = \frac{3}{2}$, $x = 5$ or $x = 2$

Tip: You can check your answers by using the calculator. Method can be found in Page 6.



CUBIC IDENTITIES

Example:

$$\begin{aligned} & 27x^3 - 64y^3 \\ &= (3x)^3 - (4y)^3 \\ &= (3x - 4y)(9x^2 + 12xy + 16y^2) \end{aligned}$$

*Take note: Cubic identity is not the same as quadratic identity, it is ***ab*** in cubic equation instead of ***2ab*** in quadratic equation.

Algebraic Identities

Quadratic Identities [E-Math]

$$(a + b)^2 = (a^2 + 2ab + b^2)$$

$$(a - b)^2 = (a^2 - 2ab + b^2)$$

$$a^2 - b^2 = (a + b)(a - b)$$

Cubic Identities [***NEW***]

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$



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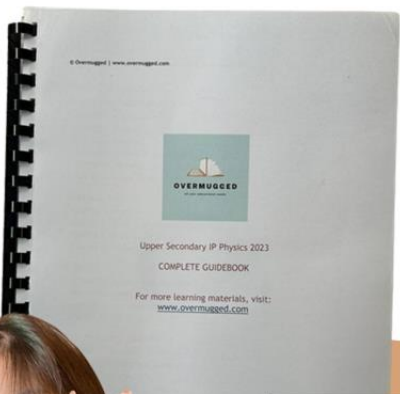
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TOPIC: KINEMATICS			
Concept	Definition	Formula	Remarks
Linear motion	<ul style="list-style-type: none"> Object that is moving in a straight line 1-D motion 	$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$	<ul style="list-style-type: none"> Motion can be represented (upwards or right) or -ve (down or left) sign. Equations can only be used if acceleration is constant.
2-D Motion	<ul style="list-style-type: none"> Object that is moving in a projectile trajectory (x and y directions) 2-D motion Acceleration is experienced in both axes Vertical and horizontal motion are independent of each other 	<p>Horizontal motion (acceleration = 0)</p> $v_x = u_x$ $s_x = u_x t$ <p>Vertical motion (uniform vertical acceleration = g)</p> $v_y = u_y + at$ $s_y = u_y t + \frac{1}{2}at^2$ $v_y^2 = u_y^2 + 2as$	<ul style="list-style-type: none"> Acceleration always act down Projectile will free fall and parabolic if resistance is negligible
2 Vectors resolution	Analyse the horizontal and vertical motion separately	<p>For a vector \vec{v} pointing at an angle θ from the horizontal:</p> $v_x = v \cos\theta \text{ (horizontal)}$ $v_y = v \sin\theta \text{ (vertical)}$ $v = \sqrt{v_x^2 + v_y^2}$ $\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$	

MARCH PRACTICE QUESTIONS 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ELEMENTARY MATHEMATICS 4048/01

Specimen Paper
Date: 3 March 2021
Candidates answer on separate writing paper

Mean THESE INSTRUCTIONS FIRST

Answer all questions.
If working is needed for any question it must be shown with the answer.
Obtain the answer by using a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π , use either your calculator value of 3.142, unless the question requires the answer in terms of π .

Topic names will be listed above each question for your benefit and revision

Upon completion of solutions:
Each candidate have exactly 2 weeks to submit their solution.
Take a picture or send the digital version of your solutions to me (Kahen) via Telegram (@Kahen_tutari) or WhatsApp (90583779).
Ensure that all workings are clear and legible.
Solutions will be marked based on your presentation, accuracy and completeness of your solution.
A markers' report and the full solutions will be provided at the end of the month.

Setter: Ong Kai Wen
This question paper consists of 2 printed pages including the cover page

at reached in a sec. ity $v_y = 0$	$v_y^2 = (u \sin\theta)^2 - 2gh$ $\Delta H = \frac{u^2 \sin^2\theta}{2g}$	<ul style="list-style-type: none"> With air resistance, Drag force acts in the same as the weight of object. Net acceleration $>> g$ Maximum height reached lower.
ity $v_y = 0$	$v_y = u \sin\theta - gt_{up}$ $\Delta t_{up} = \frac{u \sin\theta}{g}$	<ul style="list-style-type: none"> With air resistance, Drag force acts in the same as the weight of object. Net acceleration $>> g$ Final vertical speed smaller than vertical speed Average speed upwards $>> g$

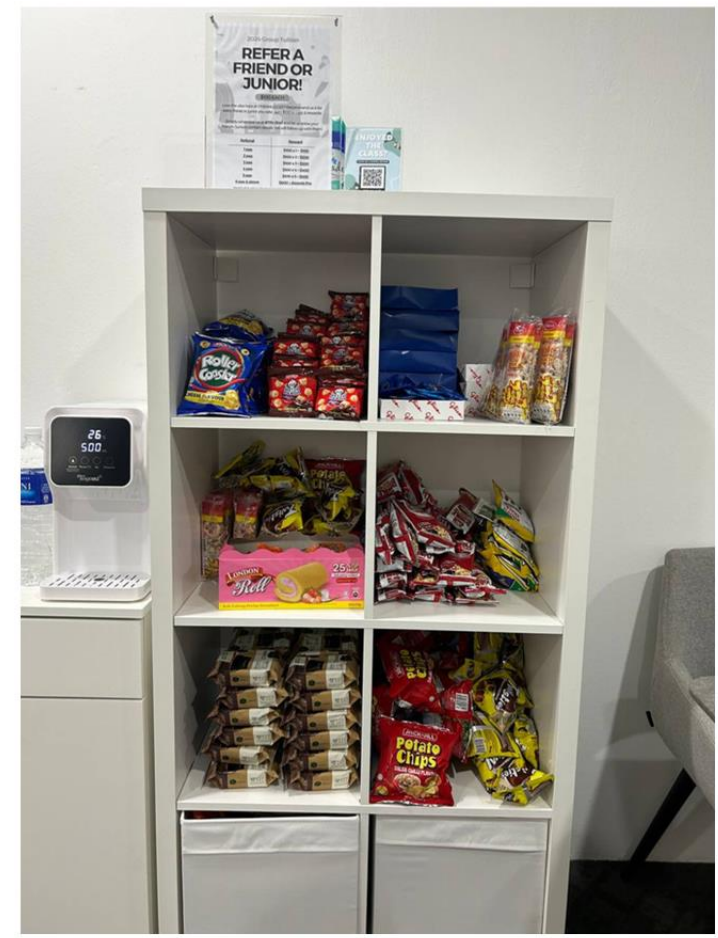




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Topical Recaps

Key highlight: Christmas Party

TERM 2: FEB – APR

Topical Mastery

Key highlight: March Holiday Cohesion Program

TERM 3: MAY – JUL

Prelim/EOY Preparation

Key highlight: Mock Prelim/EOY

TERM 4: AUG – OCT

'O' Levels / 'A' Levels Preparation

Key highlight: Mock Exams, Science Practical Assessment





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