CHAPTER 4: POLYNOMIALS

CHAPTER ANALYSIS

- Multiplication and Division of Polynomials
- Use of remainder and factor theorems, including factorizing polynomials and solving cubic equations
- Use of:

- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

WHAT IS A POLYNOMIAL?

- If the question states that the degree of the polynomial is *n*, the highest power of the polynomial will be *n.*

Example: The degree of the polynomial is 4.

Equation will be:

$$Ax^4 + Bx^3 + Cx^2 + Dx + E$$

A function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

- n / all powers must be a **positive integer**

Common Mistake

Some students have difficulty identifying which functions are polynomials

The following are NOT polynomials

$$x^2 - 2x + 3x^{\frac{1}{2}}$$

The power of $3x^{\frac{1}{2}}$ is not a **positive integer**

$$x^2 - \frac{2}{x} + \frac{5}{x^3}$$

The powers of $\frac{2}{x}$ and $\frac{5}{x^3}$ are not **positive integers**

Why?

Although at face value it does look like positive integers, the variable x is in the denominator

By indices law:

$$\frac{2}{x} = 2x^{-1} \qquad \frac{5}{x^3} = 5x^{-3}$$

MULTIPLICATION AND DIVISION OF POLYNOMIALS

- Division of polynomials involve using long division, so do recap on those!!

Steps in Long Division:

1) Divide: Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.
2) Multiply: Multiply the entire divisor by the first term of the quotient
3) Subtract: Subtract the result from the dividend to get a remainder
4) Repeat: Repeat the process with the remainder as the new dividend until the remainder is zero or of a lower degree than the divisor.

Division Algorithm

Suppose that $f(x)$ and $g(x)$ are 2 polynomials where $g(x) \neq 0$. We can find 2 polynomials $q(x)$ and $r(x)$ such that

$$f(x) = g(x) \cdot q(x) + r(x)$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Polynomial	Terminology
$f(x)$	Dividend
$g(x)$	Divisor
$q(x)$	Quotient
$r(x)$	Remainder

The degree of $g(x)$ is always less than the degree of $f(x)$

$$\begin{array}{r} q(x)\ \text{[Quotient]} \\ g(x)\ \text{[Divisor]}\ \overline{)\ f(x)\ \text{[Dividend]}} \\ \hline r(x)\ \text{[Remainder]} \end{array}$$

REMAINDER AND FACTOR THEOREM

- If we are dividing by $(x - b)$, we will have to sub in $x = b$ and write $f(b)$.

*Tip: Make the bracket $(x - b)$ to be equals to 0.

- The main idea of Remainder Theorem and Factor Theorem are the same. The only difference is that for Factor Theorem, the remainder is always equals to 0.

- As it is a factor, it will be exactly divisible, therefore leaving no remainder.

The Remainder Theorem states that when a polynomial $f(x)$ (of degree ≥ 1) is divided by $(ax \mp b)$ where $a \neq 0$, the remainder R is given by

$$R = f\left(\pm\frac{b}{a}\right)$$

Reasoning and Methodology:

$$f(x) = (ax \mp b) \cdot q(x) + R$$

$q(x)$ **(Quotient)** is a polynomial that is usually not given in questions. We want to get rid of $q(x)$ and force the whole term to be 0 so that the only thing remaining is $f(x) = R$. To force $(ax \mp b) \cdot q(x)$ to be 0, we will find values of x that will cause $(ax \mp b) = 0$

$$(ax \mp b) = 0 \Rightarrow x = \pm\frac{b}{a}$$

FACTORISATION OF POLYNOMIALS

- To determine factors of a polynomial, you can use the calculator to check.

*For Casio fx-96 SG PLUS, key in**: MODE,3,4** and key in the coefficients and constant, respectively. The calculator will solve the equation and you will be able to derive the factors from there.

***For Casio fx-97 SG PLUS, key in**: MENU,5,2,SELECT DEGREE** and key in the coefficients and constant, respectively. The calculator will solve the equation and you will be able to derive the factors from there.

Example

When I solve the equation using the calculator, if one of the solutions is $x = 1$, the factor will be $(x - 1)$.

- Determine one of the factors of $f(x)$ using **"trial and error"**
  - if more factors can be determined using "trial and error", then perform this method for the other factors as well
  - Although the method of "trial and error" is most typically **frowned upon**, there is a smart way to perform this "trial and error" method
    - Key into the calculator the polynomial to check for all the factors
    - Choose one of the factors that gives a beautiful solution ($\mathbb{Z}$ or $\mathbb{Q}$) and present that factor as the "trialed" factor

Presentation of solutions

Follow the following template to show the **smart "trial and error"**.

Let the factor be α and

$$f(x) = x^3 - 2x^2 - 5 + 6$$

- Claim that x is the factor

$$\text{Let } x = \alpha$$

- Show that the substitution results in 0

$$f(\alpha) = (\alpha)^3 - 2(\alpha)^2 - 5(\alpha) + 6 = 0$$

- Write a concluding statement

Hence, by the factor theorem, $(x - \alpha)$ is a factor

FACTORISATION OF POLYNOMIALS

- Once we have derived one of the factors of the polynomials,

Step 1: You will "let $(x - 1)$ be a factor" and use Factor Theorem to prove that remainder is equals to 0.

Step 2: Find the remaining factors using either of the TWO methods below.

Method 1: Apply Long Division, using $f(x) \div factor\ found\ in\ step\ 1$.

Method 2: Comparison Method.

$f(x) = (x - 1)(Ax^2 + Bx + C)$, whereby $f(x)$ is the polynomial you have.

TIP: Always start by comparing the highest power and the constant. You will be able to derive A and C.

Step 3: Factorise the quadratic equation found in Step 2 into two brackets.

Step 3 is applicable regardless of the method used in Step 2.

SOLVING OF POLYNOMIALS

- Once we have derived all the factors of the polynomials, express it in the form of 3 brackets.

Example:

$$f(x) = 2x^3 - 11x^2 - x + 30 = (2x + 3)(x - 5)(x - 2)$$

$$\text{Since f(x)} = 0, (2x + 3)(x - 5)(x - 2) = 0.$$

Therefore, the solutions are : $x = \frac{3}{2}, x = 5 \; or \; x = 2$

Tip: You can check your answers by using the calculator. Method can be found in Page 6.

CUBIC IDENTITIES

Example:

$$27x^3 - 64y^3$$
$$= (3x)^3 - (4y)^3$$
$$= (3x - 4y)(9x^2 + 12xy + 16y^2)$$

*Take note: Cubic identity is not the same as quadratic identity, it is $\boldsymbol{ab}$ in cubic equation instead of $\boldsymbol{2ab}$ in quadratic equation.

Algebraic Identities

Quadratic Identities [E-Math]

$$(a+b)^2 = (a^2 + 2ab + b^2)$$

$$(a-b)^2 = (a^2 - 2ab + b^2)$$

$$a^2 - b^2 = (a+b)(a-b)$$

Cubic Identities [*NEW*]

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

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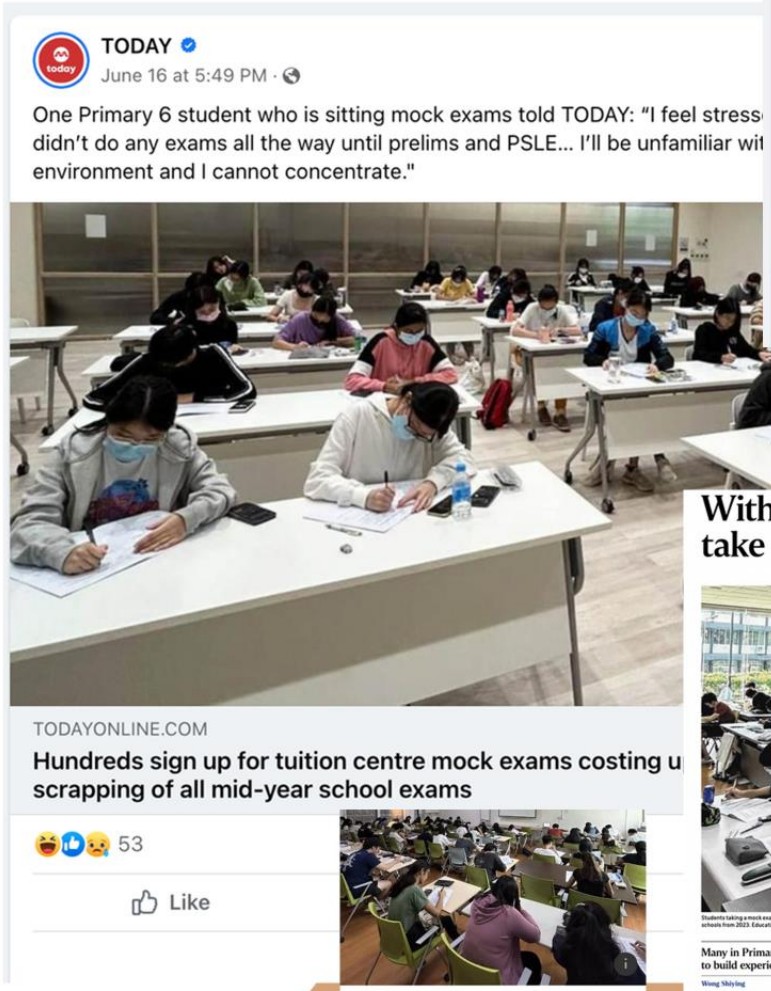

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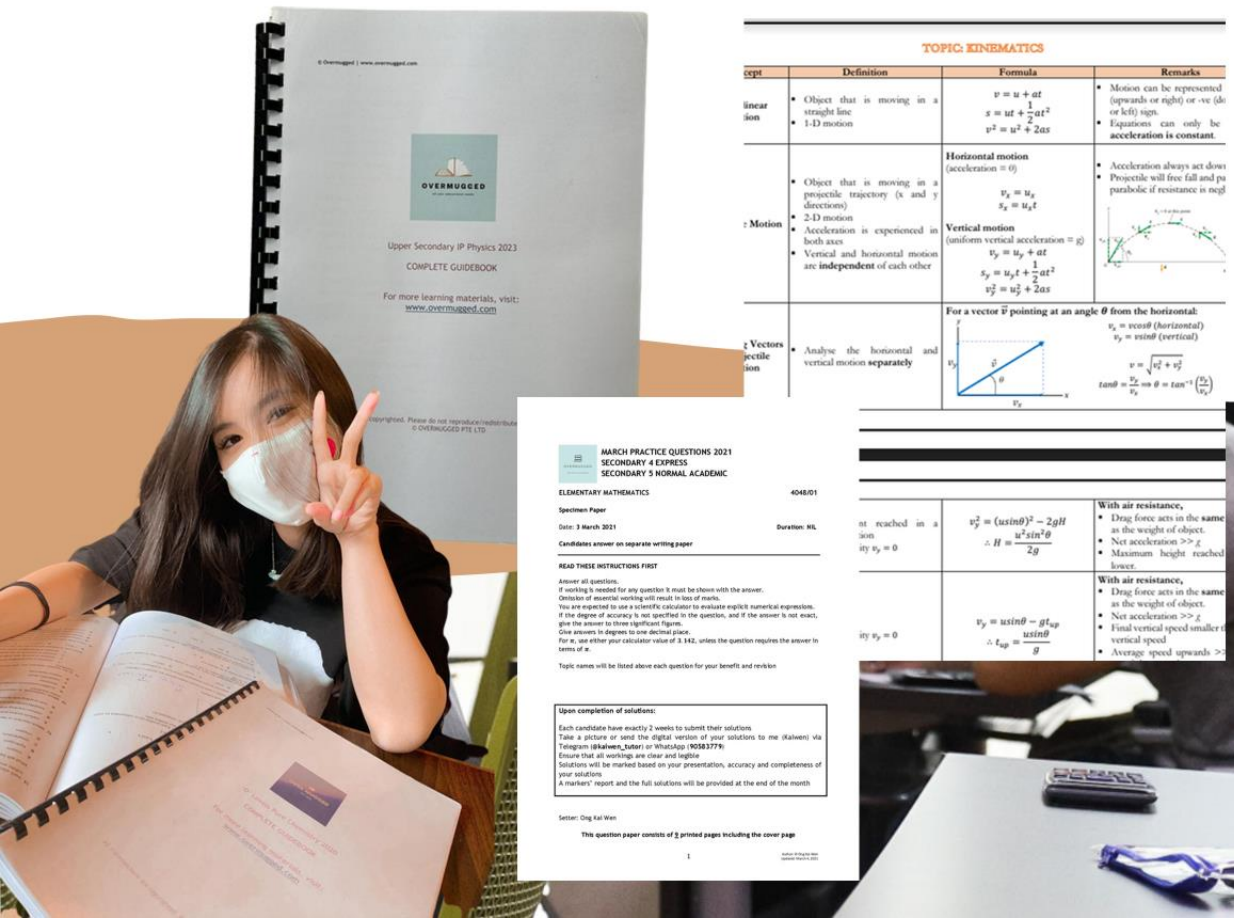

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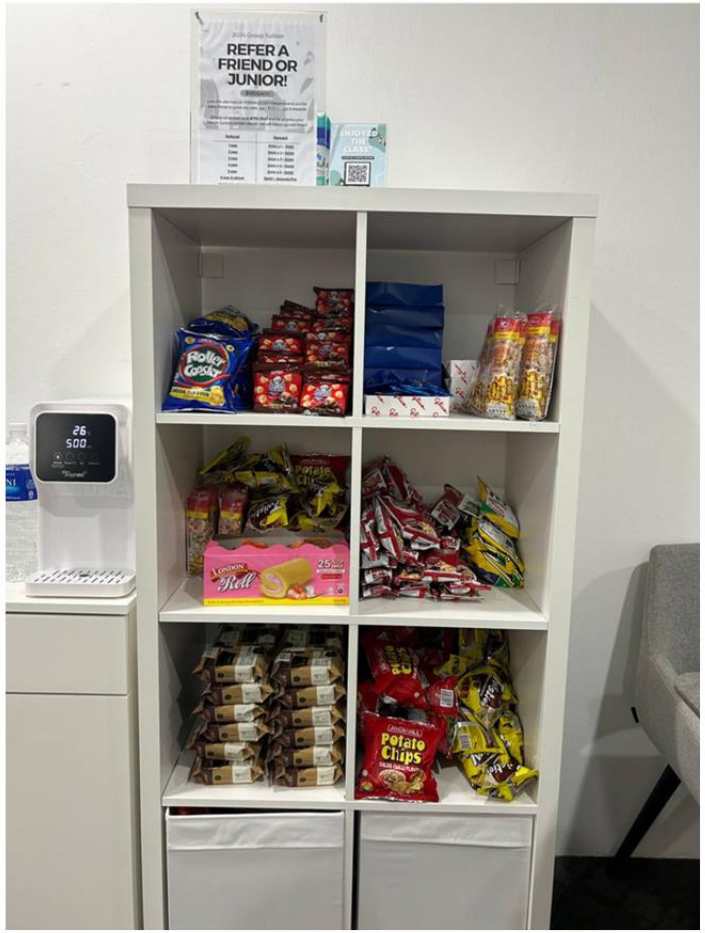

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