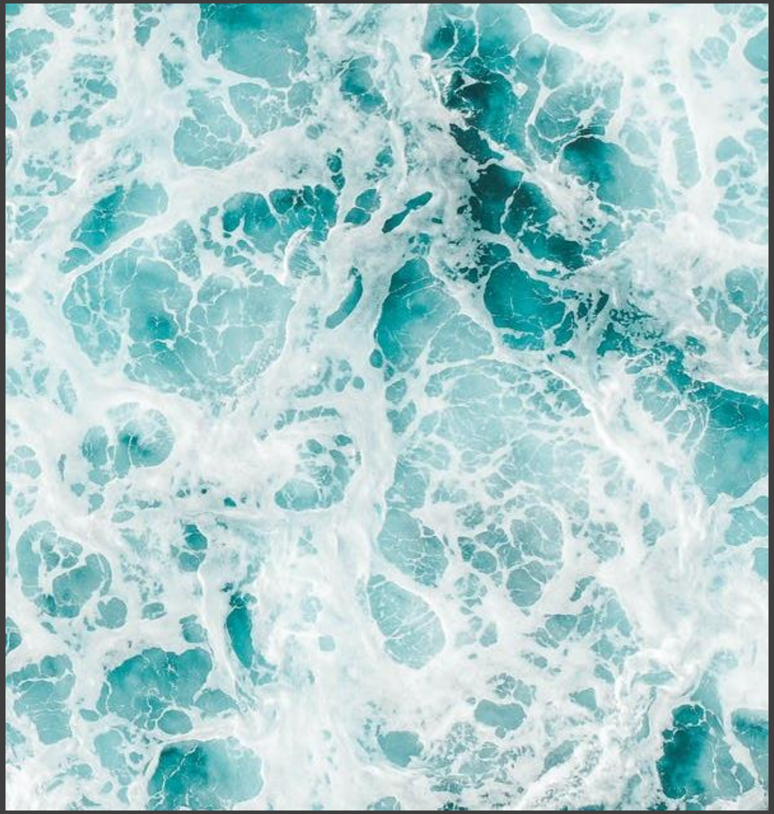
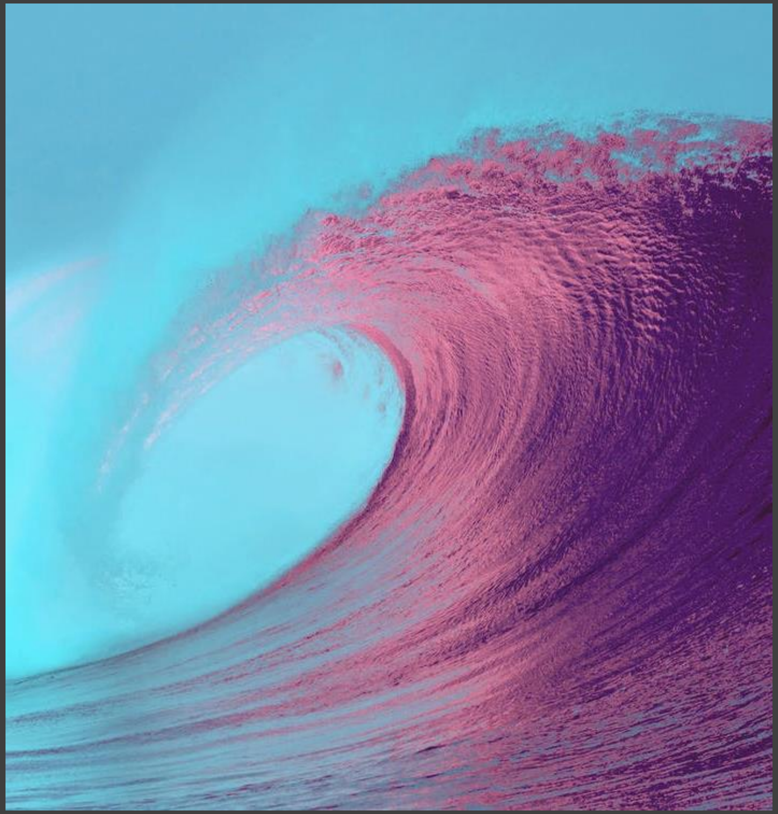
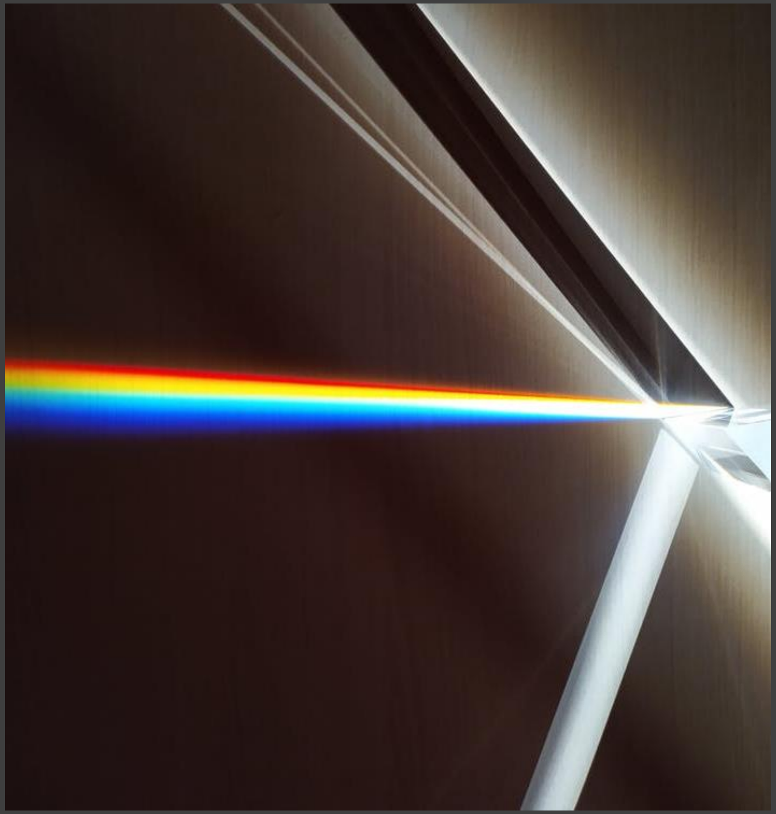


SUPERPOSITION

Overmugged

Content

- Principle of Superposition
- Stationary Waves
- Diffraction
- Single-Slit and Multiple-Slit Diffraction

Wave

- is a disturbance that travels through a medium or vacuum.

Examples

Mechanical Waves

- examples include sound and water waves.
- the disturbance refers to the displacement of particles of the media form their equilibrium position.

Electromagnetic Waves

- the disturbance refers to the varying electric and magnetic field.

Principle of Superposition

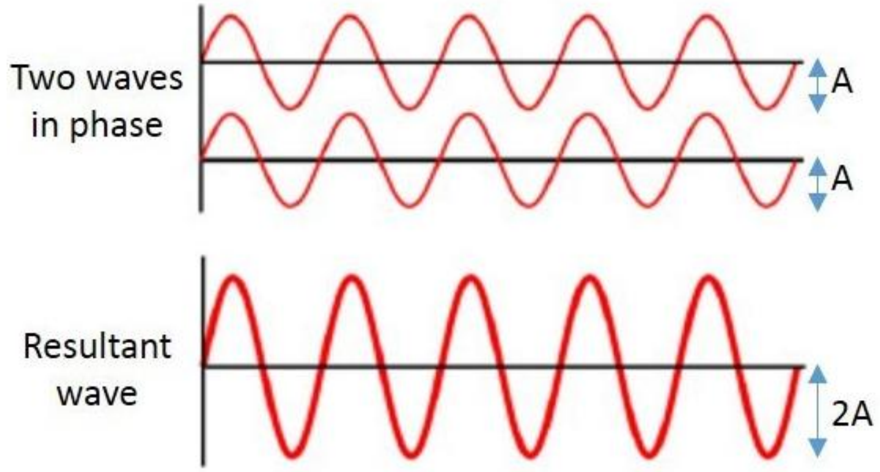
- When two or more waves of the same nature meet at a point, the resultant displacement is the **vector sum** of the individual displacements at that point.

Interference

- Interference is the phenomenon where 2 or more waves of the same type **superpose** to produce a resultant wave.

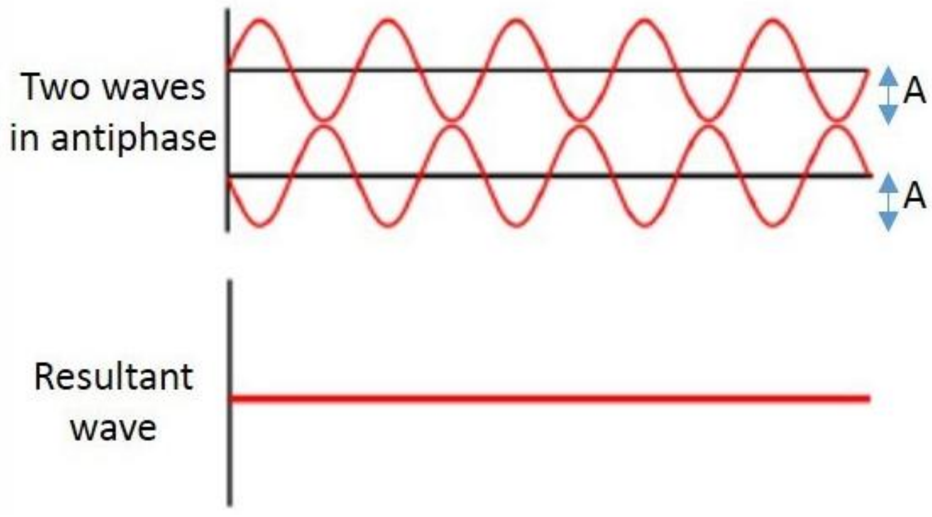
*all waves of the same **nature** can superpose and interfere with one another.

Constructive Interference



- occurs when 2 waves meet **in phase with one another,** so that there is **reinforcement** of the waves at that point.
- The **resultant displacement is larger** than the individual displacements.

Destructive Interference



- occurs when 2 waves meet in antiphase (180 degrees or π radians out of phase), so that there is a **weakening** of the waves at that point.
- The resultant displacement will be equal to 0 or **less than** the larger value of the individual displacements.

Stationary Waves

- results from the superposition of two progressive waves of the same frequency, amplitude and speed, travelling along the same line but in opposite directions

General conditions to form a Stationary Wave

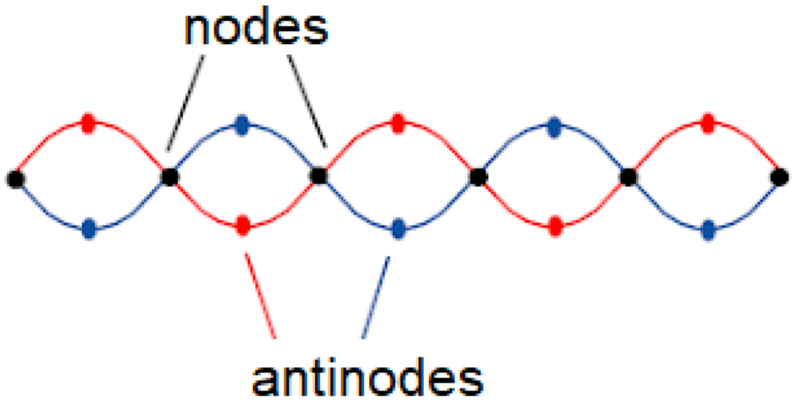
VFAT:

1. **V (**speed)**:** The individual waves must be travelling in opposite direction with the same speed.
2. **F** (frequency): The frequency of the waves must be the same.
3. **A** (amplitude): The amplitudes must be the same.
4. **T** (type): The two waves must be of the same type. (i.e. water wave, sound wave, EM wave).

Stationary Waves

Properties of a Resultant Waveform

1. It does not propagate. As such, it is also called **standing wave.**
2. Every particle **oscillates** (expect at the nodes) with the **same frequency,** but different amplitudes.



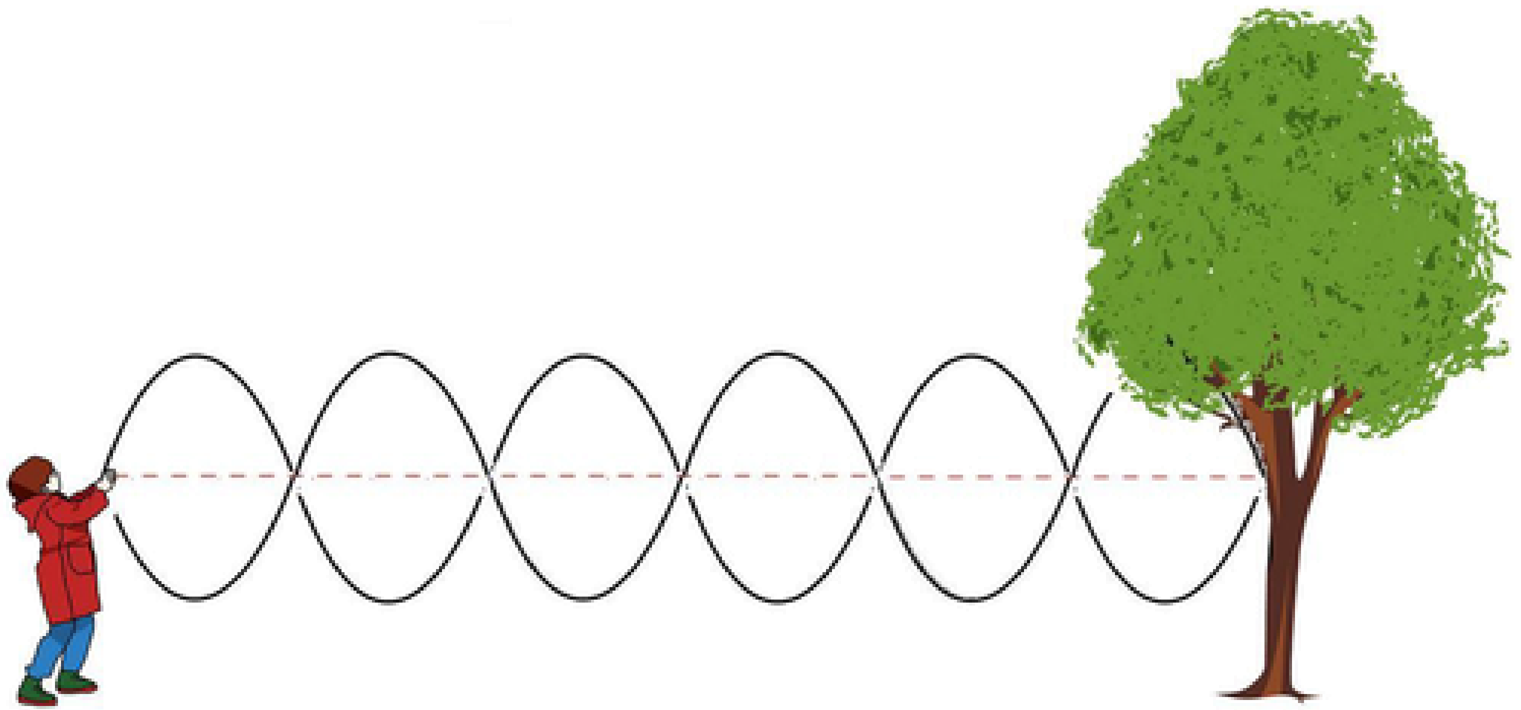
3. The **amplitude** of oscillation at the antinodes is **double** that of the component waves.
4. Within two consecutive nodes, every particle oscillates in phase. (Note that these particles do have the same amplitude.)
5. **Distance** between two adjacent nodes (or antinodes) is

$$\frac{1}{2}\lambda$$

6. Particles in neighboring segments vibrate 180° (or π rad) out of phase with each other.

Practice Example 1

As seen in the figure above, explain how the lady above managed to create a stationary wave from the string.



Practice Example 2

Progressive waves of frequency 300 Hz are superposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive wave?

Stationary Waves in Strings

- By plucking a string at different points along the wire, stationary waves can be set up at various frequencies.

Resonant/Natural Frequency

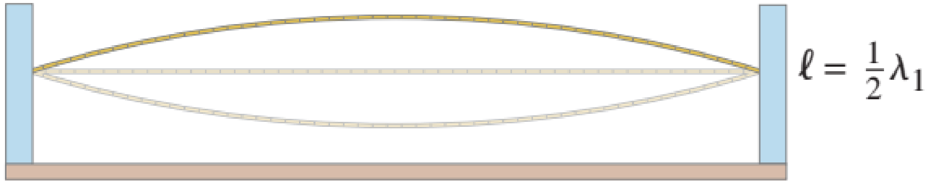
- frequency at which standing wave are produced.

Resonance

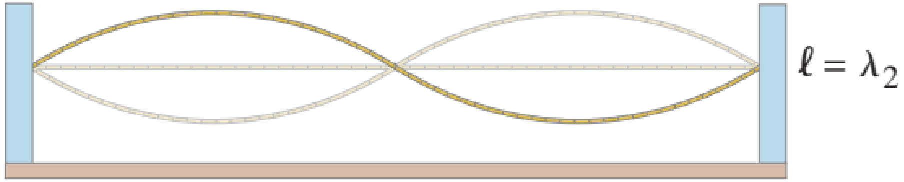
- phenomenon when an object vibrates at a frequency equal to its natural frequency.

Stationary Waves in Strings

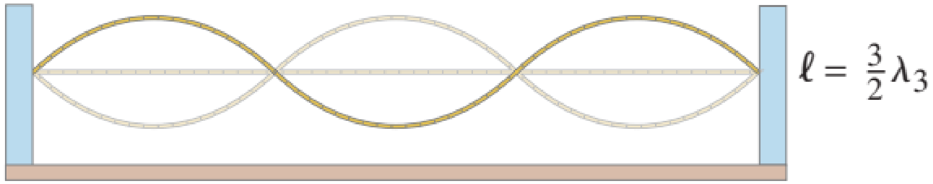
There will be different standing wave patterns as shown below. These are the different **resonant modes of vibration.**



Fundamental or first harmonic, f_1



First overtone or second harmonic, $f_2 = 2f_1$



Second overtone or third harmonic, $f_3 = 3f_1$

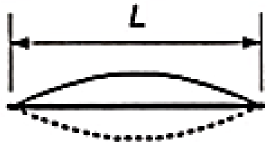
Fundamental Frequency

- lowest resonant frequency at which a standing wave is produced.
- The other (higher) natural frequencies are called **overtones.**

Harmonic Frequency

- When overtones are whole-number (integral) multiples of the fundamental, they are called **harmonics.**
  - examples include vibrating string

Stationary Waves in Strings

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental mode/frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
1st overtone		$L = 2\left(\frac{\lambda_2}{2}\right)$ $\Rightarrow \lambda_2 = L$	$f_2 = 2\left(\frac{v}{2L}\right)$	2nd harmonic
2nd overtone		$L = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2L}{3}$	$f_3 = 3\left(\frac{v}{2L}\right)$	3rd harmonic

(n-1)th overtone or **nth harmonic**

$$\boxed{\lambda_n = \frac{2L}{n}}$$

$$\boxed{f_n = nf_1 = n\left(\frac{v}{2L}\right)}$$

where λ = wavelength of the stationary

f = frequency of the stationary wave

f_1 = fundamental frequency of the string = $\frac{v}{2L}$

L = length of the string

v = speed of the progressive wave

Stationary Waves in Strings

- At the fixed ends, there must be nodes (since the strings cannot vibrate).
- In reality, multiple harmonics give the timbre or characteristics of an instrument.

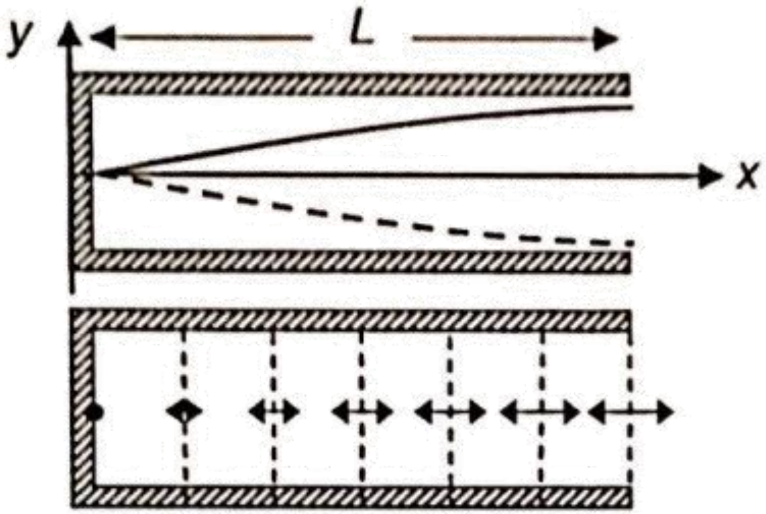
Stationary Waves in Air Columns or Pipes

- By sending sound waves into an air column or pipe, it is possible to generate stationary waves in the pipe.

- A **closed pipe** is a pipe with one end open while the other end is closed.
- In an **open pipe**, both ends are open.

Stationary Waves in Closed Pipes

- Wave propagates to the end of the pipe and is reflected.
- The reflected superposes with the incident wave and forms a stationary wave.

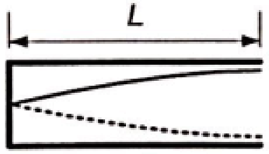
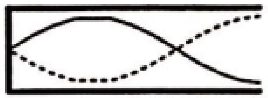
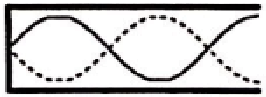


- **displacement node** is formed at the closed end, while a displacement **antinode** is formed at the open end
- The bottom figure shows arrows indicating the amplitudes of vibration of air molecules.

Harmonic Frequency

- For stationary waves in closed pipes, there are **no even harmonics**

Stationary Waves in Closed Pipes

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{4}\right)$ $\Rightarrow \lambda_1 = 4L$	$f_1 = \frac{v}{4L}$	1st harmonic
1st overtone		$L = 3\left(\frac{\lambda_3}{4}\right)$ $\Rightarrow \lambda_3 = \frac{4L}{3}$	$f_3 = 3\left(\frac{v}{4L}\right)$	3rd harmonic
2nd overtone		$L = 5\left(\frac{\lambda_5}{4}\right)$ $\Rightarrow \lambda_5 = \frac{4L}{5}$	$f_5 = 5\left(\frac{v}{4L}\right)$	5th harmonic

(n-1)th overtone or **(2n-1)th harmonic**

$$\lambda_{2n-1} = \frac{4L}{2n-1}$$

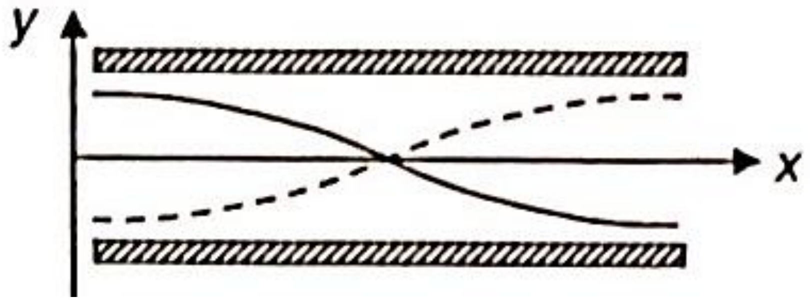
$$f_n = (2n-1)f_1 = (2n-1)\left(\frac{v}{4L}\right)$$

where $\lambda =$ wavelength of the stationary wave
$f =$ frequency of the stationary wave

$f_1 =$ fundamental frequency of the string $= \frac{v}{4L}$
$L =$ length of the string
$v =$ speed of the progressive wave

Stationary Waves in Open Pipes

- As sound wave travels down an open pipe and reaches the other end, part of the wave is reflected.



- A **displacement antinode** is formed at both ends.

Harmonic Frequency

- Overtones and harmonic frequencies are the same as in the strings.

Practice Example 3

A 1.0-m long string of a piano weighs 8.00 g . The string vibrates at a fundamental frequency of 125 Hz.

(a)What is the speed of the wave produced by the string?

(b)Find the frequencies of the first five harmonics

Practice Example 4

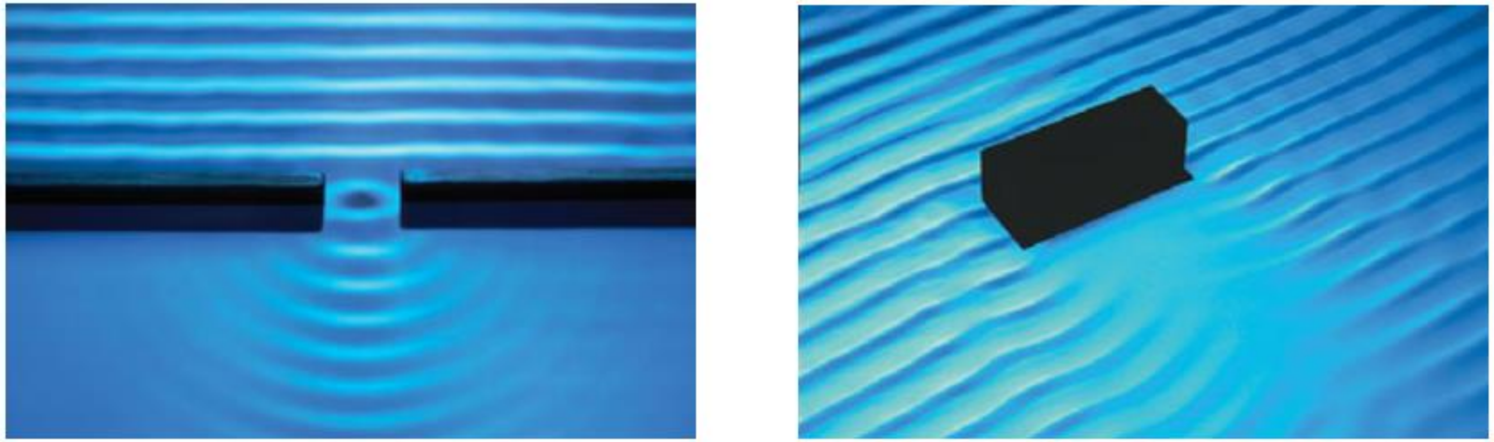
An organ pipe, 0.33 long, is open at one end and closed at the other. The speed of sound in air is 330 m/s. Assuming that the end corrections are negligible, calculate

(a)the frequencies of the fundamental and the first overtone,

(b)the length of a pipe which is open at both ends and which has a fundamental frequency equal to the difference of those calculated in (a).

Diffraction

- is the bending of waves after passing through an aperture or around an obstacle.



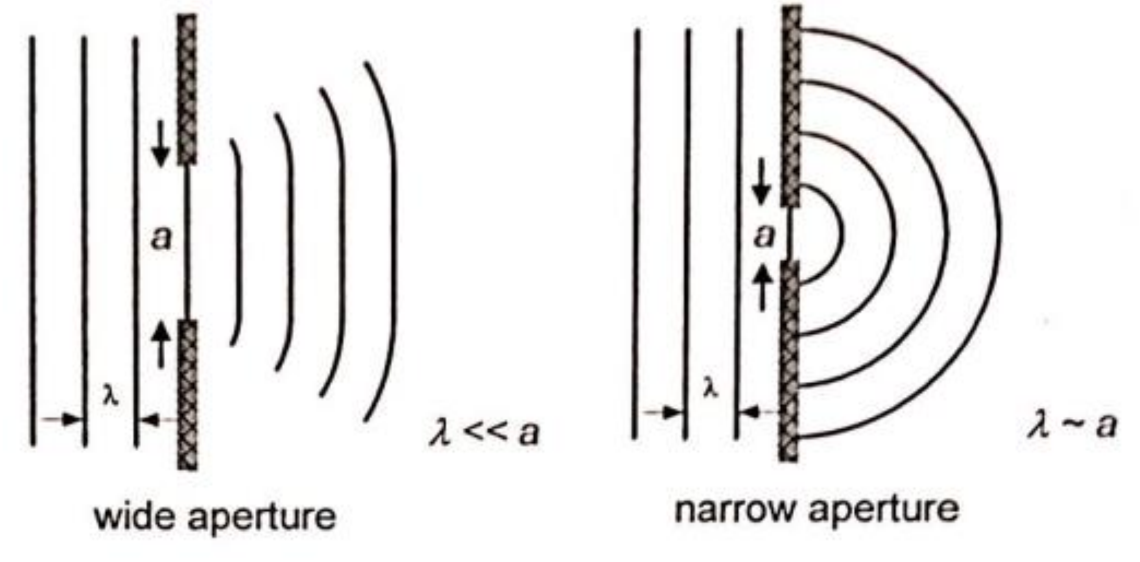
(a) (b)

Water waves passing through a (a) slit, and an (b) obstacle.

Diffraction

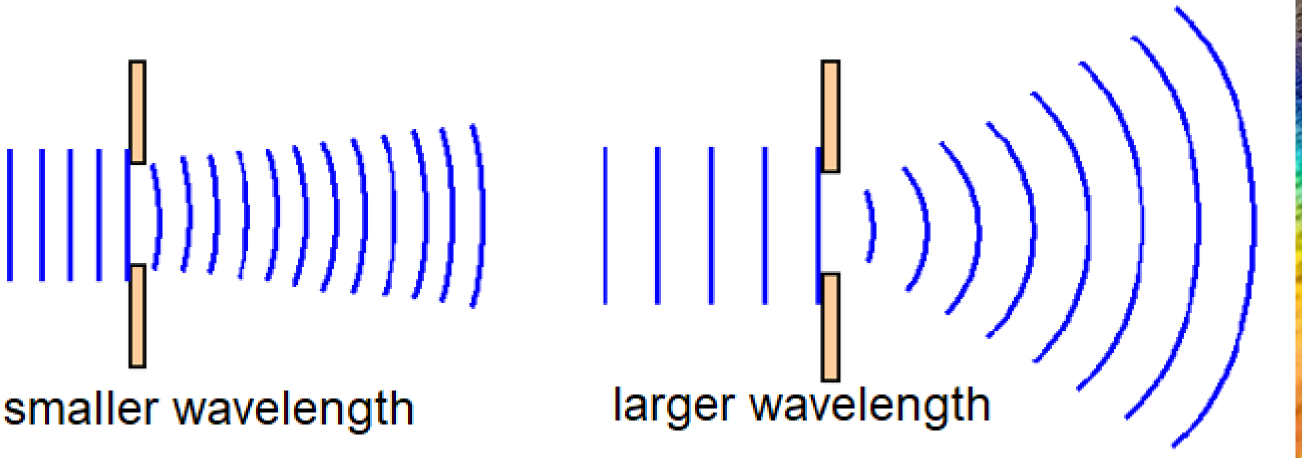
Effect of Aperture

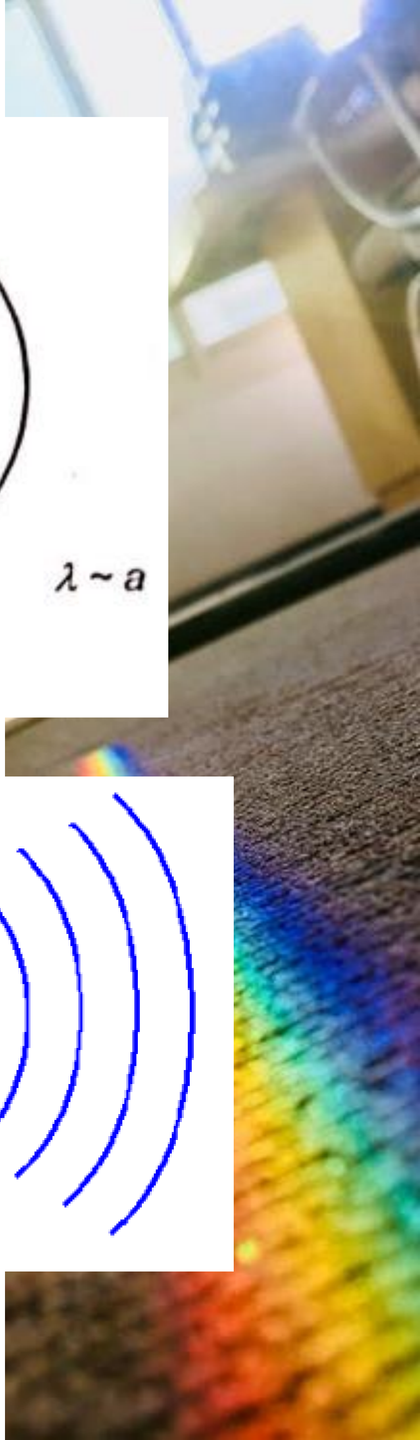
- Diffraction is more pronounce when the **aperture** is **narrower**.



Effect of Wavelength

- Diffraction is more pronounced when the **wavelength** of the wave is **larger.**





Diffraction

Generally

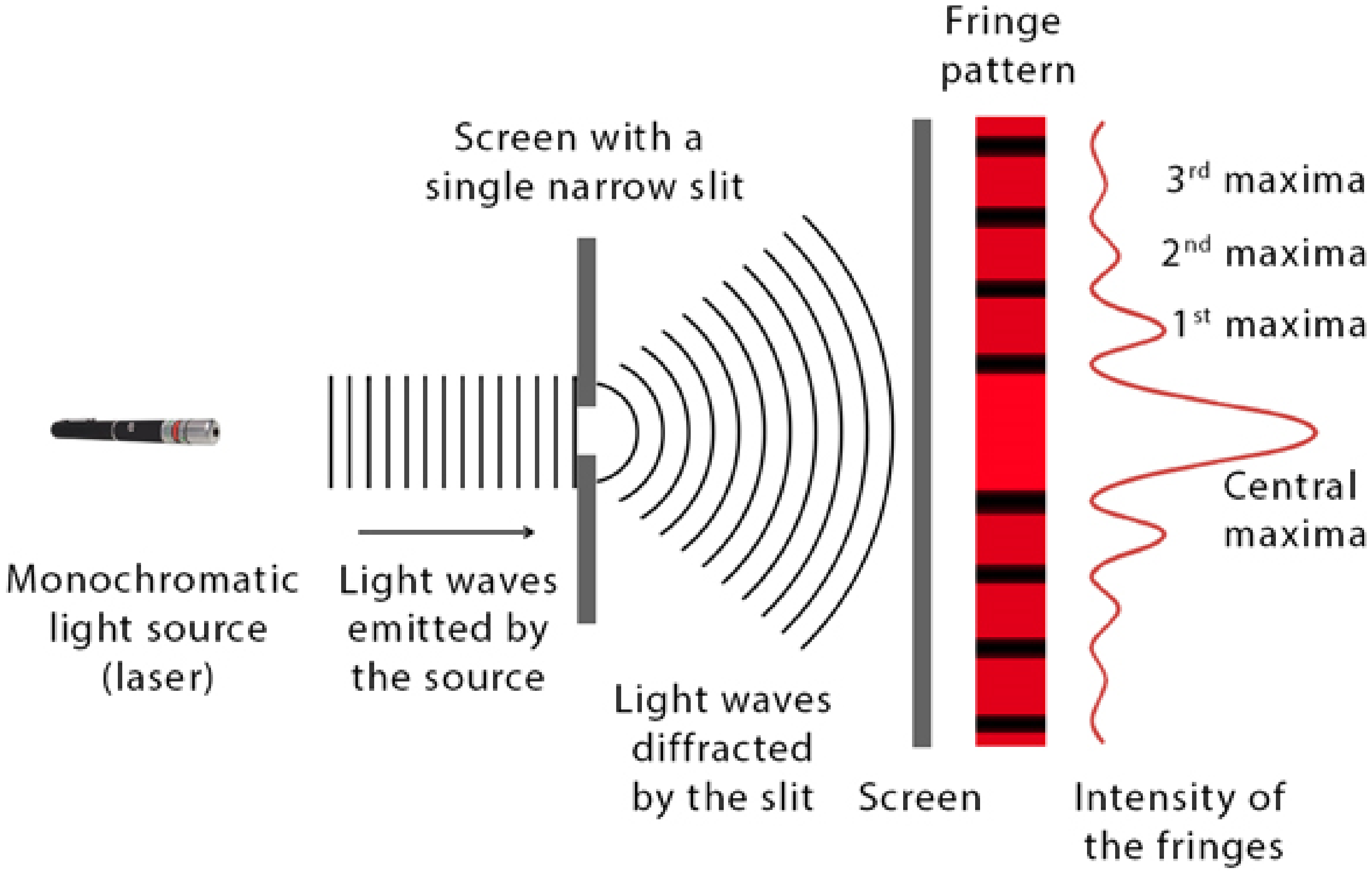
- Diffraction is pronounced when the wavelength if the wave is the same order of magnitude as the with of the aperture or obstacle.

$$\lambda \sim a$$

where $a =$ gap width or length of the obstacle.

Single-Slit Diffraction

As a light beam from a laser, passes through the single aperture, the light **diffracts**, and an intensity pattern is seen on the screen.



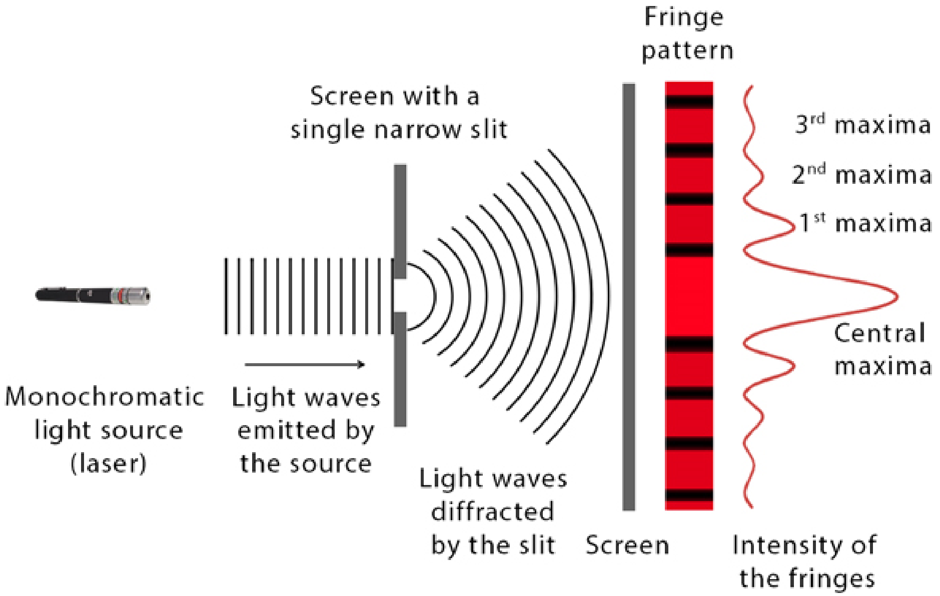
The **bright regions (maxima)** in the interference fringes are where each fray from the slit interfere constructively, whereas the **dark regions (minima)** are where each ray interfere destructively.

Single-Slit Diffraction

$$d \sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \pm 3$$

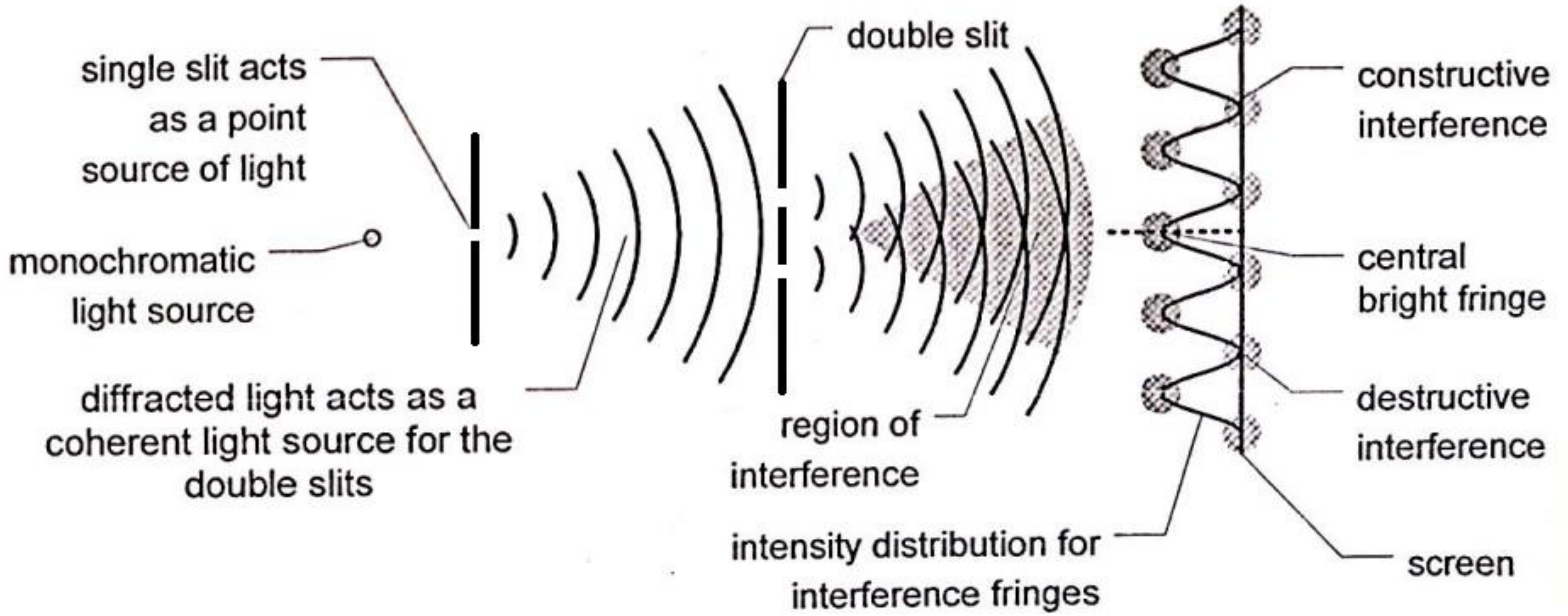
(minima/dark fringes)

where d = slit width,
λ = light's wavelength,
θ = angle relative to the original direction of light, m = order of the minimum



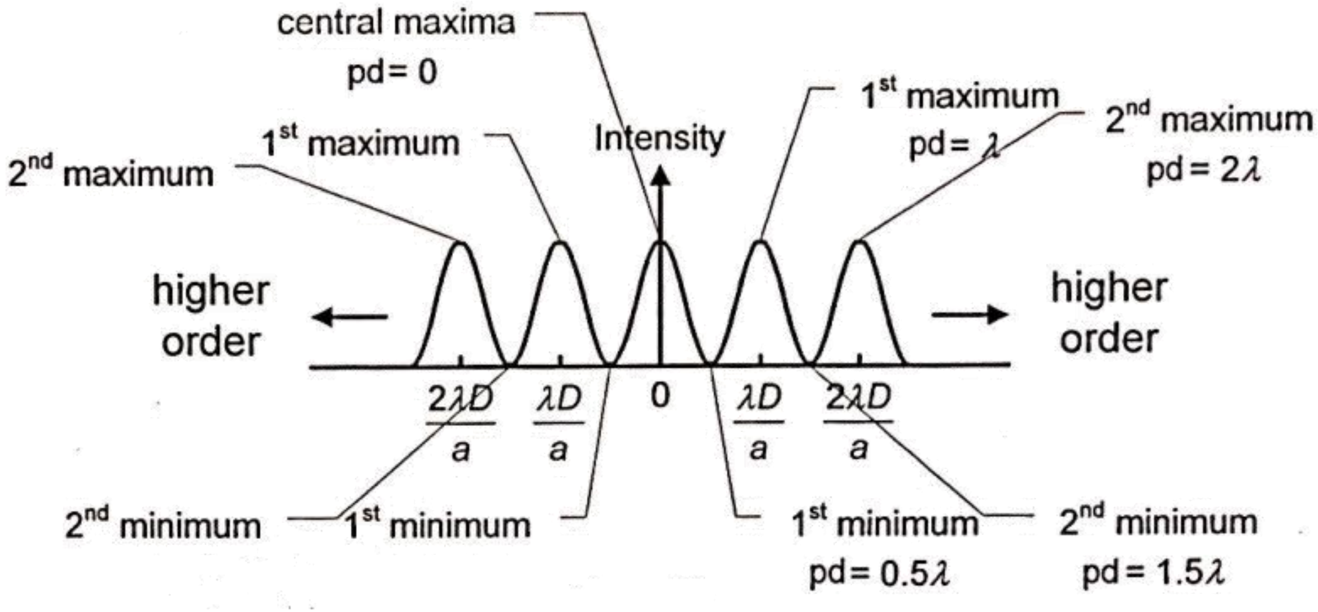
Two-Source Interference Pattern

By having 2 point sources, **S1** and **S2**, the interference patterns of waves can be studied. In Young's double slit experiment, we are able to observe the interference pattern of light, as alternating bright and dark fringes are seen on the screen.



- At points of constructive interference (maxima), bright fringes are observed.
- At points of destructive interference (minima), dark fringes are observed

Two-Source Interference Pattern



$$x = \frac{\lambda D}{a}$$

x = fringe separation
a = slit separation,
λ = light's wavelength,
D = distance between slits and screen

Formula is valid only if rays are parallel ($\boldsymbol{a} \ll \boldsymbol{D}$) and slit width is much greater than light wavelength ($\boldsymbol{a} \gg \boldsymbol{\lambda}$).

Practice Example 5

Explain why we can hear a person round a corner but not see a person.

Hint: Compare the wavelengths of sound & light.

Practice Example 6

A red laser (750 nm) is shone onto a single-slit set-up. Upon passing through the slit, it creates its second diffraction minimum at ang angle 30° with respect to the direction of the incident light.

(a)Find the width of slit.

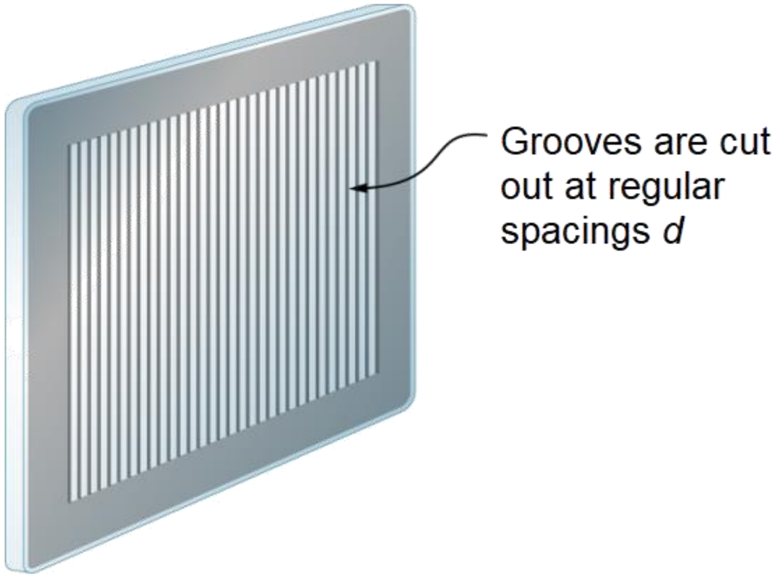
(b)At what angle is the first minimum produced?

Practice Example 7

In Young’s double-slit experiment, the separation between the first and the fifth bright fringe is 2.5 mm when the wavelength used is 620 nm. If the distance from the slit to the screen is 0.80 m, calculate the separation of the two slits.

Diffraction Grating

- consists of a large number of parallel, equally spaced lines (or slits) of equal width.
- typically consists of 100 to 1000 lines per mm.



$$\boxed{d \sin\theta = n\lambda}$$

where $d = \frac{1}{N}$ = slit separation (N= number of lines per meter)

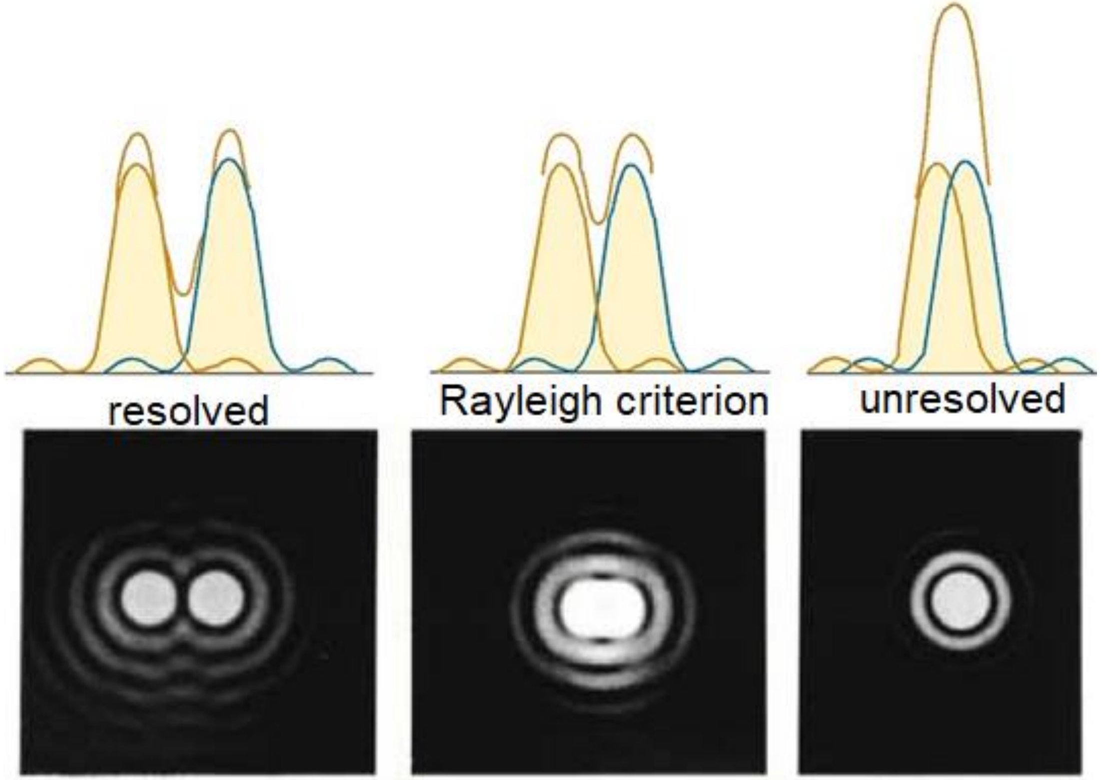
n = nth order of bright fringe (maxima)

λ = light's wavelength

θ = angle relative to the original direction of light

Rayleigh’s Criterion

- When two light sources are **very far away** from the observer, due to the diffraction pattern, the two light sources may **overlap** and appear as **one point source.**
- Rayleigh’s criterion talks about the minimum angle such that these 2 point sources can be told apart from one another and identified as **individual distinct sources.**



$$\boxed{\theta = \frac{\lambda}{b}}$$

where $\theta =$ minimum resolution angle
$\lambda =$ light’s wavelength,
$b =$ slit width

Practice Example 8

How many bright fringes can we observe using a diffraction grating of 600 lines per mm illuminated normally with light of wavelength 633 nm?

Practice Example 9

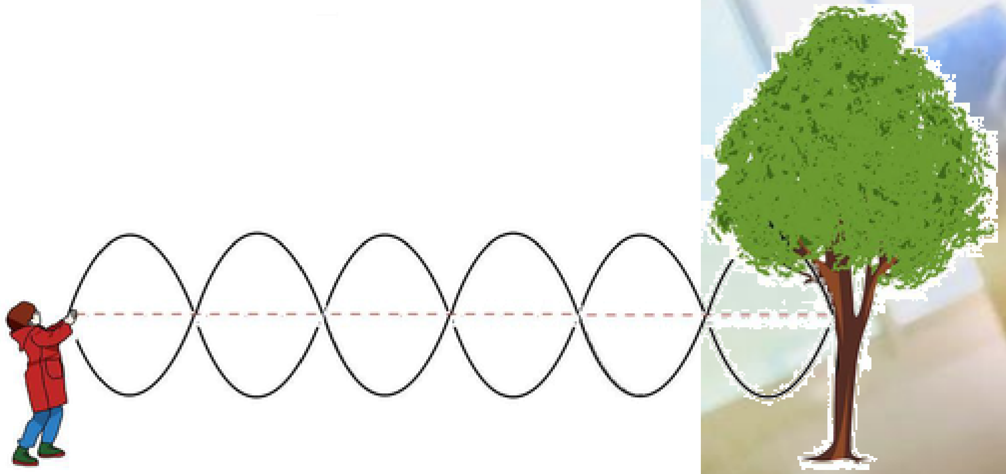
Two boats are stranded off in an island. Without any binoculars, the coast guards observe the boats, and their separation is just resolvable. If their pupils have diameter of 2.0 mm, what is the limit of resolution of the eyes? (Let the wavelength of light be 600 nm)

Write answer in degree and radian.

Suggested Solutions to Practice Example

Practice Example 1

As seen in the figure above, explain how the lady above managed to create a stationary wave from the string.



Answer:

As the woman moves her hand up and down, she creates a wave that travels to the direction of the tree. The tree is a boundary, so the wave is reflected back to the direction of the woman then superposing with the incident wave. The reflected wave has the same frequency, type, and amplitude as the incident wave. These conditions satisfy the VFAT conditions, thus a standing or stationary wave must be produced by the travelling waves generated by the woman.

Practice Example 2

Progressive waves of frequency 300 Hz are superposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive wave?

Solution:

$$v = f\lambda$$

$$v = (300)(1.5)$$

$$v = 450\ m/s$$

Practice Example 3

A 1.0-m long string of a piano weighs 8.00 g . The string vibrates at a fundamental frequency of 125 Hz.

(a) What is the speed of the wave produced by the string?

(b) Find the frequencies of the first five harmonics

Solution:

(a) At the fundamental, $\lambda = 2L = 2(1) = 2.0\, m$

$$v = f\lambda$$

$$v = (125)(2)$$

$$v = 250\, m/s$$

(b) 125 Hz, 250 Hz, 375 Hz, 500 Hz, 625 Hz.

Practice Example 4

An organ pipe, 0.33 long, is open at one end and closed at the other. The speed of sound in air is 330 m/s. Assuming that the end corrections are negligible, calculate

(a) the frequencies of the fundamental and the first overtone,

(b) the length of a pipe which is open at both ends and which has a fundamental frequency equal to the difference of those calculated in (a).

Solution:

(a) At the fundamental,

$$f_1 = \frac{v}{4L} = \frac{330}{4(0.33)} = 250\ Hz$$

First overtone,

$$f_3 = 3f_1 = 750\ Hz$$

(b) $f_1 = 750 - 250 = 500\ Hz$

$$\lambda_1 = \frac{v}{f_1} = \frac{330}{500} = 0.66\ m$$

$$\lambda_1 = 2L \rightarrow \qquad L = \frac{\lambda_1}{2} = \frac{0.66}{2} = 0.33\ m$$

Practice Example 5

Explain why we can hear a person round a corner but not see a person.

Hint: Compare the wavelengths of sound & light.

Answer:

Diffraction is more pronounced for sound waves because the wavelength of sound is much greater than the wavelength of light. This means sound waves ‘bend’ more to the extent that it reaches the person round a corner.

Practice Example 6

A red laser (750 nm) is shone onto a single-slit set-up. Upon passing through the slit, it creates its second diffraction minimum at ang angle 30° with respect to the direction of the incident light.

(a) Find the width of slit.

(b) At what angle is the first minimum produced?

Solution:

(a) $m = 2,\ \lambda = 750 \times 10^{-9}$ m, $\theta_2 = 30°$

$$d = \frac{m\lambda}{\sin\theta} = \frac{(2)(750 \times 10^{-9})}{\sin 30} = 3 \times 10^{-6} m = 3.0 \mu m$$

(b) $m = 1,\ \lambda = 750 \times 10^{-9}$ m, $d = 3 \times 10^{-6}$m

$$\sin\theta = \frac{m\lambda}{d} = \frac{(1)(750 \times 10^{-9})}{3 \times 10^{-6}} = 0.25$$

$$\theta_1 = 14.5°$$

Practice Example 7

In Young's double-slit experiment, the separation between the first and the fifth bright fringe is 2.5 mm when the wavelength used is 620 nm. If the distance from the slit to the screen is 0.80 m, calculate the separation of the two slits.

Solution:

Between the 5th and the 1st bright fringe, there are 4 fringes. So, we divide 2.5 mm by 4 to get the fringe separation x.

$$x = \frac{2.5 \times 10^{-3}}{4} = 0.625\,mm$$

$$a = \frac{\lambda D}{x} = \frac{(620 \times 10^{-9})(0.8)}{0.625 \times 10^{-3}} = 0.80 \times 10^{-3} m = 0.80\,m$$

Practice Example 8

How many bright fringes can we observe using a diffraction grating of 600 lines per mm illuminated normally with light of wavelength 633 nm?

Solution:

$$d\sin\theta = n\lambda \quad \Rightarrow n = \frac{d\sin\theta}{\lambda}$$

$\Rightarrow n < \frac{d}{\lambda}$ since sine function ranges from 0 to 1 $(0 < \theta < 90)$

$$d = \frac{1}{N} = \frac{1mm}{600} = 1.67 \times 10^{-6}$$

$$n < \frac{1.67 \times 10^{-6}}{633 \times 10^{-9}} = 2.63$$

n must be an integer so n must be 2. This means that there are 2 bright fringes on either side. Including the central fringe, there is a total of **5 bright fringes.**

Practice Example 9

Two boats are stranded off in an island. Without any binoculars, the coast guards observe the boats, and their separation is just resolvable. If their pupils have diameter of 2.0 mm, what is the limit of resolution of the eyes? (Let the wavelength of light be 600 nm)

Write answer in degree and radian.

Solution:

$$\theta = \frac{\lambda}{n} = \frac{600 \times 10^{-9}}{2 \times 10^{-3}} = 0.30\, mrad\ or\ 0.017°$$

Convert radian in degree by multiplying $180/\pi$.