Mid-Year Examination Mock Paper 2022
SECONDARY 4 EXPRESS / NORMAL ACADEMIC

CANDIDATE NAME

CENTRE

ADDITIONAL MATHEMATICS — 4049/01

Mock Paper — April/May 2022

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name and centre in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 siginificant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
There are 2 sections in this paper. Section A will contain Paper 1 (short-answered) questions, Section B will contain Paper 2 (longer problem sum) questions
The total number of marks for this paper is 100.

Additional Instructions:

- Question **11** is a bonus question. Attempt only if you have completed the paper :)
- This is a **RESTRICTED OPEN BOOK** examination.
  Candidates are allowed **1 handwritten A6 double-sided** help-sheet

Setter: **Kaiwen :)**

This question paper consists of 25 printed pages including the cover page

Grade Tables: For Examiner's Use

Section A

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	9	13	10	10	10	6	11	9	12	10	100
Score:											

Bonus Question

Question:	11	Total
Bonus Points:	3	3
Score:		

Total Score

Total Score	Deductions	Grade

Examiner's Comments:

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

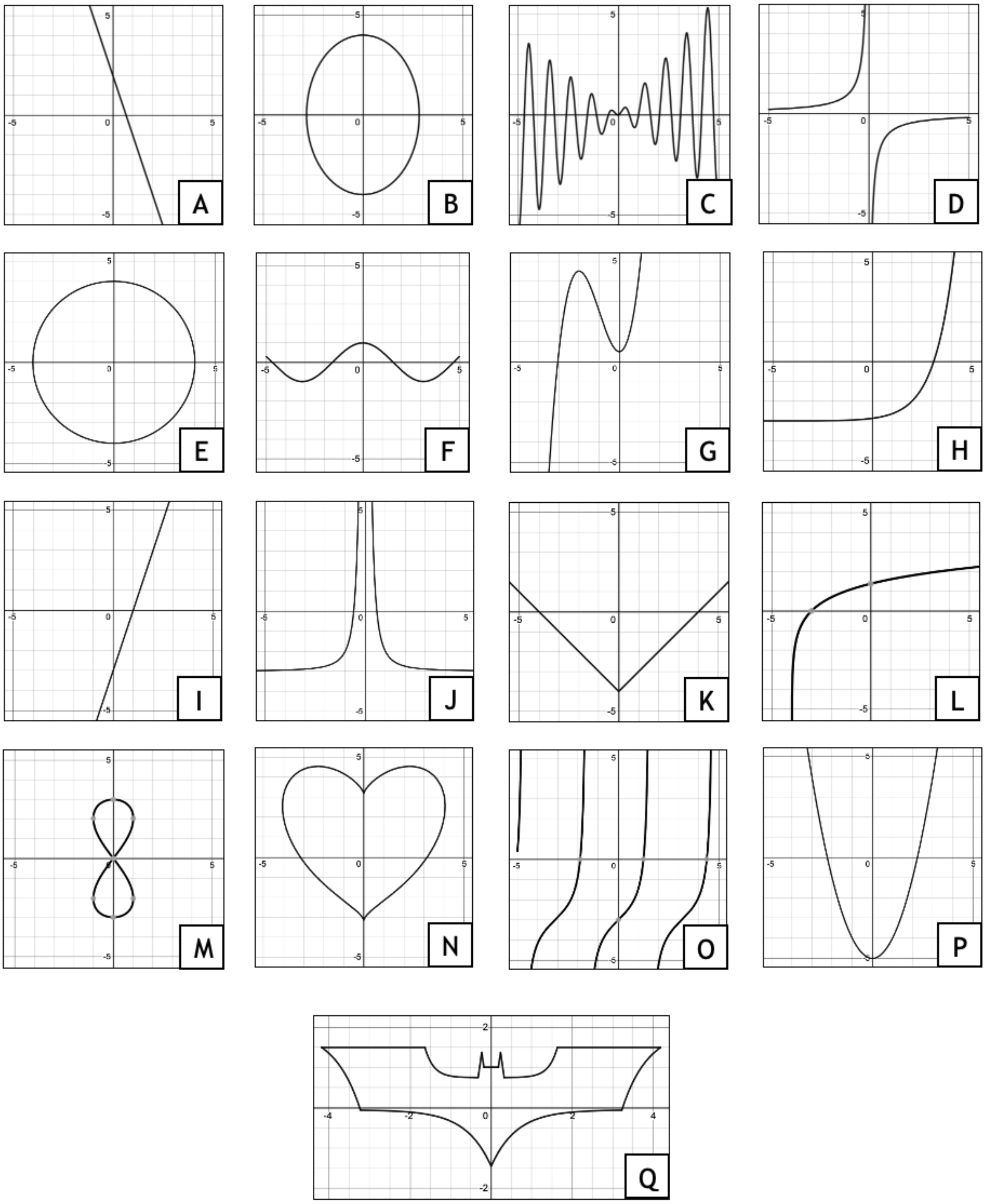
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. Each of the **17** plots shown below, labelled A to Q, show graphs of varying equations.
   Answer the following questions on the next page



(a) Identify which of the following graphs satisfy the following

(i) Which of the following graphs have only **1** line of symmetry?
[Hint: 5 graphs]

Answer ________________________ [1]

(ii) Which of the following graphs have asymptotes?
[Hint: 5 graphs]

Answer ________________________ [1]

(b) (i) Identify which of the following graphs correspond to the general form equations provided

(a) $y = ax^3 + bx^2 + cx + d$

Answer ________________________ [1]

(b) $y = \tan x$

Answer ________________________ [1]

(c) $y = -mx + c, \quad m < 0$

Answer ________________________ [1]

(ii) Identify the general form equations with the graphs provided

(a) Graph E

Answer ________________________ [1]

(b) Graph F

Answer ________________________ [1]

(c) Graph H

Answer ________________________ [1]

(d) Graph L

Answer ________________________ [1]

2. (a) Find the range of values of the constant k for which the curve lies entirely below the line

$$y = -x^2 + (1 - k)x - 2$$

$$x + y = 0$$

Answer ________________________ [4]

(b) Find the values of x and y which satisfy the following equations

$$\frac{1}{\sqrt{e^{2x-4y}}} = \frac{\sqrt[3]{e}}{e}$$

$$\frac{10^y}{2^x} = 2\left(5^{x+1}\right)$$

Answer ________________________ [4]

(c) Express the following in partial fractions

$$\frac{2x^3 - 20x^2 - 17x - 10}{(x^2 - 4)(2x^2 + 1)}$$

Answer ________________________ [5]

Credit: **[S4 ANDSS 2020 PRELIM P1: Qn 2, 10(a); P2: 6(a)]**

3. (a) Given that

$$p = 3^x \qquad q = 3^y$$

Express the following in terms of x and y

$$\log_3 \frac{p^7 q}{243}$$

Answer ________________________ [5]

(b) Find the value of x given that

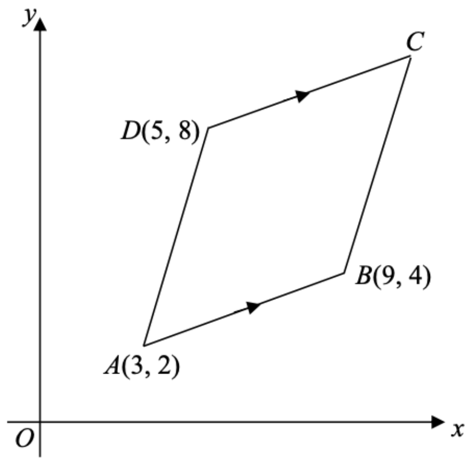
$$\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$$

Leave your answer in exact form

Answer ________________________ [5]

Credit: **[S4 AHS P2/2020 PRELIM Qn 3]**

4. The diagram shows a quadrilateral with vertices $A(3, 2)$, $B(9, 4)$, C and $D(5, 8)$.



The sides AB and DC are parallel

(a) Find the equation of DC

Answer ______________________ [2]

(b) Find the equation of the perpendicular bisector of BD

Answer ______________________ [3]

(c) The coordinates of C are equidistant from B and D. Find the coordinates of C

Answer ________________________ [3]

(d) Find the area of the quadrilateral $ABCD$

Answer ________________________ [2]

Credit: **[S4 BGSS P2/2020 PRELIM Qn 9]**

5. (a) It is given that

$$f(x) = \ln \sqrt[3]{\frac{5+x}{5-x}}$$

(i) Find $f'(x)$ and $f''(x)$

Answer ______________________ [4]

(ii) Hence, determine the range of values of x for which both $f'(x)$ and $f''(x)$ are positive

Answer ________________________ [3]

(b) Find the value of k

$$\frac{d}{dx}\left[4\sin^2\left(\frac{x}{2}+\pi\right)\right] = k\sin x$$

Answer ________________________ [3]

Credit: **[S4 BPGHS P2/2020 PRELIM Qn 5]**

6. (a) For all x and y values, show that

$$x^{\frac{1}{3}} - y^{\frac{1}{3}} = \frac{x - y}{x^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}}}$$

[Hint: Cubic Identities]

[2]

(b) **Without the use of a calculator**, evaluate the following

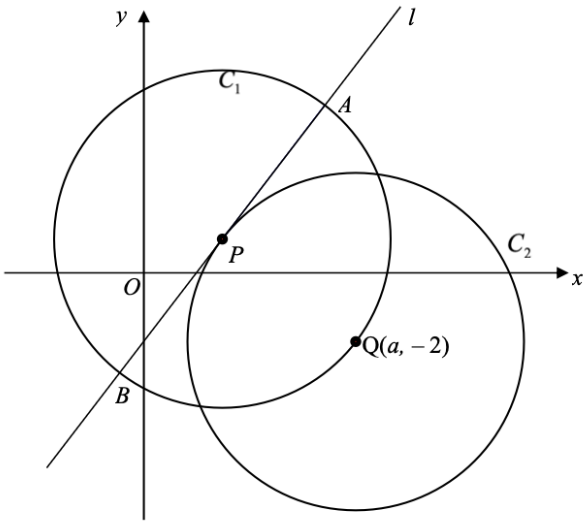
$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{2021}+\sqrt{2022}}$$

Leave your answer in the simplest and exact form

[Hint: Rationalisation]

Answer ________________________ [4]

7. The diagram shows two circles, C_1 and C_2.



Circle C_1 has its centre at P which lies on the circumference of circle C_2. Circle C_2 has its centre at $Q(a, -2)$, where a is a constant and Q lies on the circumference of circle C_1. Line l is a tangent to circle C_2 at P. The line l cuts the circle C_1 at points A and B. The equation of circle C_1 is

$$x^2 + y^2 - 4x - 2y - 20 = 0$$

(a) Find the coordinates of P and the radius of C_1

Answer ________________________ [3]

(b) Find the value of a

Answer ________________________ [2]

(c) Find the equation of C_2

Answer ________________________ [1]

(d) Find the equation of l

Answer ________________________ [2]

(e) Hence, or otherwise, find the x-coordinates of A and B

Answer ______________________ [3]

Credit: **[S4 HIHS P2/2020 PRELIM Qn 10]**

8. A polynomial has the following equation, where a and b are constants

$$f(x) = 2x^3 - 5x^2 + ax + b$$

This polynomial leaves a remainder of -4 when divided by $(x - 1)$. The graph of $y = f(x)$ has a stationary point at $x = 2$

(a) Show that $a = -4$ and $b = 3$

[5]

(b) Show that $(x + 1)$ is a factor of $f(x)$

[1]

(c) Solve the equation $f(x) = 0$

Answer ________________________ [3]

Credit: **[S4 CCHS(M) P1/2020 PRELIM Qn 3]**

9. (a) A is an obtuse angle and B is a reflex angle such that

$$\tan A = -\frac{3}{4} \qquad \cos B = \frac{15}{17}$$

(i) State which quadrants are A and B in

Answer ______________________ [1]

Without finding the values of A and B, evaluate

(ii) $\sec A$

Answer ______________________ [2]

(iii) $\sin(A + B)$

Answer ______________________ [2]

(b) (i) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x$$

[3]

(ii) Hence, for $0 \le x \le \pi$, solve the equation

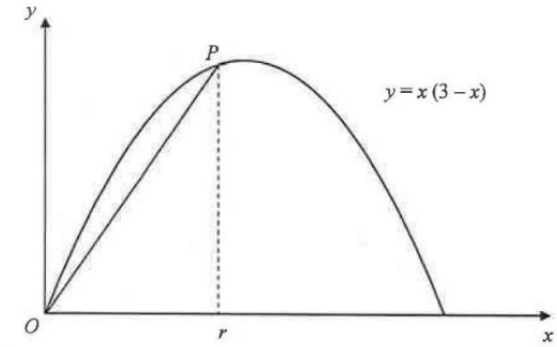
$$\frac{1}{2}(\operatorname{cosec} 2x + \cot 2x) = \cos x$$

Answer ________________________ [4]

Credit: **[S4 HSS 2020 PRELIM P1: Qn 5; P2: Qn 1]**

10. The diagram shows the trajectory of an athelete during a long jump which can be represented by the equation, where x and y are the horizontal distance and vertical height of the jump respectively

$$y = x(3 - x)$$



O is the point where the athlete takes-off from the ground. When $x = r$ m, the athlete is at point P

(a) Show that the distance, s m, between O and P is given by

$$s = \sqrt{r^4 - 6r^3 + 10r^2}$$

[3]

(b) Show that

$$\frac{ds}{dr} = \frac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}}$$

[2]

(c) Given that r can vary, find the values of r for which s is stationary

Answer ______________________ [3]

(d) Determine the nature of the smaller of these values of r

Answer ______________________ [2]

Credit: **[S4 P1/JWSS 2020 PRELIM Qn 1]**

11. **BONUS QUESTION**

Completing the Cube (Vieta's Substitution)
The cubic formula gives the roots of any cubic equation

$$ax^3 + bx^2 + cx + d = 0, \qquad a \neq 0$$

Just like completing the square, these roots can be found by "completing the cube" using Vieta's Substitution. Let the roots of the cubic be x_1, x_2 and x_3. A system of 3 equations can be written as the following

$$x_1 + x_2 + x_3 = -\frac{b}{a} \qquad x_1x_2 + x_1x_3 + x_2x_3 = \frac{c}{a} \qquad x_1 \cdot x_2 \cdot x_3 = -\frac{d}{a}$$

Let x, y and z be positive real numbers with $1 < x < y < z$ such that

$$\log_x y + \log_y z + \log_z x = 8 \qquad \log_x z + \log_z y + \log_y x = \frac{25}{2}$$

The value of $\log_y z$ can be written as $\frac{p + \sqrt{q}}{r}$ for positive integers p, q and r and such that q is not divisible by the square of any primes. Show that

$$p + q + r = 42$$

[Hint: To solve the above question, you will need Vieta's Substitution]

[3 (bonus)]

Credit: **SMT 2021 Algebra Paper Qn 7 (Modified)**

END OF PAPER