GCSE Preliminary Examination Mock Paper 2022
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

CANDIDATE NAME

CENTRE

ADDITIONAL MATHEMATICS 4049/02

Mock Paper 2 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name and centre in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 siginificant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

Grade Tables: For Examiner's Use

Total Score	**Deductions**			**Grade**
	Rounding	Units	Presentation	

Setter: **Kaiwen :)**

This question paper consists of 25 printed pages including the cover page

Grade Tables: For Examiner's Use

Question	Points	Score
1	6	
2	12	
3	11	
4	6	
5	10	
6	6	

Question	Points	Score
7	9	
8	6	
9	6	
10	12	
11	6	
Total:	90	

Examiner's Comments

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Answer all questions: 90 marks

1. (a) Find a positive number, x when added to its reciprocal gives a minimum sum

Answer _____ [3]

(b) Calculate the maximum value of $3uv^2$ given that u and v are two variables such that $u + v = 15$

Answer _____ [3]

2. (a) Given that m is a positive integer, find the value of m where

$$\frac{15^{2k} \times 9^{4k} \times 5^{6k}}{3^{2k}} = m^{8k}$$

Answer _____ [3]

(b) Without using a calculator, find the value of k such that

$$\left(\frac{4}{\sqrt{3}} + \frac{2\sqrt{15}}{3} - \frac{8}{\sqrt{12}}\right) \times \sqrt{6} = 2\sqrt{k}$$

Answer _____ [3]

(c) The lengths of the diagonals PR and QS of a rhombus $PQRS$ are $\left(4 + 2\sqrt{3}\right)$ cm and $\left(6 + \frac{4}{\sqrt{3}}\right)$ cm respectively. Leaving your answers in surd form, find

(i) the value of PQ^2

Answer _____ [3]

(ii) the area of the triangle PQR

Answer _____ [3]

3. (a) Solve the following equations

(i)

$$4^x = 7^{x-1}$$

Answer _____ [3]

(ii)

$$2\log_4 5x^2 - \log_8(4-x)^3 = 1 + \log_2(1-x)$$

Answer _____ [4]

(b) Sketch the graph of $y = 2e^{-x}$ and $y = 3 - e^{x}$ on the same diagram. Find the x-coordinate of the points of intersection of the two graphs

Answer _____ [4]

4. (a) The coefficient of x^3 in a cubic polynomial $f(x)$ is -3 and the roots of the equation $f(x) = 0$ are $\frac{1}{3}$, -1 and k. Given that $f(x)$ has a remainder of 28 when divided by $(x+2)$, find the value of k

Answer _____ [3]

(b) When a cubic polynomial $g(x)$ is divided by $(x - 1)$ and $(x + 4)$, it leaves a remainder of -12 and -37. When $g(x)$ is divided by $x^2 + 3x - 4$, the remainder is $(ax + b)$. Find the value of a and b

Answer _____ [3]

5. (a) A curve has the equation

$$y = (ax + 3)\ln x \quad x > 0, a > 0$$

The normal to the curve at the point $x = e$ is parallel to the line $y + x = 2$. Find the value of a, giving your answer in terms of e

Answer _____ [4]

(b) (i) Express the following as a sum of 3 partial fractions

$$\frac{8x + 1}{(2x - 5)(x + 1)^2}$$

Answer _____ [3]

(ii) Express y in terms of x given that $\frac{dy}{dx} = \frac{8x + 1}{(2x - 5)(x + 1)^2}$ and $y = \frac{5}{14}$ when $x = 6$

Answer _____ [3]

6. (a) The displacement, at time t, of a particle moving along the x-axis is given by

$$s = 2e^{3t} + 5e^{-3t}$$

Find the value of t when the particle comes to instantaneous rest

Answer _____ [3]

(b) Given that

$$y = (\ln x)^2$$

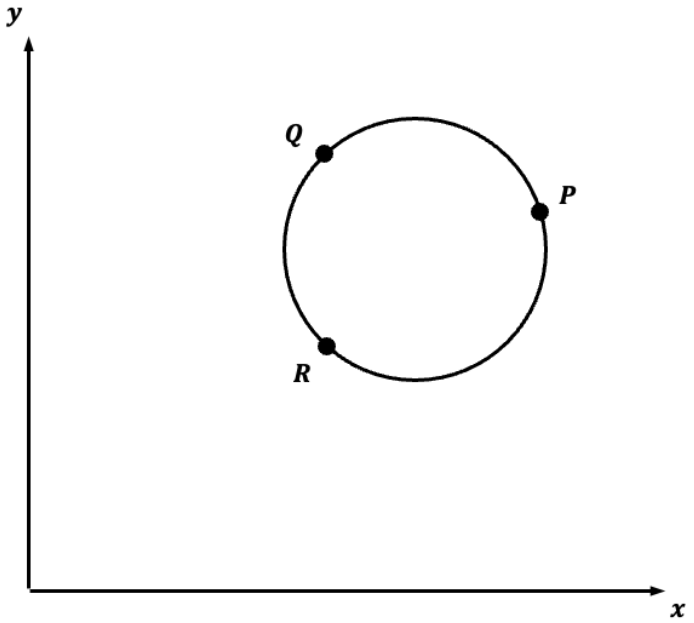
show that

$$x^2\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right) - 2 = 0$$

[3]

7. P, Q and R are three points on a circle as shown, with coordinates given by

$$P\left(5+\sqrt{2}\cos\theta, 5+\sqrt{2}\sin\theta\right) \qquad Q\left(5-\sqrt{2}\cos\theta, 5+\sqrt{2}\cos\theta\right) \qquad R(4,4)$$



(a) Show that

$$RP^2 = 4 + 2\sqrt{2}\sin\theta + 2\sqrt{2}\cos\theta$$

[3]

(b) Express $2\sqrt{2}\sin\theta + 2\sqrt{2}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Answer _____ [2]

(c) **Hence**, determine the maximum value of RP

Answer _____ [1]

(d) Given that

$$RQ^2 = 4 - 4\sin\left(\theta - \frac{\pi}{4}\right)$$

find all values of θ satisfying $0 \leq \theta < 2\pi$ such that

$$RP = RQ$$

Answer _____ [3]

8. (a) Show that

$$\frac{d}{dx}\left(\tan x \sin^2 x\right) = 2\sin^2 x + \sec^2 x - 1$$

[3]

(b) **Hence**, find the value of the following, leaving your answer in exact form

$$\int_{\frac{\pi}{4}}^{\pi} \sin^2 x \, dx$$

Answer _____ [3]

9. The equation of a curve is given as the following, where k is a constant

$$y = -2x^2 - 3kx + k$$

(a) Find the set of values of k, for which the curve lies completely below the line $y = 5k$

Answer _____ [3]

(b) In the case where $k = 1$, find the shortest distance between the curve and the line

Answer _____ [3]

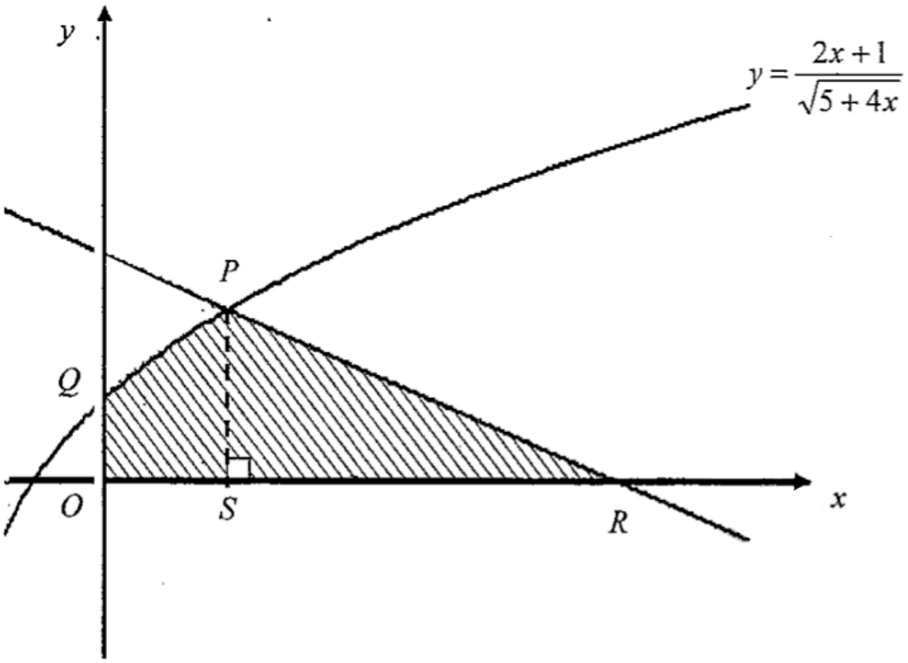
10. (a) Show that

$$\frac{d}{dx}\left[(x-1)\sqrt{5+4x}\right] = \frac{6x+3}{\sqrt{5+4x}}$$

[3]

(b) The diagram shows part of the curve

$$y = \frac{2x + 1}{\sqrt{5 + 4x}}$$



The line PR is a normal to the curve at P. Q is the point where the curve cuts the y-axis and S is a point directly below P

(i) Given that the x-coordinate of P is 1, find the equation of the line PR

Answer _____ [4]

(ii) Without calculating the area under the curve from $x = 0$ to $x = 1$, explain briefly why

$$\int_0^1 \frac{2x+1}{\sqrt{5+4x}}\,dx > \frac{1}{2}\left(1+\frac{1}{\sqrt{5}}\right)$$

[2]

(iii) Find the area of the shaded region

Answer _____ [3]

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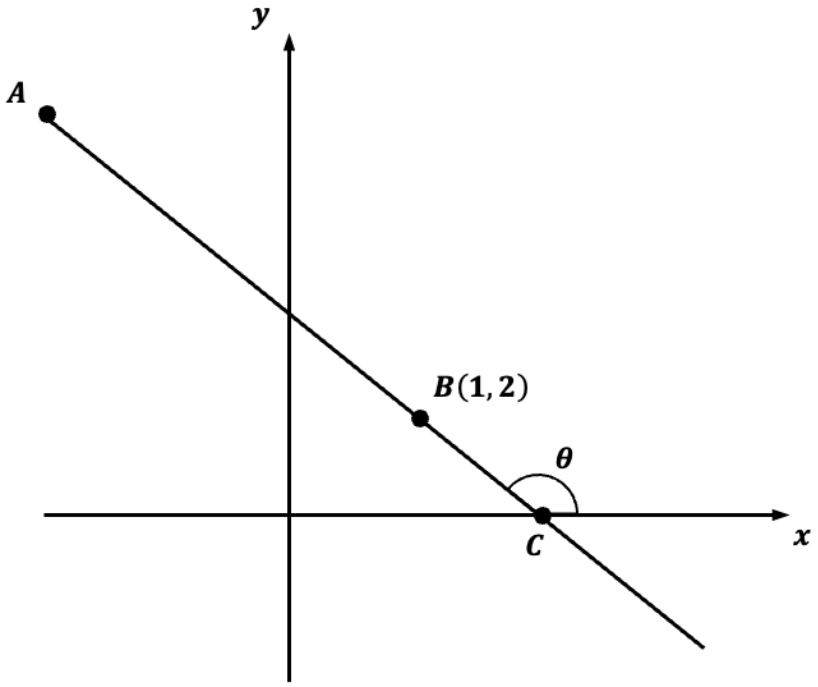
11. The diagram shows a straight line ABC such that $AB : BC = 3 : 1$



The point B is $(1, 2)$ and the point C lies on the x-axis. θ is the angle between the positive x-axis and the line AC. Given that $\tan\theta = -2$, find

(a) the equation of the line AC

Answer _____ [1]

(b) the coordinates of C and A

Answer _____ [3]

The point D is such that $ABOD$ is a parallelogram

(c) Find the coordinates of D

Answer _____ [2]

END OF PAPER

Question Source

Question	Credit	Remarks
1(a)	S4 P1/AHS 2017 PRELIM Qn 11	Modified
1(b)	S4 P2/CHIJ Sec 2017 PRELIM Qn 5	-
2	S4 P2/WGSS 2017 PRELIM Qn 9	Modified
3	S4 P2/FMS(S) 2017 PRELIM Qn 9	-
4	S4 P2/NTSS 2017 PRELIM Qn 8	-
5	S4 P2/PHS 2017 PRELIM Qn 5	-
6	S4 P1/BPGHS 2016 PRELIM Qn 6	-
7	S4 P2/RI 2012 PRELIM Qn 12	Modified
8	S4 P2/XMSS 2017 PRELIM Qn 2	-
9	S4 P1/YHSS 2017 PRELIM Qn 6	-
10	S4 P2/BHSS 2016 PRELIM Qn 9	-
11	S4 P1/HIHS 2016 PRELIM Qn 4	-