Topic 14: Differentiation (4049)

THE ABOUT

CHAPTER ANALYSIS

- Algebraic, Exponential, Trigonometric and Logarithmic first and second derivatives
- Application of Differentiation
    - Gradient, Tangent & Normal
    - Rate of Change
    - Increasing & Decreasing Functions
    - Stationary Points
    - Maxima & Minima

MASTERY

- Moderate difficulty chapter, students must get their fundamentals right with the basic differentiation rules
- 6 **key** concepts

EXAM

- Concepts usually tested as a stand-alone topic
- Generally, many students will make mistakes during the derivative, check all work carefully

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, 40% of the papers will be Calculus topics

KEY CONCEPT

General Rules of Differentiation
Trigonometric Differentiation
Exponential & Logarithmic Differentiation
Second or Higher Derivatives

General Rules of Differentiation

If $y = ax^n$ and a is a constant, n is an integer or rational number	
Rules	**Formulae**
Power Function	$\frac{d}{dx}(ax^n) = anx^{n-1}$
Linear Function	$\frac{d}{dx}(ax) = a$
Constant Function	$\frac{d}{dx}(a) = 0$
Chain Rule	$\frac{d}{dx}[k(ax+b)^n] = kn(ax+b)^{n-1} \cdot (a)$

If u and v are functions of x	
Rules	**Formulae**
Sum Rule	$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
Difference Rule	$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$
Product Rule	$\frac{d}{dx}(uv) = u\left(\frac{dv}{dx}\right) + v\left(\frac{du}{dx}\right)$
Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$

Trigonometric Differentiation

$y = \sin x$
Rules
$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}[a\sin(bx+c)] = ab\cos(bx+c)$
$\frac{d}{dx}[a\sin^n(bx+c)] = anb\sin^{n-1}(bx+c)\cos(bx+c)$

$y = \cos x$
Rules
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}[a\cos(bx+c)] = -ab\sin(bx+c)$
$\frac{d}{dx}[a\cos^n(bx+c)] = -anb\cos^{n-1}(bx+c)\sin(bx+c)$

$y = \tan x$
Rules
$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}[a\tan(bx+c)] = ab\sec^2(bx+c)$
$\frac{d}{dx}[a\tan^n(bx+c)] = anb\tan^{n-1}(bx+c)\sec^2(bx+c)$

Exponential & Natural Logarithmic Differentiation

$y = e^x$
Rules
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}[e^{ax+b}] = ae^{ax+b}$
$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^{f(x)}$

$y = \ln x$
Rules
$\frac{d}{dx}(\ln x) = \frac{1}{x}$
$\frac{d}{dx}[\ln(ax+b)] = \frac{a}{ax+b}$
$\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$

Second or Higher Derivatives

A second derivative of y is obtained by differentiating y twice

$$f''(x) \qquad \frac{d^2y}{dx^2}$$

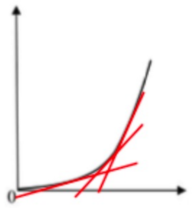
A nth derivative of y is obtained by differentiating y n times

$$f^{(n)}(x) \qquad \frac{d^ny}{dx^n}$$

KEY CONCEPT

Applications of Differentiation

Why is the gradient increasing for the graph?

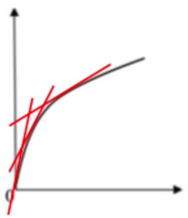


Increasing Function

The added lines to the graph are tangent lines (we add tangent lines to help find the gradient of a particular point on the curve [concept from E-Math]). Finding the gradient of these tangent lines will tell us about the overall general curvature (gradient profile) of the curve

Based on the drawn tangent lines, we can see that the gradient is slowing increasing and getting steeper, implying that the gradient is increasing

Why is the gradient decreasing for the graph?



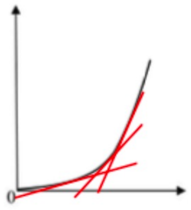
Decreasing Function

Similar reasonings above. Based on the drawn tangent lines, we can see that the gradient is slowly getting more and more shallow, implying that the gradient is decreasing

Application 1: Increasing & Decreasing Functions

A differential function $y = f(x)$ is known to be an increasing function if

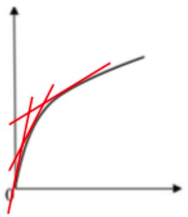
$$\frac{dy}{dx} > 0$$



Increasing Function

A differential function $y = f(x)$ is known to be an increasing function if

$$\frac{dy}{dx} < 0$$



Decreasing Function

Application 2: Stationary Point

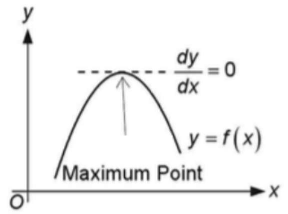
The point on the curve of a function $y = f(x)$ where

$$\frac{dy}{dx} = 0$$

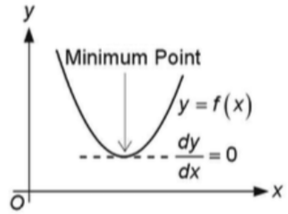
Nature of Stationary Points

There are 3 types of stationary points that we need to know for the **4049 'O' Level Syllabus**

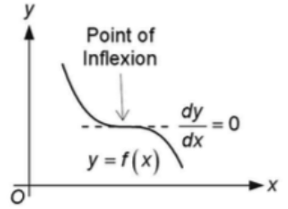
- Maximum point



- Minimum point



- Point of Inflexion



Determining Nature of Stationary Points

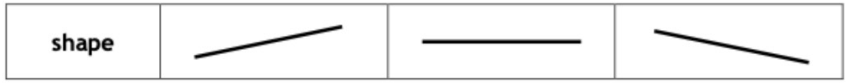
First Derivative Test

1. Draw this table into your solutions

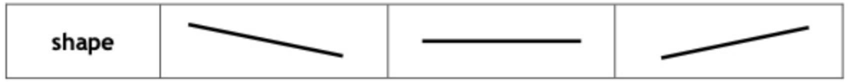
x-value	$(x - 0.01)$	x	$(x + 0.01)$
$\frac{dy}{dx}$ sign		$\frac{dy}{dx} = 0$	
shape			

2. Fill up the empty cells in the table above by substituting the x values ± 0.01 of the original x value into your derivative expression. The shape of the curve will tell you the point's nature

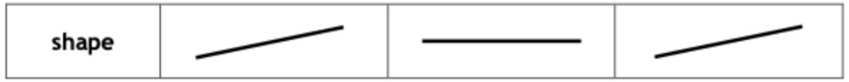
Maximum point



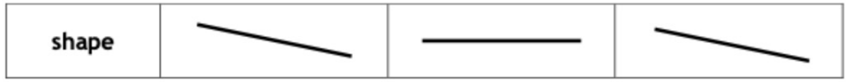
Minimum point



Point of inflexion point



Point of inflexion point



Second Derivative Test

1. Find the second derivative and substitute the x-value that you wish to test

Second Derivative value	Nature of point
$\frac{d^2y}{dx^2} > 0$	Minimum point
$\frac{d^2y}{dx^2} < 0$	Maximum point
$\frac{d^2y}{dx^2} = 0$	Inconclusive

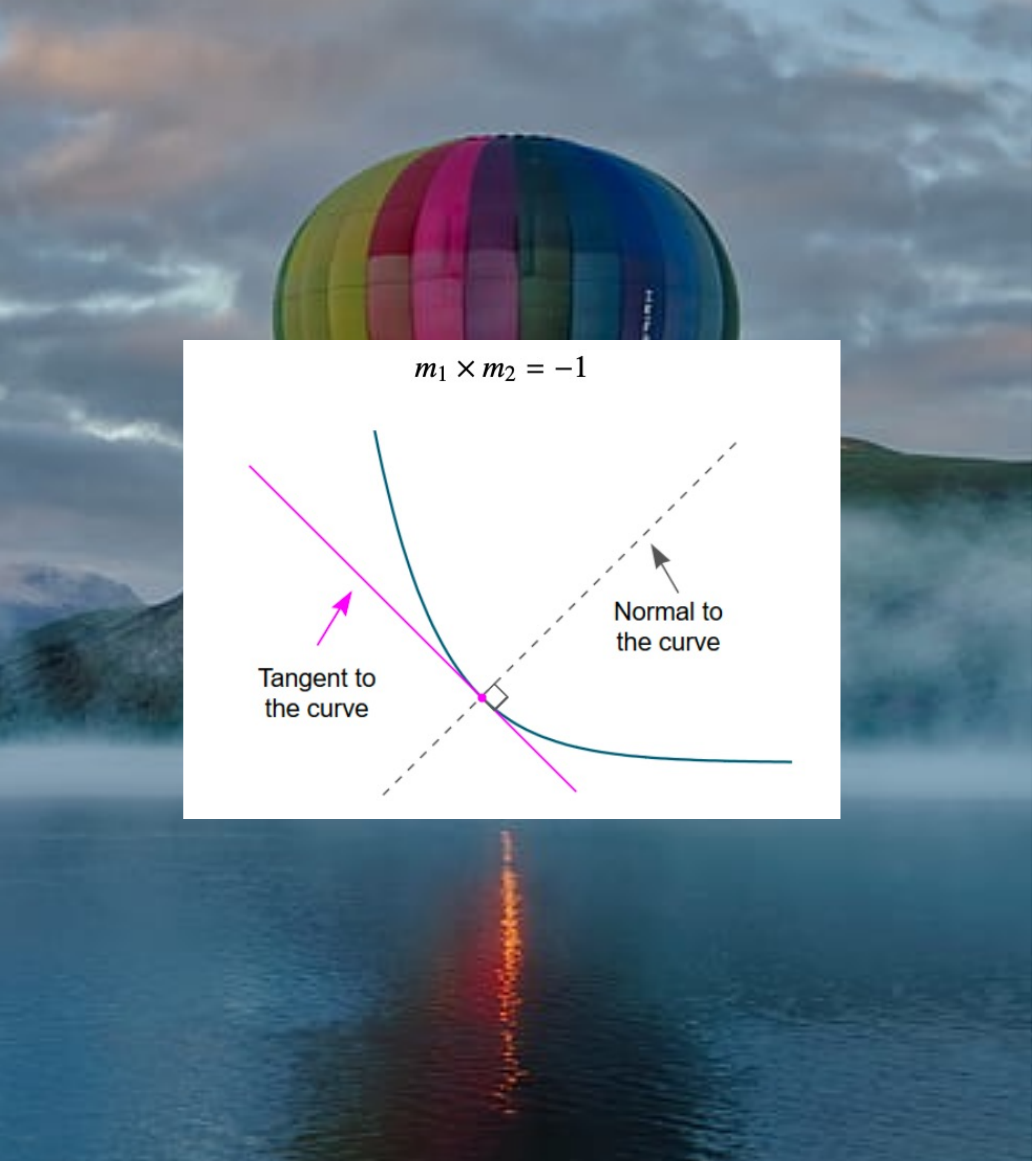
Take Note

Take note that when conducting the second derivative test, if students get that

$$\frac{d^2y}{dx^2} = 0$$

this means that the second derivative test has failed, and students must use back the first derivative test to find the nature of the point. Students must explicitly write in their answer script that the test is "inconclusive" and then shift over to the first derivative test

The first derivative test is foolproof and will always give you a result. However, the test is rather long to conduct. It is entirely to the discretion of the student as to which test he/she would like to conduct



Application 3: Gradients, Tangents & Normals

Gradient of a line can be found using 2 methods

- Reading off the equation of the line (if the equation is provided)

$$\boldsymbol{y = mx + c}$$

- Differentiating the equation of the line, and substituting the x-coordinate of the point in question, into the derivative

$$\left.\frac{dy}{dx}\right|_{x=\alpha}$$

A normal to a point cuts the same point as to where the tangent is taken from. This normal is also perpendicular to the tangent to the line of that point

Let the gradient of the tangent be $\boldsymbol{m}$ and the tangential point be $(\boldsymbol{x_1}, \boldsymbol{y_1})$	
Type of line	Equation of line
Tangent Line	$y - y_1 = m(x - x_1)$
Normal	$y - y_1 = -\dfrac{1}{m}(x - x_1)$

Application 4: Maxima & Minima

All concepts are same (as stationary point) where

$$\frac{dy}{dx} = 0$$

Important

- Do note that (unless the question states that it is unnecessary), students must prove that the value they have calculated will indeed give the maximum/minimum value. This is to get the full credit of the question
- Students can choose to use either the first derivative test, or second derivative test to prove the maxima/minima nature of the value

Application 5: Connected Rate of Change

Rate of Change questions are also very distinctive. Questions will always have the element of time, and have rates introduced in them

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

Tips on solving such questions:

Start the process by writing the term on the left and expanding it out to the right. The denominator of the first fraction on the right, and the numerator of the second fraction should be left blank

$$\frac{dy}{dx} = \frac{dy}{\quad} \times \frac{\quad}{dx}$$

Do note that the blanked term must be the same. Always read around the question and find the elements that have not been used yet. Most of the time it will be the time-element or one of the (not-used) expressions in the question

Important

- **Decreasing Rate of Change**
  - If the question states that the rate of change is decreasing, students must take into account that the value is **negative**. However, if the question is asking for the rate of decrease of something the value should be **positive**. **DO NOT** confuse between the two
- **Final Statements**
  - For all rate of change questions, a final concluding statement should be written (with the necessary units) as this is to answer the question

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