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Topic 13:
Proofs in Plane Geometry (4049)

THE ABOUT

CHAPTER ANALYSIS

- Use of:
  - Properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles
  - Congruent and Similar Triangles

- Use of:
  - Midpoint Theorem
  - Alternate Segment Theorem

Note that there are 3 chapters from E-Math that are pre-requisites
- Chapter 11: Angles Triangles and Polygons
- Chapter 12: Congruency & Similarity
- Chapter 13: Properties of Circles

MASTERY

- Challenging chapter (without practice)
- 2 **key** concepts

EXAM

- Concepts usually tested as a stand-alone topic
- Difficult especially if students, for the E-Math Chapters, do not have a strong foundation

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, an 10m question, 1 question in one of the papers

KEY CONCEPT

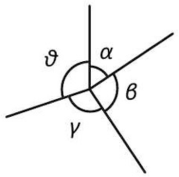
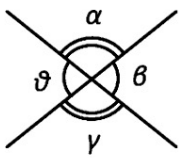
Geometrical Properties of Angles

Angle Properties of Triangles, Quadrilaterals

Geometrical Properties of Angles

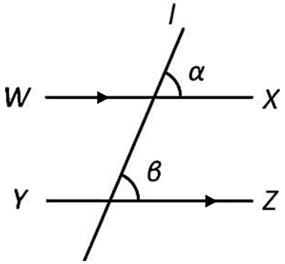
Branch of Mathematics that deals with the properties, measurements and relationships of points, lines, angles, surfaces and solids

Type of Angles	
Name	**Characteristics**
Right Angle	Angle that is equal to $90°$
Acute Angle	Angle that is less than $90°$
Obtuse Angle	Angle that is more than $90°$ but less than $180°$
Reflex Angle	Angle that is between $180°$ and $360°$
Complementary Angle	2 angles that sum to $90°$
Supplementary Angle	2 angles that sum to $180°$ (on a straight line)

Angles about a point		
Angles at a point		$\boldsymbol{\alpha+\beta+\gamma+\vartheta=360°}$
Vertically Opposite Angles		$\boldsymbol{\alpha=\gamma}$ $\boldsymbol{\beta=\vartheta}$

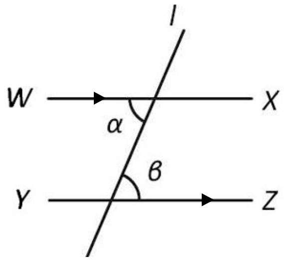
Angles formed by parallel lines and a transversal

Corresponding Angles



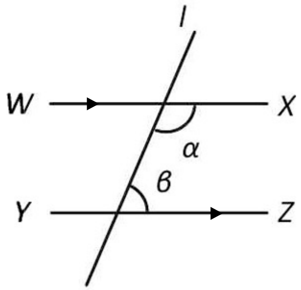
$$\boldsymbol{\alpha=\beta}$$

Alternate Angles



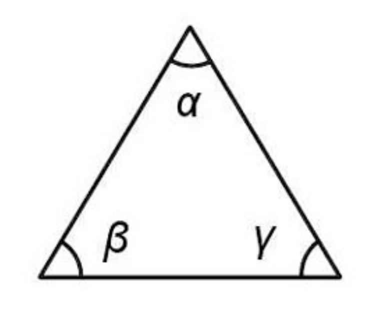
$$\boldsymbol{\alpha=\beta}$$

Interior Angles



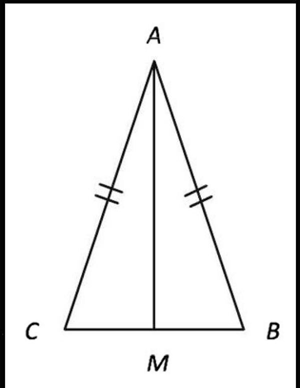
$$\boldsymbol{\alpha+\beta=180°}$$

Angle Properties of Triangles



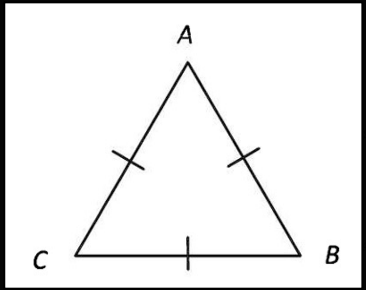
The sum of 3 angles in a triangle is $180°$

$$\alpha + \beta + \gamma = 180°$$



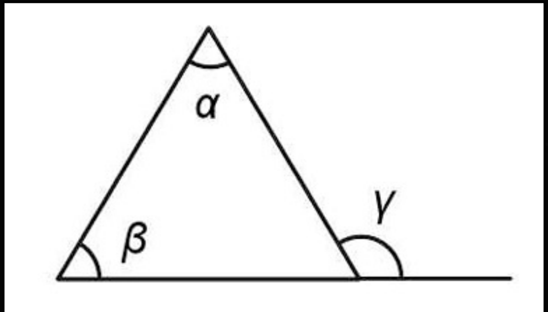
An **isosceles triangle** is a triangle with <u>2 equal sides</u>

$$\angle ACB = \angle ABC$$
$$AC = AB$$



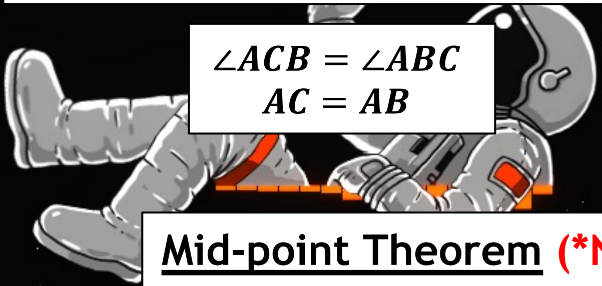
An **equilateral triangle** is a triangle with <u>3 equal sides</u>

$$\angle ACB = \angle CBA = \angle BAC = 60°$$
$$AC = CB = BA$$



Exterior angle of an angle is equal to the sum of the interior opposite angles

$$\alpha + \beta = \gamma$$

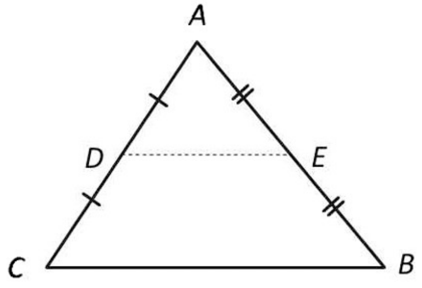


Mid-point Theorem (*NEW*)

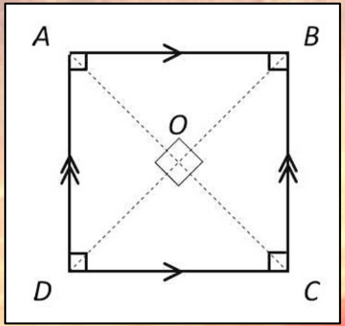
It states that the line segment connecting the midpoints of 2 sides of a triangle is parallel to the last side and is congruent to $\frac{1}{2}$ of the third side

In ΔABC, if $AD = DC$ and $AE = EB$, then

- DE must be parallel to CB
- $DE = \frac{1}{2}CB$

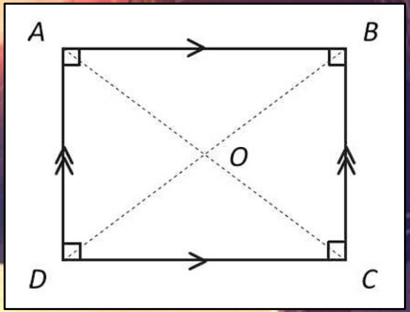


Properties of Quadrilaterals (Part 1)



A **square** is a quadrilateral where:
- All sides are equal
- All angles are right-angles
- The diagonals are equal and cut at right-angles

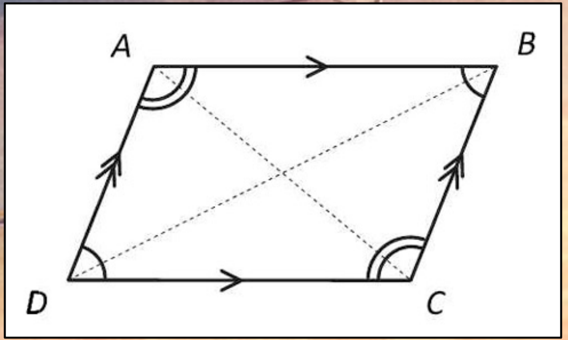
$$AB = BC = CD = DA$$
$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$
$$OA = OB = OC = OD$$



A **rectangle** is a quadrilateral where:
- Opposite sides are equal
- All angles are right-angles
- The diagonals are equal

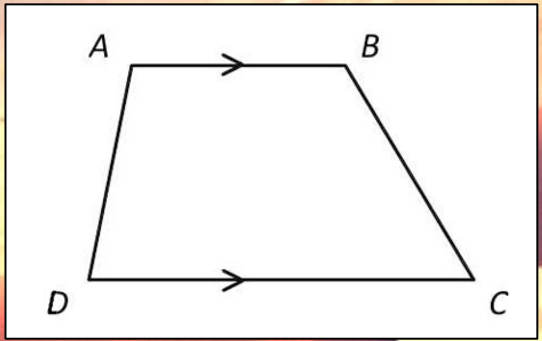
$$AB = DC \text{ and } BC = AD$$
$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$
$$OA = OB = OC = OD$$

Properties of Quadrilaterals (Part 2)

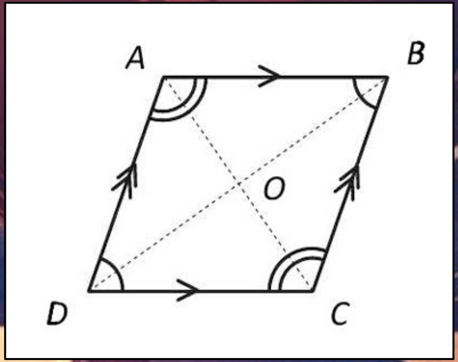


A **parallelogram** is a quadrilateral where:
- Opposite sides are equal
- Opposite angles are equal

$AB = CD$ and $AD = BC$
$\angle A = \angle C$ and $\angle B = \angle D$



A **trapezium** is a quadrilateral where:
- One pair of parallel sides
- **2** obtuse angles and **2** acute angles

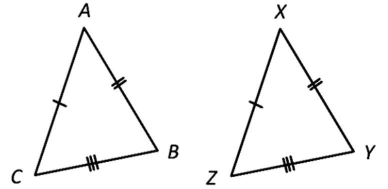
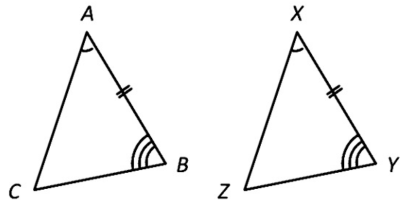
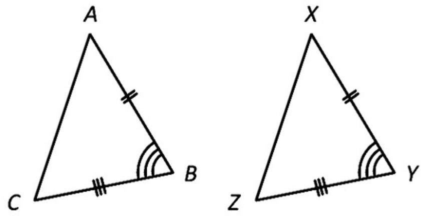
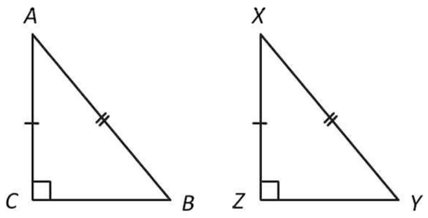


A **rhombus** is a quadrilateral where:
- All sides are equal
- Opposite angles are equal
- The diagonals are equal, cut at right-angles and bisect its interior angles

$\boldsymbol{AB = BC = CD = DA}$
$\boldsymbol{\angle A = \angle C}$ **and** $\boldsymbol{\angle B = \angle D}$
$\boldsymbol{OA = OB = OC = OD}$

KEY CONCEPT

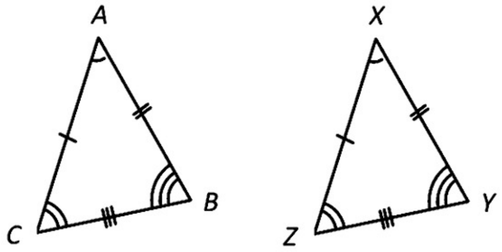
Congruency & Similarity of Triangles

Congruency Tests		
Name	**Test**	**Diagram**
SSS	$AB = XY$ $BC = YZ$ $AC = XZ$	
ASA	$AB = XY$ $\angle CAB = \angle ZXY$ $\angle ABC = \angle XYZ$	
SAS	$AB = XY$ $BC = YZ$ $\angle ABC = \angle XYZ$	
RHS	$AB = XY$ $BC = YZ$ $\angle ACB = \angle XZY = 90°$	

Symbol for Congruency: $\equiv$

Congruency

Figures that are identical in every aspect

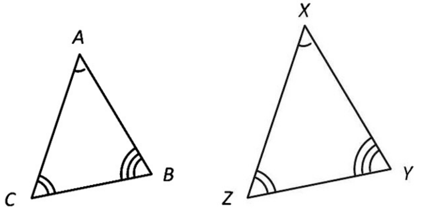
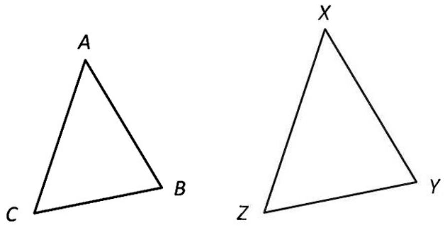
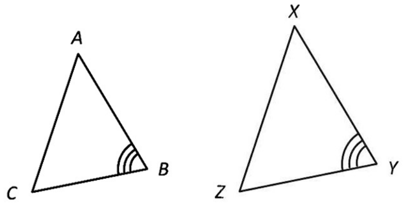


For **2** triangles to be congruent, their corresponding sides and angles **MUST** be equal

4 tests for Congruency:

1. **'SSS' or 'side-side-side' test**
2. **'ASA' or 'angle-side-angle' test**
3. **'SAS' or 'side-angle-side' test**
4. **'RHS' or 'right angle-hypotenuse-side' test**

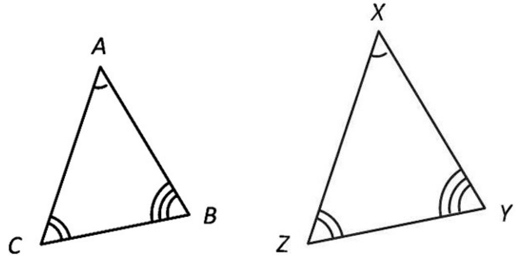
Similarity Tests

Name	Test	Diagram
AAA	$\angle CAB = \angle ZXY$ $\angle ABC = \angle XYZ$ $\angle BCA = \angle YZX$	
Corresponding sides same ratio	$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$	
Ratio of 2 sides same + 1 angle	$\angle ABC = \angle XYZ$ $\frac{AB}{XY} = \frac{BC}{YZ}$	

Make sure that all Congruency & Similarity Tests have <u>3 lines of justifications</u> each to fulfil all test conditions

Similarity

Figures that have the same shape but different sizes



For 2 triangles to be similar, their ratio of the corresponding sides are the same for all lengths, and the corresponding angles are equal

3 tests for Congruency:

1. **'AAA' or 'angle-angle-angle' test**

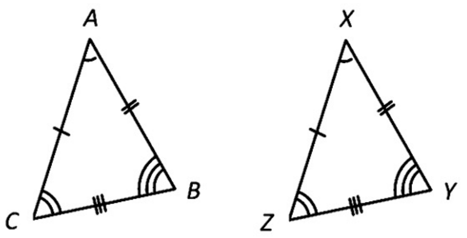
2. **Corresponding sides same ratio**

3. **Ratio of 2 pairs of corresponding sides is the same and the included angles <u>between</u> them are equal**

Take Note:

The naming conventions of Congruency & Similarity Questions are important! Many students make mistake when defining the sides that they want to use in their arguments

EX: To show that $\Delta ABC \equiv \Delta XYZ$



To show that **2** sides are equal in length, the direction of how students define the length must stay consistent (Either all Clockwise or Anti-Clockwise)

$$AC = XZ \qquad \textbf{OR} \qquad CA = ZX$$

To show that **2** angles are equal in size, the direction of how students define the angles must stay consistent (Either all Clockwise or Anti-Clockwise)

$$\angle ABC = \angle XYZ \qquad \textbf{OR} \qquad \angle CBA = \angle ZYX$$

Justifications for each argument

Formal Justifications must be provided at each step, no matter how trivial

1. If the question states that the lengths/angles are equal, simply write the equal lengths/angles and state that it is "**Given**"

$$AC = XZ \quad (\textbf{given})$$

* Note that you can only use "Given" if the question explicitly states that the lengths/angles are equal! If you require to perform some calculations to find the lengths, you are NOT allowed to write "Given"

2. If **2** triangles have a common side/angle (means that **2** triangles are stuck together and joined with a side/angle that both triangles have), simply write that the side/angle is common

$$AC \textbf{ is a common side}$$

3. If the question requires some calculations for the angles, reasons must be explicitly stated as to how you come about with said calculations. Use reasons from **Topic 11:** Angles, Triangles & Polygons, and **Topic 13:** Properties of Circles to help justify all your arguments

$$\angle ABC = \angle XYZ \ (\textbf{alternate angles})$$

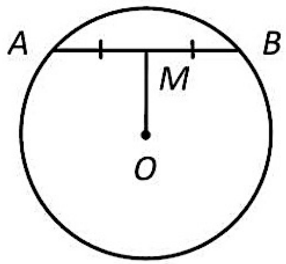
KEY CONCEPT

Symmetry Properties of Circles

Angle Properties of Circles

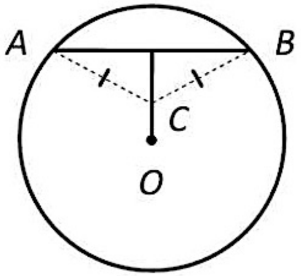
Additional Useful Theorems:

The line segment drawn from the centre to the midpoint of the chord is perpendicular to the chord



If $AM = MB$, then $AB \perp OM$

Every point on the perpendicular bisector of a line segment is equidistant from the endpoints of the segment

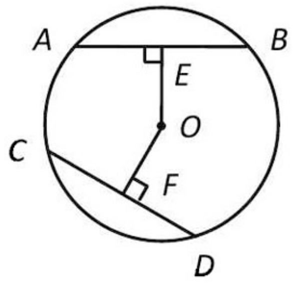


Symmetry Properties of Circles

4 Theorems to remember:

1. **Chord Theorem**

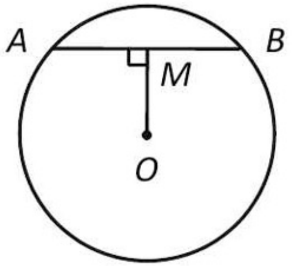
Chords equidistant from the centre of the circle are equal



If $AB = CD$, then $OE \perp AB$ and $OF \perp CD$

2. **Perpendicular Bisector Theorem**

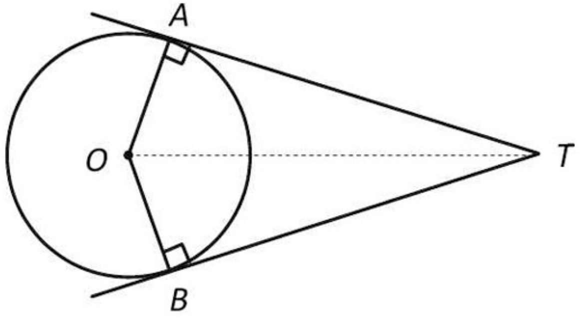
A line from the centre, perpendicular to a chord that bisects the chord is known as the perpendicular bisector



If $AB \perp OM$, then $AM = MB$

Additional Useful Theorems:

The line joining the external point to the centre of the circle bisects the angle between the tangents

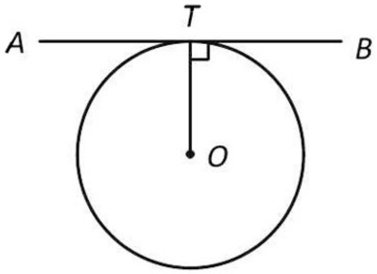


$$\angle AOT = \angle BOT$$

Symmetry Properties of Circles

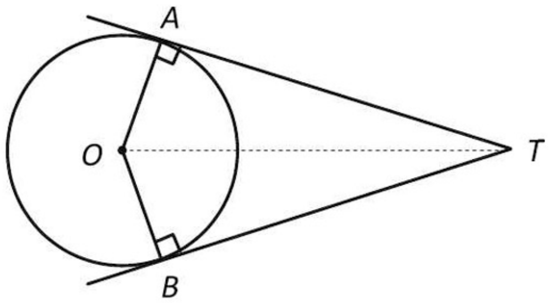
3. Tangent Theorem

The line perpendicular to the tangent at the point of contact passes through the centre of the circle



$$\angle OTA = \angle OTB = 90°$$

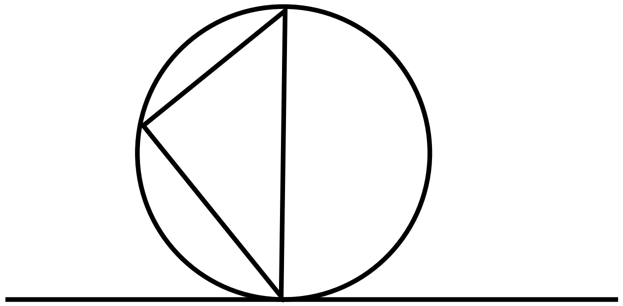
Tangents drawn from an external point to a circle are equal



$$AT = BT$$

Take Note:

This is a highly tested theorem! Many students struggle to find and use this Theorem in their solutions.

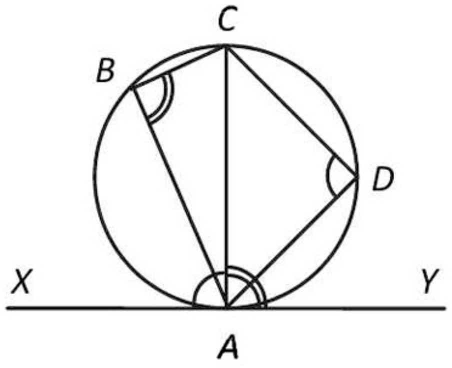


Always look out for a triangle with all 3 points touching the circle with one of the points at a tangent to the circle

Symmetry Properties of Circles

4. Alternate Segment Theorem

An angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment

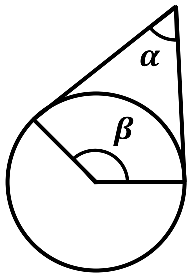


$$\angle XAC = \angle ADC$$

$$\angle CAY = \angle ABC$$

Take Note:

Many students get <u>tricked</u> by this figure



Many students think that $\boldsymbol{\alpha} = \mathbf{2}\boldsymbol{\beta}$ when in actual fact there is no special relationship between $\boldsymbol{\alpha}$ **and** $\boldsymbol{\beta}$

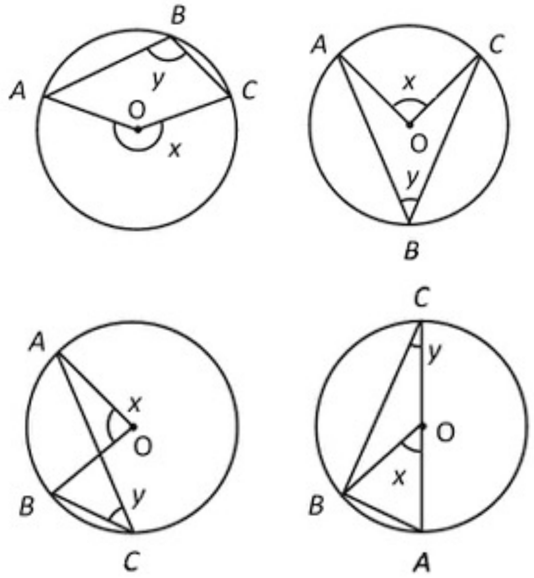
UNLESS: If the 2 lines above are tangents that extend to a point, then

$$\boldsymbol{\alpha} + \boldsymbol{\beta} = \mathbf{180°}$$

Angle Properties of Circles

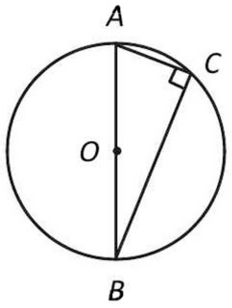
4 Angle Properties to remember:

1. Angle at centre is **2** times the angle at the circumference



$\boldsymbol{x} = \mathbf{2}\boldsymbol{y}$

2. Angles in a semicircle

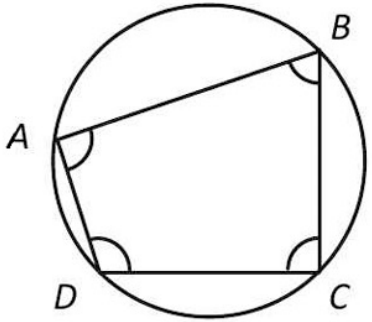


$\angle \boldsymbol{ACB} = \mathbf{90°}$

Additional Useful Properties:

A **cyclic quadrilateral** is a quadrilateral drawn inside a circle such that all its 4 vertices lie on the circumference of the circle

The sum of the opposite angles of a cyclic quadrilateral is $\mathbf{180°}$

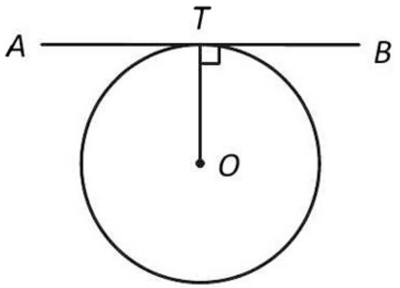


$$\angle CDA + \angle ABC = 180°$$

$$\angle DAB + \angle BCD = 180°$$

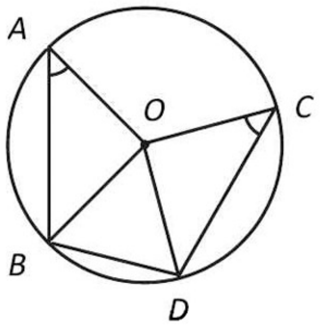
Angle Properties of Circles

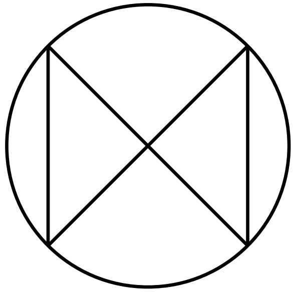
3. Angle between the tangent and radius is $90°$



$$\angle OTA = \angle OTB = 90°$$

4. Angles in same segment are equal





$$\angle BAO = \angle DCO$$

Always look for this “butterfly” shape

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