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March Practice Questions 2022 Full Solutions (E-Math)

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Question Source

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level E-Math Examination. All questions compiled here are from **2009 - 2021 School Mid-Year / Prelim Papers**. Questions are categorised into the various topics and range in varying difficulties. If questions are sourced from respective sources, credit will be given when appropriate.

How to read:

[S4 ABCSS P1/2011 PRELIM Qn 1]

Secondary 4, ABC Secondary School, Paper 1, 2011, Prelim, Question 1

Syllabus (4048)

Algebra	**Geometry & Measurement**	**Statistics & Probability**
Numbers & their Operations	Angles, Triangles & Polygons	Data Analysis & Handling
Ratio & Propotion	Congruency & Similarity	Probability
Percentage	Properties of Circles	
Rate & Speed	Trigonometry	
Algebraic Expressions & Formulae	Mensuration	
Functions & Graphs	Coordinate Geometry	
Equations & Inequalities	Vectors in 2 Dimensions	
Set Language & Notation		
Matrices		
Problems in Real-World Context		

Contents

1 Numbers & their Operations

1.1 Full Solutions

1. (a)

$$\mathbf{1.50 \times 10^{11} \ m}$$

(b)

$$\begin{aligned}\text{Time taken} &= \frac{1.4959 \times 10^{11}}{343} \\ &= \mathbf{4.36 \times 10^{8} \ s \ (3.s.f.)}\end{aligned}$$

(c)

$$\begin{aligned}\text{Percentage} &= \frac{384400 \times 10^{3}}{1.4959 \times 10^{11}} \times 100\% \\ &= \mathbf{0.257\% \ (3.s.f.)}\end{aligned}$$

(d)

$$\begin{aligned}\text{Time taken} &= \left(1.4959 \times 10^{11}\right) \times \left(8000 \times 10^{-9}\right) \\ &= \mathbf{1.20 \times 10^{6} \ s \ (3.s.f.)}\end{aligned}$$

2. (a)

$$\begin{aligned}\left(\frac{x^6}{25y^4}\right)^{-\frac{1}{2}} &= \left(\frac{25y^4}{x^6}\right)^{\frac{1}{2}} \\ &= \boldsymbol{\frac{5y^2}{x^3}}\end{aligned}$$

(b)

$$\begin{aligned}9\left(\sqrt[3]{3^{3x}}\right) &= \frac{1}{3^{3(2-x)}} \\ 3^2\left(3^x\right) &= 3^{-6+3x} \\ 3^{2+x} &= 3^{3x-6}\end{aligned}$$

Comparing powers,

$$\begin{aligned}2 + x &= 3x - 6 \\ x &= \mathbf{4}\end{aligned}$$

(c)

$$\begin{aligned}ax^n - x &= 0 \\ x\left(ax^{n-1} - 1\right) &= 0 \\ x = 0 \quad \text{or} \quad ax^{n-1} - 1 &= 0\end{aligned}$$

For the second equation, since n is even, $(n-1)$ is odd

$$\begin{aligned}ax^{n-1} - 1 &= 0 \\ ax^{n-1} &= 1 \\ x^{n-1} &= \frac{1}{a} \\ x &= \sqrt[n-1]{\frac{1}{a}}\end{aligned}$$

Since $(n-1)$ is odd, there is only one solution for $\sqrt[n-1]{\frac{1}{a}}$

$$\therefore \mathbf{2 \ solutions}$$

3. (a) (i)

$$27^{\frac{1}{3}} = 3.0000 \qquad \pi = 3.1415 \qquad \frac{22}{7} = 3.1428$$
$$3.\dot{1} = 3.1111 \qquad 3.33 = 3.3300 \qquad \sqrt{10} = 3.1622$$

$$\mathbf{27^{\frac{1}{3}} \qquad 3.\dot{1} \qquad \boldsymbol{\pi} \qquad \frac{22}{7} \qquad \sqrt{10} \qquad 3.33}$$

(ii)

$$\boldsymbol{\pi} \qquad \mathbf{\sqrt{10}}$$

(b) (i)

$$\mathbf{0.0923}$$

(ii)

$$\begin{aligned} \text{Time} &= \frac{750 \times 10^6}{500} \\ &= 1.5 \times 10^6 \text{ sec} \\ &= \mathbf{2.5 \times 10^4 \text{ min}} \end{aligned}$$

2 Ratio & Proportion

2.1 Full Solutions

1. (a)

$\underline{5}$ men takes $\underline{9}$ days to build a house

$\underline{1}$ man takes $\underline{45}$ days to build a house

$\underline{3}$ men takes $\underline{\mathbf{15}}$ days to build a house

(b)

$$V \propto R^2 \qquad \Rightarrow \qquad V = kR^2$$

Since the radius of the pipe increased by 150%,

$$R_{\text{New}} = 2.5R$$

$$\begin{aligned} V_{\text{New}} &= \frac{V}{R^2} \times (2.5R)^2 \\ &= 6.25V \end{aligned}$$

$$\begin{aligned} \text{Percentage increase} &= \frac{V_{\text{New}} - V}{V} \times 100\% \\ &= \frac{6.25 - V}{V} \times 100\% \\ &= \mathbf{525\%} \end{aligned}$$

2. (a) Case 1: Online platform excluding the 5% discount

$$\begin{aligned} \text{Selling price} &= 110\% \times 95\% \times 2020 \\ &= \$2111 \text{ (nearest dollar)} \end{aligned}$$

Case 2: Online platform factoring the 5% discount

$$\begin{aligned} \text{Selling price} &= \frac{110\% \times 2020}{95\%} \\ &= \mathbf{\$2339 \text{ (nearest dollar)}} \end{aligned}$$

(b)

$\underline{45}$ staff takes $\underline{8}$ hours a day to complete a project

$\underline{1}$ staff takes $\underline{360}$ hours a day to complete a project

$\underline{40}$ staff takes $\underline{\mathbf{9}}$ hours a day to complete a project

(c)

$$\begin{aligned} \text{SGD } 1 &= \text{AUD } \frac{1}{12} \\ &= \text{AUD } 0.89285 \end{aligned}$$

Hence, the money changer because 1 Singapore dollar worth more than **in the bank**

(d)

$$\begin{aligned} \text{Number of coins required} &= \frac{1000000}{21} \\ &= 47620 \text{ (nearest coins)} \end{aligned}$$

$$\begin{aligned} \text{Amount of money raised} &= (0.2 - 0.12) \times 47620 \\ &= \mathbf{\$3809.60 \text{ (2.d.p.)}} \end{aligned}$$

3 Percentage

3.1 Full Solutions

1. (a)

$$\begin{aligned}\text{Number of children in 2019} &= \frac{3714 + 8096 + 6516}{88\%} \times 12\% \\ &= \mathbf{2499}\end{aligned}$$

$$\begin{aligned}\text{Number of adults in 2018} &= \frac{8096}{128\%} \times 100\% \\ &= \mathbf{6325}\end{aligned}$$

(b) (i)

$$\begin{aligned}k &= \frac{1.49 \times 10^6}{878} \\ &= 1697.0387... \\ &= \mathbf{1.70 \times 10^3}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Mean number of days} &= \frac{5.08}{1.49} \\ &= 3.409395... \\ &= \mathbf{3.41\ days}\end{aligned}$$

(c)

$$\text{After 6 years} = 9600\left(1 + \frac{2.4}{100}\right)^6$$

$$\begin{aligned}\text{Total he received} &= 9600\left(1 + \frac{2.4}{100}\right)^6 \times \left(\frac{1}{0.72}\right) \times 98.5\% \\ &= 15141.702... \\ &= \mathbf{S\$15141.70\ (2.d.p.)}\end{aligned}$$

2. (a) (i)

$$\begin{aligned}\text{Deposit} &= \$5200 \times 20\% \\ &= \mathbf{\$1040}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Hire purchase price} &= \$5200 + 3\left[(\$5200 - \$1040) \times 3.6\%\right] \\ &= \mathbf{\$5649.28\ (exact)}\end{aligned}$$

(iii)

$$\begin{aligned}\text{Monthly instalment} &= \frac{\$5649.28 - \$1040}{36} \\ &= 128.035555... \\ &= \mathbf{\$128.04\ (2.d.p.)}\end{aligned}$$

(b) (i)

- Interest Rate: 1.45 should be divided by 4 and not 12, as it is compounded every 3 months
- Value of n: Power n should be 12 and not 3, as the value of n is the number of times it is compounded within the timeframe

(ii)

$$\boldsymbol{\$x\left(1+\frac{\left(\frac{1.45}{4}\right)}{100}\right)^{12}}$$

(iii) We are solving for x

$$\begin{aligned}\$12533 &= \$x\left(1+\frac{\left(\frac{1.45}{4}\right)}{100}\right)^{12} \\ x &= \frac{\$12533}{\left(1+\frac{\left(\frac{1.45}{4}\right)}{100}\right)^{12}} \\ &= \$12000.446... \\ &= \mathbf{\$12000\ (nearest\ dollar)}\end{aligned}$$

(iv)

$$\begin{aligned}\text{Alternative rate} &= \left[\frac{\$12533}{\left(1+\frac{\left(\frac{1.45}{4}\right)}{100}\right)^{12}}\right]\left(1+\frac{1.7}{100}\right)^{3} \\ &= 12622.93214... \\ &= \$12623\ (\text{nearest dollar})\end{aligned}$$

Yes, he should have invested the money compounded on a **yearly basis** as he would have earned more interest

3. (a)

$$\begin{aligned}\text{Total discount from Shop } B &= 0.2p + \frac{x}{100}(0.8p)\\ &= p\left(0.2 + \frac{0.8}{100}x\right)\\ &= \frac{p}{125}(25 + x)\end{aligned}$$

$$\therefore \boldsymbol{a = 25}$$

(b)

$$\begin{aligned}0.35p &< \frac{p}{125}(25 + x)\\ 0.35(125) &< 25 + x\\ x &> 18\frac{3}{4}\end{aligned}$$

$$\therefore \boldsymbol{k = 18\frac{3}{4}}$$

4 Rate & Speed

4.1 Full Solutions

1. (a) Since the distance travelled is 1140 m, this corresponds to the area under the graph

$$\frac{1}{2}(t)(40) = 1140$$
$$t = \mathbf{57\ s}$$

(b) To find the acceleration, we are looking for the gradient of the line

$$\text{Acceleration} = \frac{40-0}{25-75}$$
$$= \mathbf{-1.25\ m/s^2}$$

(c)

$$60 \text{ km/h} = \frac{60 \times 1000}{3600}$$
$$= \mathbf{16\frac{2}{3}\ m/s}$$

(d)

$$\text{Time taken (second car)} = \frac{1140}{\left(16\frac{2}{3}\right)}$$
$$= 68.4 \text{ s}$$

Disagree, as the second car will take a longer time

2.

$$\text{Time to varnish 1 jar} = \frac{1}{\left(\frac{3}{5}\right) + \left(\frac{2}{3}\right)}$$
$$= \frac{15}{19} \text{ hr}$$

$$\therefore 20 \text{ jars} = 20\left(\frac{15}{19}\right)$$
$$= 15\frac{15}{19} \text{ hrs}$$
$$= \mathbf{15\ hrs\ 47\ min}$$

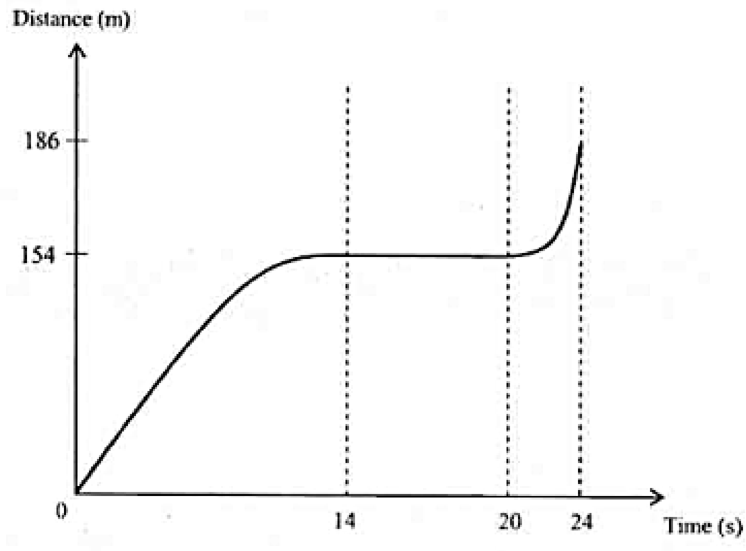
3. (a)

$$\begin{aligned}\text{Acceleration} &= \text{Gradient of line}\\ &= \frac{16-0}{24-20}\\ &= \mathbf{4\ m/s^2}\end{aligned}$$

(b)

$$\begin{aligned}\text{Total distance} &= \text{Area under graph}\\ &= A_{\text{arc }AB} + A_{\text{trapezium}}\\ &= \frac{1}{4}\left(\frac{22}{7}\right)(14)^2 + \frac{1}{2}(16)(8+12)\\ &= \mathbf{314\ m}\end{aligned}$$

(c) Distance-time graph



5 Algebraic Expressions & Formulae

5.1 Full Solutions

1. (a) (i)

$$\boldsymbol{x = 3} \qquad \boldsymbol{y = 11} \qquad \boldsymbol{z = 15}$$

(ii)

$$\begin{aligned} T_n &= 3 + 4(n-1) \\ &= \mathbf{4}\boldsymbol{n} - \mathbf{1} \end{aligned}$$

(iii)

$$\begin{aligned} T_{109} &= 4(109) - 1 \\ &= \mathbf{435} \end{aligned}$$

(b) (i)

Number of lines per dot = **5**

(ii)

Number of lines = **15**

(iii)

Number of lines per dot (n) = $\boldsymbol{n - 1}$

(iv) Pattern:

0 1 3 6 9 ...

Note that these numbers follow the pattern of

$$\frac{n(n-1)}{2}$$

$$\begin{aligned} \therefore K_{256} &= \frac{256(256-1)}{2} \\ &= \mathbf{32540} \end{aligned}$$

(v) Based on the formulae found in part (b)(iv), 71 must be written in as a product of 2 consecutive integers, however this is absurd as 71 is prime

2. (a)

$$S_{35} = \frac{35(35+1)}{2}$$
$$= \mathbf{630}$$

(b)

$$\frac{n(n+1)}{2} = 1378$$
$$n^2 + n - 2756 = 0$$
$$(n+53)(n-52) = 0$$

$$n = \mathbf{52} \quad \text{or} \quad n = 53 \text{ (N.A.)}$$

(c)

$$T = (1+2+3+...+99) + 99(100)$$
$$= \frac{99(99+1)}{2} + 9900$$
$$= \mathbf{14850}$$

(d)

$$P = 2(1+2+...+50)$$
$$= 2\left(\frac{50(50+1)}{2}\right)$$
$$= \mathbf{2550}$$

(e)

$$\text{Sum of all odd integers} = \frac{100(101)}{2} - 2550$$
$$= \mathbf{2500}$$

3. (a)

$$T_5 = \mathbf{6 + (5-2)^2 - 10 = 5}$$

(b)

$$\begin{aligned} T_n &= 6 + (n-2)^2 - 2n \\ &= 6 + n^2 - 4n + 4 - 2n \\ &= \boldsymbol{n^2 - 6n + 10} \end{aligned}$$

(c)

$$\begin{aligned} \frac{(3k)^2 - 6(3k) + 10}{k^2 - 6k + 10} &= 17 \\ 9k^2 - 18k + 10 &= 17k^2 - 102k + 170 \\ 8k^2 - 84k + 160 &= 0 \\ 2k^2 - 21k + 40 &= 0 \ \textbf{(shown)} \end{aligned}$$

□

(d)

$$\begin{aligned} 2k^2 - 21k + 40 &= 0 \\ (2k-5)(k-8) &= 0 \end{aligned}$$

$$k = \mathbf{2\frac{1}{2}} \quad \text{or} \quad k = \mathbf{8}$$

(e)

$$\boldsymbol{k = 2\frac{1}{2} \notin \mathbb{Z}^+} \quad \Rightarrow\Leftarrow$$

4. (a) (i)

$$\begin{aligned} S &= \frac{8}{2}\sqrt{(3)^2 - (-7)} \\ &= \mathbf{16} \end{aligned}$$

(ii)

$$\begin{aligned} S &= \frac{a}{2}\sqrt{n^2 - b} \\ \frac{2S}{a} &= \sqrt{n^2 - b} \\ n^2 - b &= \frac{4S^2}{a^2} \\ n^2 &= \frac{4S^2}{a^2} + b \\ \boldsymbol{n} &= \boldsymbol{\pm\sqrt{\frac{4S^2}{a^2} + b}} \end{aligned}$$

(b)

$$\begin{aligned} \frac{5}{\sqrt[3]{5}} &= 5^{x-1} \\ 5^{\frac{2}{3}} &= 5^{x-1} \\ \therefore \frac{2}{3} &= x - 1 \\ x &= \mathbf{1\frac{2}{3}} \end{aligned}$$

(c)

$$\begin{aligned} 9x^2 - 25 + 12xy - 20y &= (3x-5)(3x+5) + 4y(3x-5) \\ &= \boldsymbol{(3x-5)(3x+5+4y)} \end{aligned}$$

(d)

$$\begin{aligned} \frac{x^2+3x-7}{x-4} + \frac{9+3x}{4-x} &= \frac{x^2+3x-7}{x-4} - \left(\frac{9+3x}{x-4}\right) \\ &= \frac{x^2+3x-7-9-3x}{x-4} \\ &= \frac{x^2-16}{x-4} \\ &= \frac{(x+4)(x-4)}{x-4} \\ &= \boldsymbol{x+4} \end{aligned}$$

6 Functions & Graphs

6.1 Full Solutions

1. (a) For A and B, substitute $y = 0$,

$$0 = (x-4)(x+3)$$

$$x = 4 \quad \text{or} \quad x = -3$$
$$\boldsymbol{A(-3, 0)} \quad \text{or} \quad \boldsymbol{B(4, 0)}$$

For C, substitute $x = 0$,

$$y = 12$$

$$\therefore \boldsymbol{C(0, -12)}$$

(b)

$$\begin{aligned}\text{Length of } AC &= \sqrt{(-3-0)^2 + (0-(-12))^2} \\ &= \sqrt{153} \\ &= 12.369316... \\ &= \textbf{12.4 units (3.s.f.)}\end{aligned}$$

(c)

$$\begin{aligned}\text{Gradient of } BC &= \frac{0-(-12)}{4-0} \\ &= 3\end{aligned}$$

$$\therefore \boldsymbol{y = 3x - 12}$$

(d)

$$\begin{aligned}x\text{-coordinate of minimum point} &= \frac{4+(-3)}{2} \\ &= \frac{1}{2}\end{aligned}$$

Substitute $x = \frac{1}{2}$ into equation of the curve,

$$\begin{aligned}y &= \left(\frac{1}{2} - 4\right)\left(\frac{1}{2} + 3\right) \\ &= -12\frac{1}{4}\end{aligned}$$

$$\text{Minimum point} = \left(\frac{1}{2}, -12\frac{1}{4}\right)$$

(e) Below the minimum point $\left(\frac{1}{2}, -12\frac{1}{4}\right)$, there will be no solutions for the equation as there will be no intersection points between the curve and the line $y = k$, where

$$k < -12\frac{1}{4}$$

2. Substitute the point $A(0.5, 5)$ into the curve,

$$\begin{aligned} 5 &= \frac{p}{0.5} + 0.5q + 1 \\ 5 &= 2p + 0.5q + 1 \\ 10 &= 4p + q + 2 \\ 4p + q &= 8 \text{ ......(1)} \end{aligned}$$

Substitute the point $B(3, -10)$ into the curve,

$$\begin{aligned} -10 &= \frac{p}{3} + 3q + 1 \\ -30 &= p + 9q + 3 \\ p + 9q &= -33 \\ p &= -33 - 9q \text{ ......(2)} \end{aligned}$$

Substitute Equation (2) into Equation (1),

$$\begin{aligned} 4(-33 - 9q) + q &= 8 \\ 35q &= -140 \\ q &= -4 \end{aligned}$$

Substitute $q = -4$ into Equation (2),

$$\begin{aligned} p &= -33 - 9(-4) \\ &= 3 \end{aligned}$$

$$\therefore p = \mathbf{3} \qquad \text{or} \qquad q = \mathbf{-4}$$

3. (a) When $x = 3$,

$$\begin{aligned} q &= \frac{3}{10}\left[15 - (3)^2\right] \\ &= \mathbf{1\frac{4}{5}} \end{aligned}$$

(b)

Graph is drawn on the next page

(c)

$$\frac{x}{10}\left(15 - x^2\right) = -1$$

We are sketching the line $y = -1$ (blue)

$$\therefore \boldsymbol{x = -0.69} \qquad \text{or} \qquad \boldsymbol{x = 4.17}$$

(d) We draw a tangent at $x = 3.5$ (green)

$$\begin{aligned} \text{Gradient} &= \frac{2.91 - (-0.78)}{2.6 - 4.3} \\ &= \mathbf{-2.17\ (3.s.f.)} \end{aligned}$$

(e) (i)

Graph is drawn on the next page

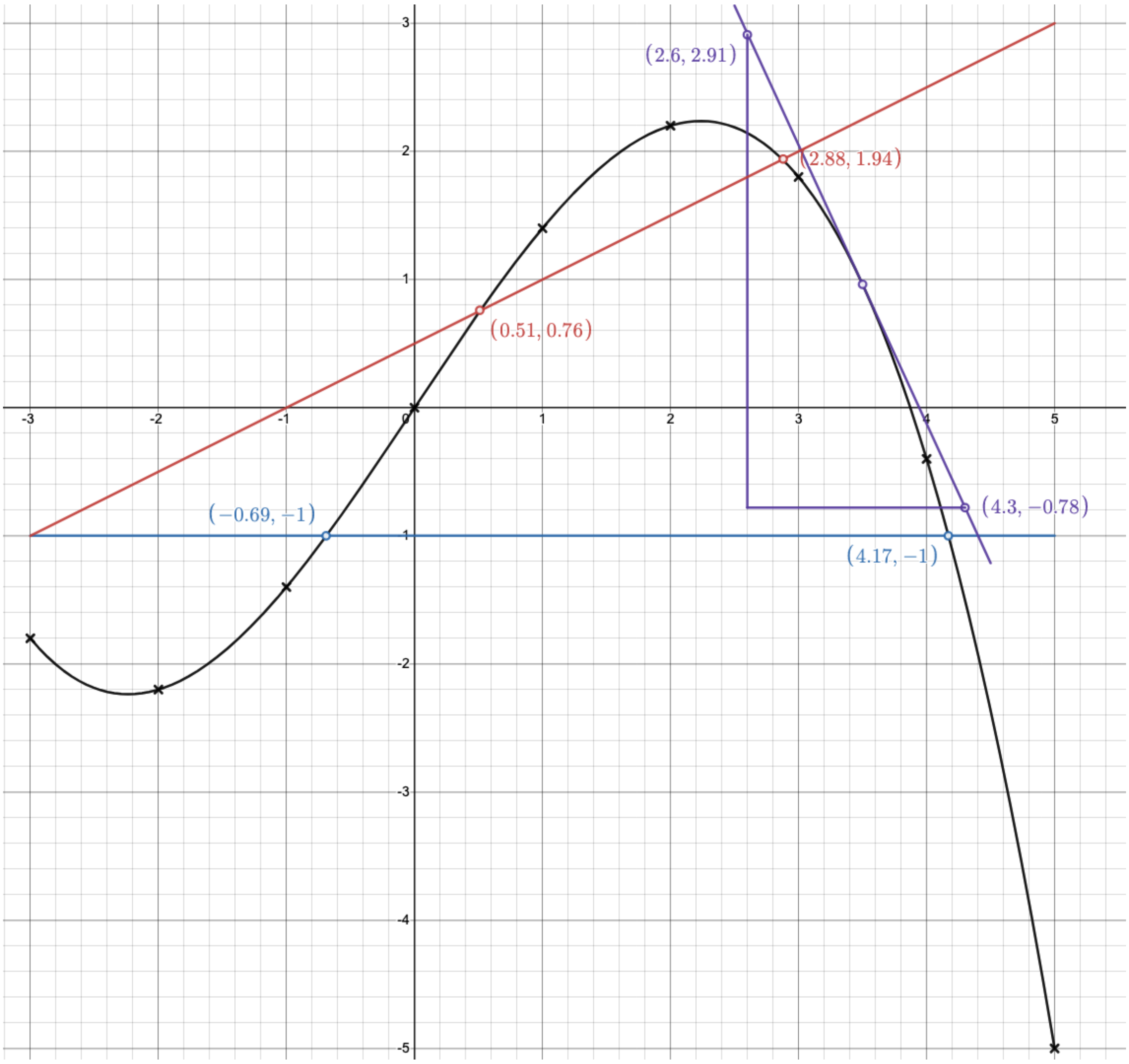
(ii)

$$\boldsymbol{2y = x + 1}$$

(iii)

$$\mathbf{(0.51, 0.76)} \qquad \mathbf{(2.88, 1.94)}$$

Graph for Question 3



4. (a) When $t = 2.5$,

$$\begin{aligned} k &= 30 \times 2^{2.5} \\ &= \mathbf{169.7} \end{aligned}$$

(b) When $t = 0$,

$$N = \mathbf{30}$$

(c)

Graph is drawn on the next page

(d) When $N = 250$,

$$t = \mathbf{3.1\ s\ (2.s.f.)}$$

(e)

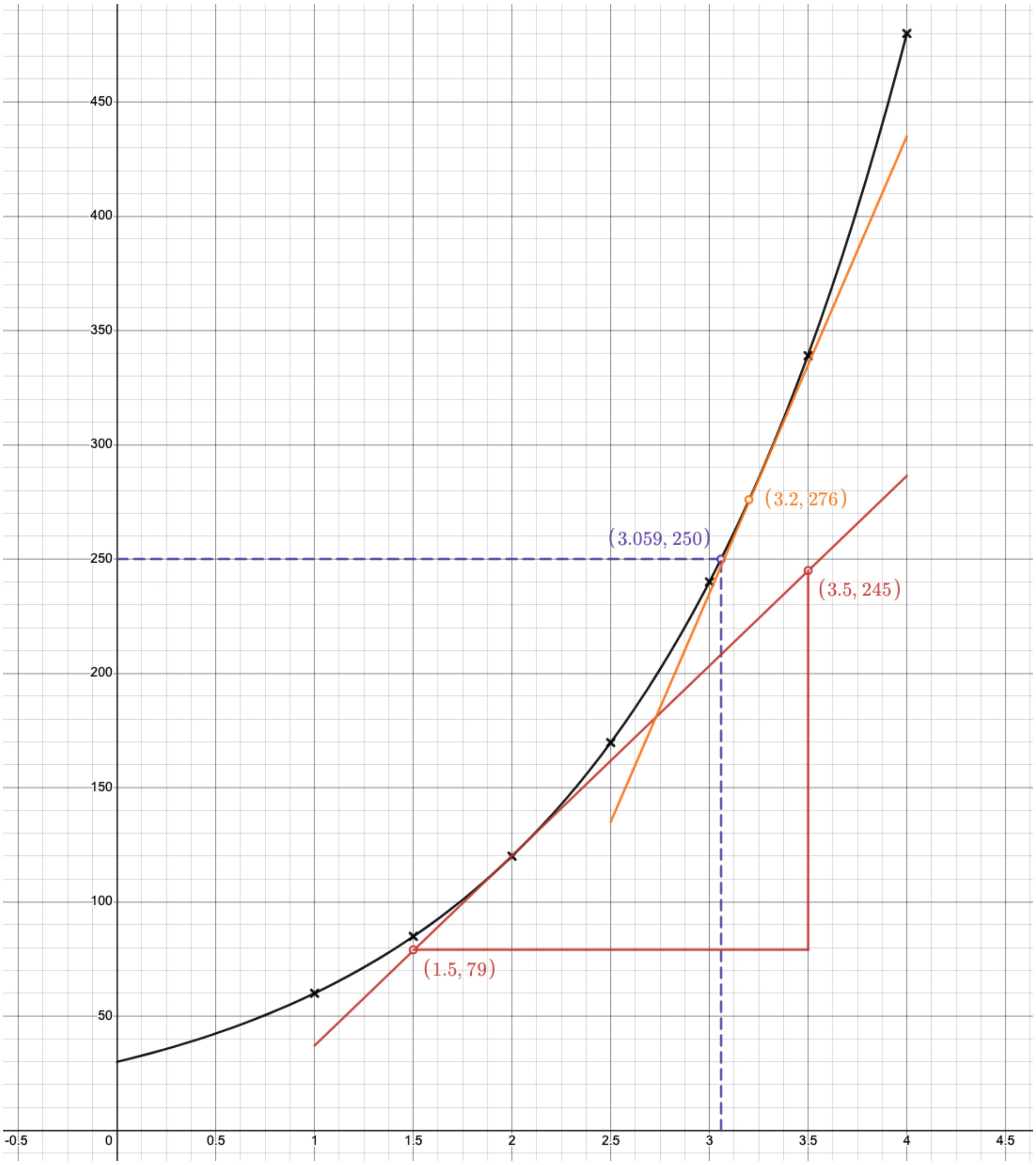
$$\begin{aligned} \text{Gradient} &= \frac{245 - 79}{3.5 - 1.5} \\ &= \mathbf{83} \end{aligned}$$

At $t = 2$, the flies are **increasing at 83 flies per day**

(f)

$$t = \mathbf{3.2\ s\ (2.s.f.)}$$

Graph for Question 4



7 Equations & Inequalities

7.1 Full Solutions

1. (a)

$$\left(\boldsymbol{\frac{900}{n}}\right) \textbf{ jars of cookies}$$

(b)

$$\boldsymbol{\$\left(\frac{900}{n} - 2\right)(n+3)}$$

(c)

$$\begin{aligned}\left(\frac{900}{n} - 2\right)(n+3) - 900 &= 92\\ 900 + \frac{2700}{n} - 2n - 6 - 900 &= 92\\ \frac{2700}{n} - 2n - 6 &= 92\\ 2700 - 2n^2 - 98n &= 0\\ m^2 + 49n - 1350 &= 0 \textbf{ (shown)}\end{aligned}$$

□

(d)

$$n^2 + 49n - 1350 = 0$$

$$\begin{aligned}n &= \frac{-49 \pm \sqrt{(49)^2 - 4(1)(-1350)}}{2(1)}\\ &= \frac{-49 \pm \sqrt{7801}}{2}\end{aligned}$$

$$n = 19.662 \textbf{ (3.d.p.)} \qquad \text{or} \qquad n = -68.662 \textbf{ (3.d.p.)}$$

(e)

$$\begin{aligned}\text{Selling price} &= \left(\frac{-49 + \sqrt{7801}}{2}\right)\\ &= \textbf{\$22.66 (nearest cents)}\end{aligned}$$

2. (a) (i)

$$\begin{aligned} 3x^2 - 507 &= 0 \\ x^2 &= 169 \\ \boldsymbol{x} &\boldsymbol{= \pm 13} \end{aligned}$$

(ii)

$$\begin{aligned} \frac{x-2}{x+3} - \frac{x-3}{12+4x} &= 5 \\ \frac{x-2}{x+3} - \frac{x-3}{4(x+3)} &= 5 \\ 4(x-2) - (x-3) &= 5[4(x+3)] \\ 4x - 8 - x + 3 &= 20x + 60 \end{aligned}$$

$$\boldsymbol{x = -3\frac{14}{17}}$$

(b) (i)

$$\begin{aligned} \frac{1}{9c^{-2}} \times \frac{(3d)^{-1}}{c^3} &= \frac{c^2}{9} \times \frac{1}{3c^3 d} \\ &\boldsymbol{= \frac{1}{27cd}} \end{aligned}$$

(ii)

$$\begin{aligned} \frac{5f^2g^3}{21gh} \div \frac{40f^5g^2}{7h^3} &= \frac{5f^2g^2}{21h} \times \frac{7h^3}{40f^5g^2} \\ &= \frac{35f^2g^2h^3}{840f^5g^2h} \\ &\boldsymbol{= \frac{h^2}{24f^3}} \end{aligned}$$

(c) (i)

$$\begin{aligned} (x+y)_{max} &= x_{max} + y_{max} \\ &= 2 + 3 \\ &= \mathbf{5} \end{aligned}$$

(ii)

$$\begin{aligned} xy_{min} &= (x_{min})\,(y_{min}) \\ &= -7 \times 3 \\ &= \mathbf{-21} \end{aligned}$$

(iii)

$$\begin{aligned} (x^2 - y^2)_{max} &= (x_{max})^2 - (y_{min})^2 \\ &= (-7)^2 - 0 \\ &= \mathbf{49} \end{aligned}$$

3. (a)

$$\text{Outbound trip} = \mathbf{\frac{500}{\boldsymbol{x}}\ hours}$$

(b) (i)

$$\text{Return trip} = \mathbf{\frac{500}{\boldsymbol{x}-25}\ hours}$$

(ii)

$$\frac{500}{x-25} - \frac{500}{x} = \frac{15}{60}$$
$$60[500(x) - 500(x-25)] = 15(x)(x-25)$$
$$30000x - 30000x + 750000 = 15x^2 - 375x$$
$$15x^2 - 375x - 750000 = 0$$
$$x^2 - 25x - 50000 = 0 \ \textbf{(shown)}$$

(iii)

$$x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(-50000)}}{2} = \frac{25 \pm \sqrt{200625}}{2}$$
$$x = 236.455910... \quad \text{or} \quad x = -211.455910$$
$$\boldsymbol{x} = \mathbf{236.46\ (2.d.p.)} \quad \text{or} \quad \boldsymbol{x} = \mathbf{-211.46\ (2.d.p.)}$$

(iv)

$$\text{Return trip} = \frac{500}{\left(\frac{25+\sqrt{200625}}{2}\right) - 25}$$
$$= 2.364559...$$
$$= \mathbf{2\ hours\ 22\ minutes}$$

8 Set Language & Notation

8.1 Full Solutions

1. (a)

$$\mathscr{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$$
$$E = \{2, 3, 5, 7, 11, 13\}$$
$$F = \{2, 3, 6, 9\}$$
$$G = \{3, 6, 9, 12\}$$

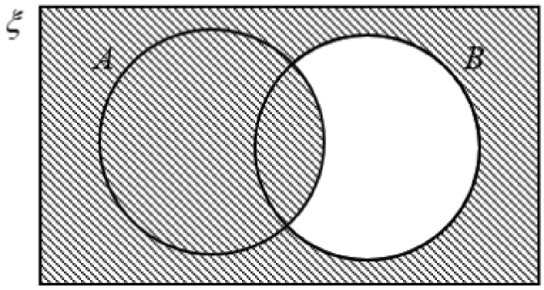
(i)

$$E \cap F = \mathbf{\{2, 3\}}$$

(ii)

$$\boldsymbol{11 \in E} \qquad \boldsymbol{F \cap G = \{3, 6, 9\}}$$

(b) Diagram



(c)

$$\boldsymbol{(A \cap B)' \cap (A \cup B)}$$

2.

$$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$$
$$A = \{3, 6, 9, 12, 15\}$$
$$B = \{1, 4, 9, 16\}$$
$$C = \{10, 11, 12, 13, 14, 15, 16\}$$

(a)

$$\boldsymbol{A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16\}}$$

(b)

$$\boldsymbol{A \cap B = \{9\}}$$

(c)

$$A \cup B \cup C = \{1, 3, 4, 6, 9, 10, 11, 12, 13, 14, 15, 16\}$$
$$\therefore \boldsymbol{(A \cup B \cup C)' = \{2, 5, 7, 8\}}$$

9 Matrices

9.1 Full Solutions

1. (a) (i)

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 20 & 21 \\ 30 & 34 \\ 16 & 15 \end{pmatrix} - \begin{pmatrix} 18 & 15 \\ 32 & 37 \\ 11 & 14 \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{2} & \mathbf{6} \\ \mathbf{-2} & \mathbf{-3} \\ \mathbf{5} & \mathbf{1} \end{pmatrix}$$

(ii) The **change** in new subscriptions **between May and June** which **Aldrick and Bryan** obtained for **packages F, G and H respectively**

(b) (i)

$$\mathbf{C} = \begin{pmatrix} \mathbf{30} & \mathbf{45} & \mathbf{60} \end{pmatrix}$$

(ii)

$$\mathbf{CA} = \begin{pmatrix} 30 & 45 & 60 \end{pmatrix} \begin{pmatrix} 18 & 15 \\ 32 & 37 \\ 11 & 14 \end{pmatrix}$$
$$= \begin{pmatrix} 30(18) + 45(32) + 60(11) & 30(15) + 45(37) + 60(14) \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{2640} & \mathbf{2955} \end{pmatrix}$$

(iii) The **total sales commissioned Aldrick and bryan received respectively in May**

2. (a)

$$\mathbf{A} = \begin{pmatrix} \mathbf{390} & \mathbf{300} & \mathbf{350} \\ \mathbf{150} & \mathbf{200} & \mathbf{180} \end{pmatrix}$$

(b)

$$\mathbf{H} = \begin{pmatrix} \mathbf{0.15} & \mathbf{0.45} \end{pmatrix}$$

(c) (i)

$$\begin{aligned} \mathbf{R} &= \mathbf{A+B} \\ &= \begin{pmatrix} 390 & 300 & 350 \\ 150 & 200 & 180 \end{pmatrix} + \begin{pmatrix} 220 & 250 & 200 \\ 260 & 230 & 170 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{610} & \mathbf{550} & \mathbf{550} \\ \mathbf{410} & \mathbf{430} & \mathbf{350} \end{pmatrix} \end{aligned}$$

(ii)

$$\begin{aligned} \mathbf{M} &= \mathbf{RL} \\ &= \begin{pmatrix} 610 & 550 & 550 \\ 410 & 430 & 350 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 610 + 550 + 550 \\ 410 + 430 + 350 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{1710} \\ \mathbf{1190} \end{pmatrix} \end{aligned}$$

(iii) The elements represent the **total weight of** newspapers and clothes collected **respectively** by the 3 classes, from the **2 collections/weeks**

(iv)

$$\begin{aligned} \mathbf{HM} &= \begin{pmatrix} 0.15 & 0.45 \end{pmatrix} \begin{pmatrix} 1710 \\ 1190 \end{pmatrix} \\ &= \begin{pmatrix} 0.15(1710) + 0.45(1190) \end{pmatrix} \\ &= \mathbf{(792)} \end{aligned}$$

(v)

$$\therefore \text{Amount of money raised} = \mathbf{\$792}$$

3. (a) (i)

$$\mathbf{B} = \begin{pmatrix} \mathbf{20} \\ \mathbf{30} \\ \mathbf{10} \end{pmatrix}$$

(ii)

$$\begin{aligned} \mathbf{C} &= \mathbf{AB} \\ &= \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \\ &= \begin{pmatrix} 5(20) + 3(30) + 4(10) \\ 2(20) + 1(30) + 2(10) \\ 2(20) + 3(30) + 2(10) \\ 1(20) + 2(30) + 3(10) \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{230} \\ \mathbf{90} \\ \mathbf{150} \\ \mathbf{110} \end{pmatrix} \end{aligned}$$

(iii) The elements of **C** represent the **total number** of buns, of toothbrushes, of packets of Milo and of packets of coffee respectively, needed to pack **all** the bags

(b) (i)

$$\mathbf{D} = \begin{pmatrix} \mathbf{1} & \mathbf{1.5} & \mathbf{6.4} & \mathbf{5.6} \end{pmatrix}$$

(ii)

$$\begin{aligned} \mathbf{E} &= \mathbf{DA} \\ &= \begin{pmatrix} 1 & 1.5 & 6.4 & 5.6 \end{pmatrix} \begin{pmatrix} 5 & 3 & 4 \\ 2 & 1 & 2 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 1(5) + 1.5(2) + 6.4(2) + 5.6(1) \\ 1(3) + 1.5(1) + 6.4(3) + 5.6(2) \\ 1(4) + 1.5(2) + 6.4(2) + 5.6(3) \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{26.4} & \mathbf{34.9} & \mathbf{36.6} \end{pmatrix} \end{aligned}$$

(c)

$$\begin{aligned} \mathbf{F} &= \mathbf{EB} \\ &= \begin{pmatrix} 26.4 & 34.9 & 36.6 \end{pmatrix} \begin{pmatrix} 20 \\ 30 \\ 10 \end{pmatrix} \\ &= \begin{pmatrix} 26.4(20) + 34.9(30) + 36.6(10) \end{pmatrix} \\ &= \mathbf{(1941)} \end{aligned}$$

(d) The element of **F** represent the **total cost** in dollars of **all the items** needed to pack **all the goodie bags** altogether

4. (a)

$$\mathbf{N} = \begin{pmatrix} \mathbf{14} & \mathbf{12} & \mathbf{10} \\ \mathbf{13} & \mathbf{21} & \mathbf{16} \end{pmatrix}$$

(b)

$$\mathbf{P} = \begin{pmatrix} \mathbf{26} & \mathbf{29} & \mathbf{18} \\ \mathbf{31} & \mathbf{36} & \mathbf{27} \end{pmatrix}$$

(c) **Total products sold on Saturdays and Sundays respectively**

(d)

$$\begin{aligned} \mathbf{Q} &= \frac{1}{2}\begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix}\begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 26(2.5) + 29(3.2) + 18(4.5) \\ 31(2.5) + 36(3.2) + 27(4.5) \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{119.4} \\ \mathbf{157.1} \end{pmatrix} \end{aligned}$$

(e) **Average sum received from selling on Saturday and Sunday**

(f)

$$\begin{aligned} \mathbf{R} &= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \\ &= \begin{pmatrix} 238.8 + 314.2 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{553} \end{pmatrix} \end{aligned}$$

Total amount earned on both Saturday and Sunday

10 Problems in Real-World Context

10.1 Full Solutions

1. CBSH Card

$$\begin{aligned}\text{Petrol savings} &= 0.14 \times 350 + 0.05 \times 350\\ &= \$66.50\end{aligned}$$

$$\begin{aligned}\text{Dining savings} &= 0.05 \times 400\\ &= \$20\end{aligned}$$

$$\begin{aligned}\text{Grocery savings} &= 0.05 \times 100\\ &= \$5\end{aligned}$$

$$\begin{aligned}\text{Total savings} &= \$66.50 + \$20 + \$5\\ &= \$91.50\end{aligned}$$

BSOP Card

$$\begin{aligned}\text{Petrol savings} &= 0.15 \times 350\\ &= \$52.50\end{aligned}$$

$$\text{Dining savings} = \$0$$

$$\begin{aligned}\text{Grocery savings} &= 0.05 \times 100\\ &= \$5\end{aligned}$$

$$\begin{aligned}\text{Total savings} &= \$52.50 + \$0 + \$5\\ &= \$57.50\end{aligned}$$

CBCO Card

$$\begin{aligned}\text{Petrol savings} &= 0.14 \times 350 + 0.043 \times 350\\ &= \$64.05\end{aligned}$$

$$\begin{aligned}\text{Dining savings} &= 0.05 \times 400\\ &= \$20\end{aligned}$$

$$\begin{aligned}\text{Grocery savings} &= 0.05 \times 100\\ &= \$5\end{aligned}$$

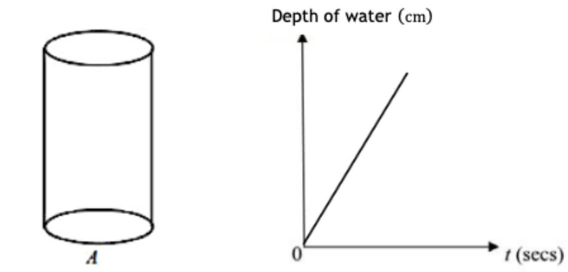
$$\begin{aligned}\text{Total savings} &= \$64.05 + \$20 + \$5\\ &= \$89.05\end{aligned}$$

Hence, he should apply for the **CBSH card**

2. (a) Graphs

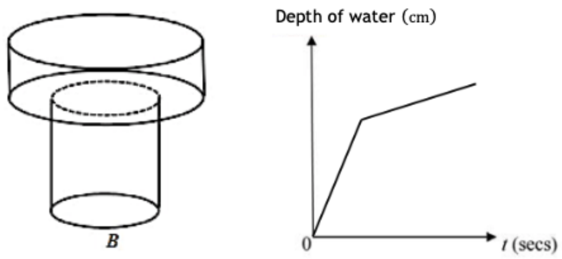
Solution

Tips / Thinking Process



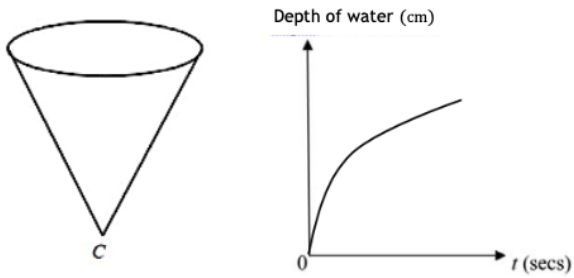
For container ***A***, the container is completely uniform. Hence, the rate of the water entering the container will be constant

The graph is a straight line with a positive gradient



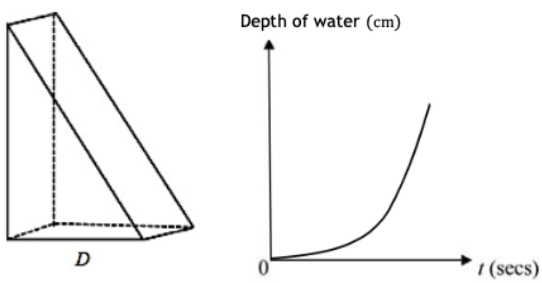
For container ***B***, the base of the bottom part of the container is significantly smaller than the top part of the container. The rate of water filling up the bottom part of the container will not only be uniform but significantly faster than the top part of the container

The graph is a straight line split into **2** segments, where the initial gradient is much steeper than the second part of the line



For container ***C***, the container is a cylinder. The initial portion (at the tip of the cone) is going to fill up so much faster than the top of the container (base of the cone). The rate of water filling up the container is decreasing

The graph is a curve with a decreasing gradient



For container ***D***, the container is a triangular prism. The base of the prism (rectangular base) is going to fill up so much slower than the top of the container (tip of the triangle). The rate of water filling up the container is increasing

The graph is a curve with an increasing gradient

Taken from 2022 Overmugged E-Math Curated Notes (Volume A)

(b)

$$\text{Ratio of volume of prism} = \frac{1}{4}$$

$$\begin{aligned}\text{Required time} &= \frac{3}{4} \times 12 \\ &= \mathbf{9\ seconds}\end{aligned}$$

3. (a)

$$\begin{aligned}\text{Mean length of time} &= \frac{6 \times 4 + 7.25 + 8 \times 2}{7} \\ &= \textbf{6.75 hours}\end{aligned}$$

(b) (i)

$$\textbf{Multiplied the given annual consumption by } \frac{\mathbf{6.75}}{\mathbf{8}}$$

(ii) Model S

$$\begin{aligned}\text{Cost of electricity} &= 25.3 \times 1755 \\ &= \$444.015\end{aligned}$$

$$\begin{aligned}\text{Cost (before discount) of servicing} &= \$35 \times 3 \times 1.07 \\ &= \$112.35\end{aligned}$$

$$\begin{aligned}\text{Cost (after discount) of servicing} &= \$112.35 \times 60\% \\ &= \$67.41\end{aligned}$$

$$\begin{aligned}\text{Total for Model S} &= \$650 + 7\,(\$444.015 + \$67.41) \\ &= \$4229.975... \\ &= \textbf{\$4229.96 (2.d.p.)}\end{aligned}$$

Model E

$$\begin{aligned}\text{Cost of electricity} &= 25.3 \times 1066.5 \\ &= \$269.8245\end{aligned}$$

$$\begin{aligned}\text{Total for Model E} &= \$1300 + 7\,(\$269.8245 + \$112.35) \\ &= \$3975.2215... \\ &= \textbf{\$3975.22 (2.d.p.)}\end{aligned}$$

Since **Model E** is obviously cheaper, Meg should get **Model E**

4. (a) (i)

$$\begin{aligned}\text{Perimeter} &= 3\left[\text{Arc} + \text{Straight line}\right]\\ &= 3\left[6 + 3\left(\frac{120}{180}\pi\right)\right]\\ &= 36.84955...\\ &= \mathbf{36.85\ cm\ (2.d.p.)\ \ (shown)}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Area} &= 3\text{ rectangle} + 3\text{ sector} + 1\text{ triangle}\\ &= 3(6)(3) + 3\left[\frac{1}{2}(3)^2\left(\frac{120}{180}\pi\right)\right] + \frac{1}{2}(6)^2\sin 60^\circ\\ &= 97.8627...\\ &= \mathbf{97.86\ cm^2\ (2.d.p.)\ \ (shown)}\end{aligned}$$

(b) The goal is to find the least surface area of the 2 containers

$$\begin{aligned}\text{Total surface area of Diagram 2} &= 2\text{ circles} + 1\text{ rectangle}\\ &= 2\pi(3)^2 + 2\pi(3)(6)(3)\\ &= 126\pi\\ &= 395.84\text{ cm}^2\text{ (5.s.f.)}\end{aligned}$$

$$\begin{aligned}\text{Total surface area of Diagram 3} &= 2(\text{part(a)(ii)}) + 1\text{ rectangle}\\ &= 2\underline{\mathbf{(a)(ii)}} + \left[\left(\underline{\mathbf{(a)(i)}}\right)\times 6\right]\\ &= 416.82\text{ cm}^2\text{ (5.s.f.)}\end{aligned}$$

Comparing the 2 surface areas, the total surface area of the cylinder is less than the total surface area of the triangular box, which needs less material, making it more environmentally friendly. Hence, it is cheaper to use the **closer cylinder design**

Note: **(a)(i)** and **(a)(ii)** are the numerical answers from part (a). Remember to use at least 5 significant figures or the exact form in your solution and computation

11 Angles, Triangles & Polygons

11.1 Full Solutions

1. (a)

$$\text{Each exterior angle} = \mathbf{\frac{360}{2n+6}}$$

(b)

$$\begin{aligned} \frac{360}{n} - \frac{180}{n+3} &= 14 \\ \frac{360(n+3) - 180n}{n(n+3)} &= 14 \\ 360n + 1080 - 180n &= 14n(n+3) \\ 14n^2 + 42n - 180n - 1080 &= 0 \\ 14n^2 - 138n - 1080 &= 0 \ \textbf{(shown)} \end{aligned}$$

□

(c)

$$\begin{aligned} 14n^2 - 138n - 1080 &= 0 \\ (n-15)(7n+36) &= 0 \end{aligned}$$

$$n = \mathbf{15} \qquad \text{or} \qquad n = \mathbf{-5\frac{1}{7}}$$

(d)

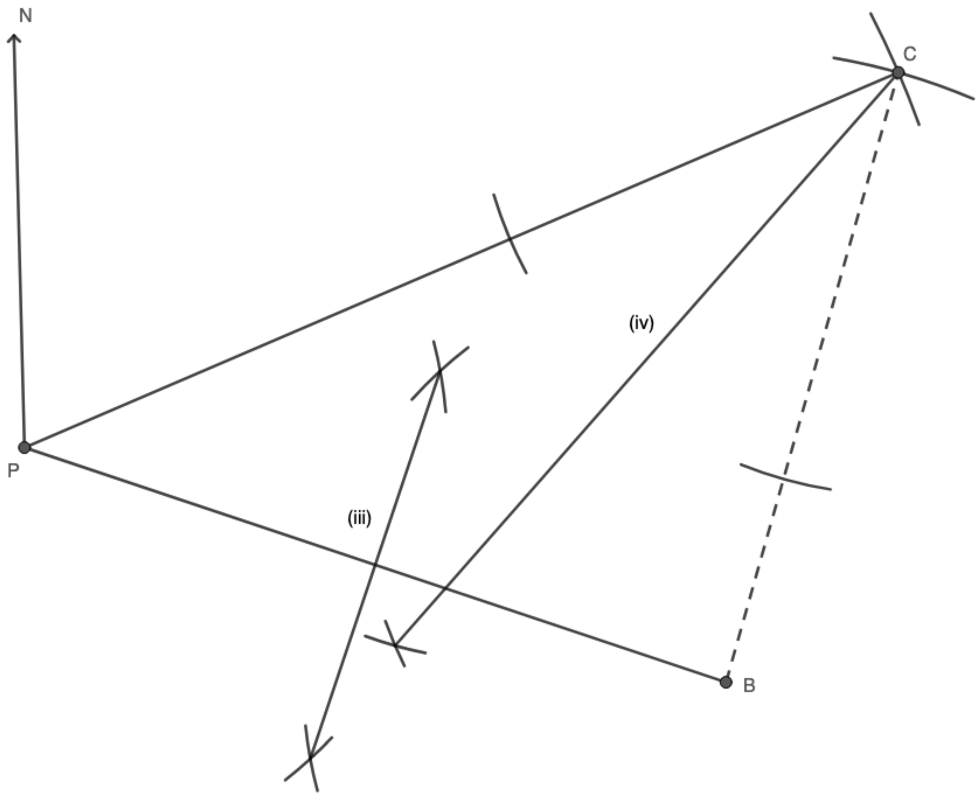
$$\boldsymbol{n = -5\frac{1}{7} \notin \mathbf{Z}^+} \qquad \Rightarrow\Leftarrow$$

n must be a positive integer since it is the number of sides of a polygon

(e)

$$\begin{aligned} \text{Each interior angle} &= 180^\circ - \left(\frac{360^\circ}{15}\right) \\ &= \mathbf{156^\circ} \end{aligned}$$

2. Diagram



3. (a)

Rhombus

(b) (i)

$$\text{Interior angle of hexagon} = \frac{(6-2)\times 180^\circ}{6} = \mathbf{120^\circ}$$

(ii)

$\angle FAB = \angle CBA = 120^\circ$ (interior angle of a regular hexagon) (A)

$FA = BC$ (sides of a regular hexagon) (S)

AB is a common side (S)

By the **SAS** congruency test, $\angle FAB = \angle CBA$

□

(iii)

$$\angle AFB = \frac{180^\circ - 120^\circ}{2} = 30^\circ \text{ (angles in an isosceles triangle)}$$

$$\angle ABF = \angle BAC = 30^\circ \text{ (congruent triangles)}$$

$$\angle AXB = 180^\circ - 30^\circ - 30^\circ = 120^\circ \text{ (angles in a triangle)}$$

$$\therefore \angle FXC = \mathbf{120^\circ} \textbf{ (vertically opposite angles)}$$

(iv)

$$\angle FAD = \frac{120^\circ}{2} = 60^\circ \text{ (angle bisector)}$$

$$\angle FAD + \angle AFE = 60^\circ + 120^\circ = 180^\circ$$

By the property: **interior angles**, FE is **parallel** to AD

4. (a) (i)

$$\text{Interior angle of a regular octagon} = \frac{(8-2)\times 180^\circ}{8}$$
$$= \mathbf{135^\circ}$$

(ii)

$$\angle CDE = 360^\circ - 135^\circ - 75^\circ$$
$$= 150^\circ \text{ (angles about a point)}$$
$$\text{Exterior angle} = 30^\circ \text{ (supplementary angles)}$$

$$\therefore n = \frac{360^\circ}{30^\circ}$$
$$= \mathbf{12}$$

(iii)

$$\angle BAC = \frac{180^\circ - 150^\circ}{2}$$
$$= \mathbf{15^\circ \text{ (angles in an isosceles triangle)}}$$

(iv)

$$\angle ACE = 150^\circ - 15^\circ - 15^\circ$$
$$= 120^\circ$$
$$\therefore \angle CAE = \frac{180^\circ - 120^\circ}{2}$$
$$= \mathbf{30^\circ \text{ (angles in an isosceles triangle)}}$$

(b)

$$\angle ABC = \angle CDE \text{ (interior angles of a polygon) (A)}$$
$$AB = BC = CD = DE \text{ (sides of a regular polygon) (S) (S)}$$
$$\therefore \text{By the } \mathbf{SAS} \text{ congruency test, } \triangle ABC \equiv \triangle CDE$$

(c) Assume that it is true to have a circle with diameter AE and point C on the circumference.This implies that

$$\angle ACE = 90^\circ \text{ (angles in a semicircle)}$$

However, from part (a)(iv), $\angle ACE = 120^\circ \neq 90^\circ$. $\Rightarrow\Leftarrow$ Hence, it is **not possible**

12 Congruency & Similarity

12.1 Full Solutions

1. (a)

$$BD \text{ is a common side (S)}$$
$$\angle BAD = \angle BCD \text{ (angles in a semicircle) (A)}$$

$$\begin{aligned}\angle ADB &= 180^\circ - 106^\circ - 37^\circ \\ &= 37^\circ \text{ (angles in opposite segment)}\end{aligned}$$

$$\therefore \angle ADB = \angle CDB \text{ (A)}$$

By the **ASA** congruency test, $\triangle ABD \equiv \triangle CBD$

□

(b)

$$\begin{aligned}\cos \angle CDB &= \frac{CD}{BD} \\ CD &= 8\cos 37^\circ \\ &= 6.389084... \\ &= \mathbf{6.39\ cm}\end{aligned}$$

(c)

$$\begin{aligned}\text{Shaded Area} &= \text{Area of sector } AECD - \text{Area of triangle } ACD \\ &= \frac{1}{2}(CD)^2\angle ADC - \frac{1}{2}(AD)(CD)\sin\angle ADC \\ &= \frac{1}{2}\left[8\cos\left(\frac{37^\circ}{180^\circ}\pi\right)\right]^2\left(\frac{74^\circ}{180^\circ}\pi\right) - \frac{1}{2}\left[8\cos\left(\frac{37^\circ}{180^\circ}\pi\right)\right]^2\sin\left(\frac{74^\circ}{180^\circ}\pi\right) \\ &= 6.741119... \\ &= \mathbf{6.74\ cm^2}\end{aligned}$$

Take Note

Since calculation of the area of a sector must be done in radians, all angles must be converted into radians

2.

$$\begin{aligned}\text{Curved surface of cylinder} &= 2\pi(2r)h \\ &= 4\pi rh\end{aligned}$$

Let s be the small cone, b be the big cone

$$\begin{aligned}\text{Curved surface of frustrum} &= \text{Curved}_{\text{ big cone}} - \text{Curved}_{\text{ small cone}} \\ &= 2\pi rl_b - \pi rl_s\end{aligned}$$

The big cone is geometrically similar to the small cone

$$\frac{r_s}{r_b} = \frac{h_s}{h_b} = \frac{l_s}{l_b} = \frac{1}{2}$$

$$\begin{aligned}\frac{A_s}{A_b} &= \left(\frac{l_s}{l_b}\right)^2 \\ &= \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4}\end{aligned}$$

$$\text{Curved}_{\text{ small cone}} = \pi r\sqrt{r^2+h^2}$$
$$\therefore \text{Curved}_{\text{ big cone}} = 3\pi r\sqrt{r^2+h^2}$$

$$\begin{aligned}3\pi r\sqrt{r^2+h^2} &= 4\pi rh \\ \sqrt{r^2+h^2} &= \frac{4}{3}h \\ h^2+r^2 &= \frac{16}{9}h^2 \\ h^2 &= \frac{9}{7}r^2 \\ h &= \pm\sqrt{\frac{9}{7}r^2} \text{ (rej -ve)} \\ &= \boldsymbol{\frac{3}{\sqrt{7}}r}\end{aligned}$$

3. (a)

$$\triangle \boldsymbol{RTS}$$

(b) (i)

$$\angle PVS = \angle UVQ \text{ (vertically opposite angles) (A)}$$
$$\angle VPS = \angle VUQ \text{ (alternate angles) (A)}$$
$$\angle VSP = \angle VQU \text{ (alternate angles) (A)}$$

By the **AAA** similarity test, $\triangle PVS$ is similar to $\triangle UVQ$

(ii)

$$\frac{QV}{QS} = \frac{1}{3}$$
$$QS = 2QT$$
$$\frac{QV}{2QT} = \frac{1}{3}$$
$$\frac{QV}{QT} = \frac{2}{3}$$

$$\therefore \boldsymbol{QV = 2VT} \textbf{ (shown)}$$

□

(c)

$$\begin{aligned}\frac{\text{Area of } \triangle QVU}{\text{Area of trapezium } PURS} &= \frac{\text{Area of } \triangle QVU}{\text{Area of } \triangle QPU} \times \frac{\text{Area of } \triangle QPU}{\text{Area of trapezium } PURS} \\ &= \frac{1}{3} \times \frac{3}{9} \\ &= \frac{1}{9}\end{aligned}$$

$$\therefore \mathbf{1 : 9}$$

13 Properties of Circles

13.1 Full Solutions

1. (a)

$$\angle OEA = \mathbf{35^\circ} \textbf{ (angle in the same segment)}$$

(b)

$$\begin{aligned}\angle AOB &= 35^\circ \times 2\\ &= 70^\circ \text{ (angle at centre is 2 times angle at circumference)}\end{aligned}$$

$$\begin{aligned}\text{Reflex}\angle AOB &= 360^\circ - 70^\circ\\ &= \mathbf{290^\circ} \textbf{ (angles about a point)}\end{aligned}$$

(c) Since OA and OB are radii of the circle, $\triangle AOB$ is an isosceles triangle

$$\begin{aligned}\angle BAO &= \frac{180^\circ - 70^\circ}{2}\\ &= 55^\circ \text{ (angles in an isosceles triangle)}\end{aligned}$$

$$\begin{aligned}\therefore \angle BAC &= 55^\circ - 40^\circ\\ &= \mathbf{15^\circ}\end{aligned}$$

(d) Since OE and OA are radii of the circle, $\triangle OAE$ is an isosceles triangle

$$\angle OAE = 35^\circ \text{ (angles in an isosceles triangle)}$$

$$\begin{aligned}\angle CDE &= 180^\circ - 40^\circ - 35^\circ\\ &= 105^\circ \text{ (angles in opposite segments)}\end{aligned}$$

$$\angle FEB = \mathbf{105^\circ} \textbf{ (corresponding angles)}$$

(e)

$$\begin{aligned}\angle OED &= 180^\circ - 105^\circ\\ &= 75^\circ \text{ (interior angles)}\end{aligned}$$

Since OE and OD are radii of the circle, $\triangle OED$ is an isosceles triangle

$$\begin{aligned}\angle DOE &= 180^\circ - 75^\circ - 75^\circ\\ &= \mathbf{15^\circ} \textbf{ (angles in an isosceles triangle)}\end{aligned}$$

2. (a) (i)

$$\begin{aligned}\angle BCA &= 30^\circ + 30^\circ \\ &= 60^\circ \text{ (exterior angle in a triangle)}\end{aligned}$$

$$\angle ABC = 90^\circ \text{ (angles in a semicircle)}$$

$$\begin{aligned}\angle BAC &= 180^\circ - 60^\circ - 90^\circ \\ &= \mathbf{30^\circ} \textbf{ (sum of angles in a triangle)}\end{aligned}$$

(ii)

$$\angle CEB = \mathbf{30^\circ} \textbf{ (angles in the same segment)}$$

(iii)

$$\begin{aligned}\angle CBE &= 180^\circ - 140^\circ \\ &= \mathbf{40^\circ} \textbf{ (angles in opposite segment)}\end{aligned}$$

(iv)

$$\angle OBX = 90^\circ \text{ (tangent is perpendicular to radius)}$$

$$\begin{aligned}\angle OBY &= 90^\circ - 40^\circ - 30^\circ \\ &= \mathbf{20^\circ} \textbf{ (complementary angles)}\end{aligned}$$

(b)

$$\begin{aligned}\angle ABE &= 20^\circ + 30^\circ \\ &= 50^\circ \text{ (base angles in an isosceles triangle)}\end{aligned}$$

$$T \textbf{ lies inside the circle} \because \angle ATE > \angle ABE$$

3. (a) (i)

$$\angle EBA = \mathbf{23^\circ} \textbf{ (angles in the same segment)}$$

(ii)

$$\begin{aligned}\angle EBC &= 90^\circ - 23^\circ \\ &= \mathbf{67^\circ} \textbf{ (angles in a semicircle)}\end{aligned}$$

(iii)

$$\begin{aligned}\angle CDE &= 180^\circ - 67^\circ \\ &= \mathbf{113} \textbf{ (angles in opposite segment)}\end{aligned}$$

(b) Let D be the centre of the circle

$$\angle DCE = 90^\circ \text{ (tangent is perpendicular to radius)}$$

$$\begin{aligned}\angle CBE &= 65^\circ \times 2 \\ &= 130^\circ \text{ (angle at centre is 2 times angle at circumference)}\end{aligned}$$

$$\begin{aligned}\text{Sum of angles in the quadrilateral} &= 90^\circ + 90^\circ + 130^\circ + 48^\circ \\ &= 358^\circ \neq 360^\circ\end{aligned}$$

Since the sum of angles in the quadrilateral is 358°, this is absurd as the total interior angles of a quadrilateral is 360° $\Rightarrow\Leftarrow$

D is **not** the centre of the circle

14 Trigonometry

14.1 Full Solutions

1. (a) (i) By Sine Rule,

$$\begin{aligned}\frac{AB}{\sin 40^\circ} &= \frac{120}{\sin 60^\circ}\\ AB &= \frac{120\sin 40^\circ}{\sin 60^\circ}\\ &= 89.067...\\ &= \mathbf{89.1^\circ\ (1.d.p.)}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Area} &= \frac{1}{2}\left(\frac{120\sin 40^\circ}{\sin 60^\circ}\right)(120)\sin 80^\circ\\ &= 5263.83...\\ &= \mathbf{5260\ cm\ (3.s.f.)}\end{aligned}$$

(iii) In $\triangle BEC$

$$\begin{aligned}BE &= 120\sin 30^\circ\\ &= 60 \text{ cm}\end{aligned}$$

The angle of depression corresponds to the angle of elevation of A from B. Let this be θ

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{100-60}{\left(\frac{120\sin 40^\circ}{\sin 60^\circ}\right)}\right)\\ &= \mathbf{26.7^\circ\ (1.d.p.)}\end{aligned}$$

(b) (i)

$$\begin{aligned}\angle ABC &= 150^\circ - (245^\circ - 180^\circ)\\ &= 85^\circ\end{aligned}$$

By Cosine Rule,

$$\begin{aligned}AC &= \sqrt{(90)^2+(160)^2-2(90)(160)\cos 85^\circ}\\ &= 176.61...\\ &= \mathbf{177\ m\ (3.s.f.)}\end{aligned}$$

(ii)

$$\begin{aligned}h &= \frac{1}{3}(28)\\ &= \frac{28}{3}\text{ m}\end{aligned}$$

$$XC = \frac{\left(\frac{28}{3}\right)}{\tan 15^\circ}$$

$$\begin{aligned}\text{Distance walked} &= \sqrt{(90)^2+(160)^2-2(90)(160)\cos 85^\circ} - \frac{\left(\frac{28}{3}\right)}{\tan 15^\circ}\\ &= 141.78 \text{ m}\\ &= \mathbf{142\ m\ (3.s.f.)}\end{aligned}$$

2. (a)

$$\begin{aligned}\angle PQS &= 180^\circ - 126^\circ \\ &= 54^\circ\end{aligned}$$

By Sine Rule,

$$\begin{aligned}\frac{QS}{\sin 68^\circ} &= \frac{460}{\sin 54^\circ} \\ QS &= \frac{460 \sin 68^\circ}{\sin 54^\circ} \\ &= 527.188... \\ &= \mathbf{527\ m\ (3.s.f.)}\end{aligned}$$

(b)

$$\begin{aligned}\angle RQS &= 126^\circ - 98^\circ \\ &= 28^\circ\end{aligned}$$

By Cosine Rule,

$$\begin{aligned}RS &= \sqrt{(562)^2 + \left(\frac{460 \sin 68^\circ}{\sin 54^\circ}\right)^2 - 2(562)\left(\frac{460 \sin 68^\circ}{\sin 54^\circ}\right)\cos 28^\circ} \\ &= 265.654... \\ &= \mathbf{266\ m\ (3.s.f.)}\end{aligned}$$

(c) Let the distance be x

$$\begin{aligned}\angle XQR &= 180^\circ - 98^\circ \\ &= 82^\circ\end{aligned}$$

$$\begin{aligned}\cos 82^\circ &= \frac{x}{562} \\ x &= 562 \cos 82^\circ \\ &= 78.215282... \\ &= \mathbf{78.2\ m\ (3.s.f.)}\end{aligned}$$

3. (a)

$$\begin{aligned}\angle PRQ &= 241^\circ - 180^\circ \\ &= 61^\circ \text{ (alternate angles)}\end{aligned}$$

By Cosine Rule,

$$\begin{aligned}PQ &= \sqrt{(72)^2 + (85)^2 - 2(72)(85)\cos 61^\circ} \\ &= 80.466951... \\ &= \mathbf{80.5\ m\ (3.s.f.)}\end{aligned}$$

(b) Let the first possible point S be S_1 and the second possible point be S_2

$$\begin{aligned}\angle QRS &= 99^\circ - 61^\circ \\ &= 38^\circ\end{aligned}$$

In $\triangle RQS$, by Sine Rule,

$$\begin{aligned}\frac{\sin \angle RSQ}{85} &= \frac{\sin 38^\circ}{54} \\ \sin \angle RSQ &= \frac{8\sin 38^\circ}{54}\end{aligned}$$

Using the first possible point S_1,

$$\begin{aligned}\angle RS_1Q &= \frac{8\sin 38^\circ}{54} \\ &= 75.7188... \\ &= \mathbf{75.7^\circ\ (1.d.p.)}\end{aligned}$$

Using the second possible point S_2,

$$\begin{aligned}\angle RS_2Q &= 180^\circ - \left(\frac{8\sin 38^\circ}{54}\right) \\ &= 104.2812... \\ &= \mathbf{104.3^\circ\ (1.d.p.)}\end{aligned}$$

(c) Let point S_1 be Temasek and let point S_2 be Novotel

<u>Team Novotel</u>

$$\begin{aligned}\angle RQS_2 &= 180° - 38° - \left[180° - \left(\frac{8\sin 38°}{54}\right)\right] \\ &= 37.7188°\end{aligned}$$

By Sine Rule,

$$\begin{aligned}\frac{RS_2}{\sin 37.7188°} &= \frac{54}{\sin 38°} \\ RS_2 &= 53.66 \text{ m} \\ &= 53.7 \text{ m}\end{aligned}$$

$$\therefore \text{Time taken} = \frac{53.66}{0.7x} = \frac{76.657}{x}$$

<u>Team Tamasek</u>

$$\begin{aligned}\text{(Team Temasek) } \angle RQS_1 &= 180° - 38° - \left[\frac{8\sin 38°}{54}\right] \\ &= 66.2812°\end{aligned}$$

By Sine Rule,

$$\begin{aligned}\frac{RS_2}{66.2812°} &= \frac{54}{\sin 38°} \\ RS_2 &= 80.3017 \text{ m} \\ &= 80.3 \text{ m}\end{aligned}$$

$$\therefore \text{Time taken} = \frac{80.3017}{x}$$

Time taken by Team Novotel is less than time taken by Team Temasek. Hence, **I agree** that Team Novotel will win the race

4. (a) (i)

$$\begin{aligned} BC &= \sqrt{(570)^2 + (540)^2 - 2(570)(540)\cos 65^\circ} \\ &= 596.939023... \\ &= \mathbf{597\ m\ (3.s.f.)} \end{aligned}$$

(ii) By Sine Rule,

$$\begin{aligned} \frac{\sin \angle BCA}{570} &= \frac{\sin 65^\circ}{\sqrt{(570)^2 + (540)^2 - 2(570)(540)\cos 65^\circ}} \\ \angle BCA &= \sin^{-1}\left(\frac{570 \sin 65^\circ}{\sqrt{(570)^2 + (540)^2 - 2(570)(540)\cos 65^\circ}}\right) \\ &= 59.929255.. \\ &= \mathbf{59.9^\circ\ (1.d.p.)} \end{aligned}$$

(iii)

$$\begin{aligned} \text{Bearing} &= 360^\circ - (180 - 79)^\circ - \sin^{-1}\left(\frac{570 \sin 65^\circ}{\sqrt{(570)^2 + (540)^2 - 2(570)(540)\cos 65^\circ}}\right) \\ &= 199.070744.. \\ &= \mathbf{199.1^\circ\ (1.d.p.)} \end{aligned}$$

(b)

$$\begin{aligned} \tan 2.6^\circ &= \frac{HD}{490} \\ HD &= 490 \tan 2.6^\circ \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(540)^2 - (490)^2} \\ &= \sqrt{51500} \end{aligned}$$

Let the angle of depression be θ,

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{490 \tan 2.6^\circ}{\sqrt{51500}}\right) \\ &= 5.599872... \\ &= \mathbf{5.6^\circ\ (1.d.p.)} \end{aligned}$$

(c)

$$\text{Area of land} = \frac{1}{2}(490)\left(\sqrt{51500}\right) + \frac{1}{2}(570)(540)\sin 65^\circ$$

$$\begin{aligned} \text{Value of land} &= \$45000\left[\frac{1}{2}(490)\left(\sqrt{51500}\right) + \frac{1}{2}(570)(540)\sin 65^\circ\right] \\ &= \$877860.524... \\ &= \mathbf{\$877860.52\ (2.d.p.)} \end{aligned}$$

15 Mensuration

15.1 Full Solutions

1. (a)

$$\begin{aligned}\text{Arc } BD &= 6(1.2)\\ &= \mathbf{7.2\ cm}\end{aligned}$$

(b)

$$\begin{aligned}\tan 1.2 &= \frac{BC}{6}\\ BC &= 6\tan 1.2\\ &= 15.4329...\\ &= \mathbf{15.4\ cm\ (3.s.f.)}\end{aligned}$$

(c)

$$\begin{aligned}\cos 1.2 &= \frac{6}{AC}\\ AC &= \frac{6}{\cos 1.2}\end{aligned}$$

$$\begin{aligned}\text{Perimeter of } Y &= 7.2 + 6\tan 1.2 + \left(\frac{6}{\cos 1.2} - 6\right)\\ &= 33.1911..\\ &= \mathbf{33.2\ cm\ (3.s.f.)}\end{aligned}$$

(d)

$$\begin{aligned}\text{Area of } X &= \frac{1}{2}(6\tan 1.2)^2\left(\frac{\pi}{2} - 1.2\right) - \frac{1}{2}(6\tan 1.2)^2 \sin\left(\frac{\pi}{2} - 1.2\right)\\ &= 1.0049...\\ &= \mathbf{1.00\ cm^2\ (3.s.f.)}\end{aligned}$$

(e)

$$\begin{aligned}\text{Area of } Y &= \frac{1}{2}(6)(6\tan 1.2) - \frac{1}{2}(6)^2(1.2)\\ &= 24.6987...\\ &= \mathbf{24.7\ cm^2\ (3.s.f.)}\end{aligned}$$

2. (a) (i)

$$\text{Surface area of the ornament} = 2\pi\left(\frac{32}{2}\right)^2 + 2\pi\left(\frac{16}{2}\right) + \left[\pi\left(\frac{32}{2}\right)^2 - \pi\left(\frac{16}{2}\right)^2\right]$$
$$= \mathbf{832\pi\ cm^2}$$

(ii)

$$\text{Volume of ornament} = \frac{2}{3}\pi\left(\frac{32}{2}\right)^3 - \frac{2}{3}\pi\left(\frac{16}{2}\right)^3$$
$$= 7506.31204...$$
$$= \mathbf{7510\ cm^3\ (3.s.f.)}$$

(b) (i) First, note that $\triangle APB$ is a right triangle (angles in a semicircle)

$$OA = \frac{\left(\frac{5}{\cos 75^\circ}\right)}{2}$$
$$= 9.66 \text{ cm } \mathbf{(3.s.f.)\ (shown)}$$

□

(ii)

$$\angle AOP = 75^\circ \times 2$$
$$= 150^\circ \text{ (angle at centre is 2 times angle at circumference)}$$

$$\text{Shaded region} = \frac{1}{2}\left(\frac{5}{2\cos\left[\frac{75^\circ}{180^\circ}\times\pi\right]}\right)^2\left(\frac{150^\circ}{180^\circ}\times\pi\right) - \frac{1}{2}\left(\frac{5}{2\cos\left[\frac{75^\circ}{180^\circ}\times\pi\right]}\right)^2 \sin\left(\frac{150^\circ}{180^\circ}\times\pi\right)$$
$$= 98.805759...$$
$$= \mathbf{98.8\ cm^2\ (3.s.f.)}$$

3.

$$\pi rl + 2\pi r^2$$
$$l = 2r$$

By Pythagoras' Theorem,

$$h = \sqrt{(2r)^2 - r^2}$$
$$= \sqrt{3}r$$

$$\text{Volume} = \pi r^2\left(\sqrt{3}r + 2r\right) - \frac{1}{3}\pi r^2\left(\sqrt{3}r\right) - \frac{2}{3}\pi r^3$$
$$= 7.81638...r^3$$
$$= \mathbf{7.82r^3\ (3.s.f.)}$$

4. (a) (i) By similar triangles,

$$\frac{r}{R} = \frac{8}{h}$$
$$\boldsymbol{r = \frac{8R}{h}}$$

(ii)

$$\begin{aligned}
\text{Volume of water} &= V_{\text{big cone}} - V_{\text{small cone}} \\
&= \frac{1}{3}\pi(R)^2(h) - \frac{1}{3}\pi(r)^2(8) \\
&= \frac{1}{3}\pi(R)^2(h) - \frac{1}{3}\pi\left(\frac{8R}{h}\right)^2(8) \\
&= \frac{1}{3}\pi R^2 h - \frac{512R^2}{3h^2}\pi \\
&= \frac{1}{3}\pi R^2\left(h - \frac{512}{h^2}\right) \ \textbf{(shown)}
\end{aligned}$$

(b) Let the new radius be r_2

$$\begin{aligned}
\frac{r_2}{R} &= \frac{h-2}{h} \\
r_2 &= \frac{R(h-2)}{h}
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Volume of water} &= \frac{1}{3}\pi(r_2)^2(h-2) \\
&= \frac{1}{3}\pi\left(\frac{R(h-2)}{h}\right)^2(h-2) \\
&= \frac{1}{3}\pi R^2\frac{(h-2)^3}{h^2} \ \textbf{(shown)}
\end{aligned}$$

(c)

$$\begin{aligned}
\therefore \frac{1}{3}\pi R^2\left(h - \frac{512}{h^2}\right) &= \frac{1}{3}\pi R^2\frac{(h-2)^3}{h^2} \\
h - \frac{512}{h^2} &= \frac{(h-2)^3}{h^2} \\
h^3 - 512 &= (h-2)\left(h^2 - 4h + 4\right) \\
&= h^3 - 4h^2 + 4h - 2h^2 + 8h - 8
\end{aligned}$$

$$\begin{aligned}
6h^2 - 12h - 504 &= 0 \\
h^2 - 2h - 84 &= 0 \ \textbf{(shown)}
\end{aligned}$$

(d)

$$
\begin{aligned}
h &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-84)}}{2(1)} \\
&= \frac{2 \pm \sqrt{340}}{2}
\end{aligned}
$$

$$h = 10.219544... \quad \text{or} \quad h = -8.219544...$$

$$\boldsymbol{h = 10.2 \text{ (1.d.p.)} \quad \text{or} \quad h = -8.2 \text{ (1.d.p.)}}$$

(e)

$$
\begin{aligned}
\therefore \text{Volume of water} &= \frac{1}{3}\pi(7)^2\left(\left(\frac{2+\sqrt{340}}{2}\right) - \frac{512}{\left(\frac{2+\sqrt{340}}{2}\right)^2}\right) \\
&= 272.838007... \\
&= \mathbf{273\ cm^3\ (3.s.f.)}
\end{aligned}
$$

16 Coordinate Geometry

16.1 Full Solutions

1. (a) Given that $5OA = 3OB$

$$\frac{OA}{OC} = \frac{3}{5}$$

$$\therefore \text{Gradient of line is } = -\frac{3}{5}$$

Hence,

$$6 = -\frac{3}{5}(5) + c$$
$$c = 9$$

$$\therefore y\text{-intercept} = 9 \textbf{ (shown)}$$

□

(b) By inspection,

$$\boldsymbol{A(0,9)} \qquad \boldsymbol{D(0,3)}$$

(c) By inspection,

$$\text{Length of } OA = 9 \text{ units}$$

$$\therefore \boldsymbol{E(5,15)}$$

(d)

$$\text{Area of parallelogram} = 9 \times 5$$
$$= \textbf{45 units}^{\mathbf{2}}$$

2. (a)

$$\begin{aligned}\text{Gradient of } PQ &= \frac{0-(-6)}{5-0}\\ &= \frac{6}{5}\end{aligned}$$

$$\therefore \boldsymbol{y = \frac{6}{5}x - 6}$$

(b)

$$\begin{aligned}\frac{1}{2}(10)(k) &= 40\\ k &= 8 \textbf{ (shown)}\end{aligned}$$

□

(c) (i)

$$\begin{aligned}\text{Length of } PQ &= \sqrt{(5-0)^2+(0+6)^2}\\ &= \sqrt{61}\\ &= \textbf{7.81 units (3.s.f.)}\end{aligned}$$

At S,

$$\begin{aligned}y &= \frac{6}{5}(8) - 6\\ &= 3.6\end{aligned}$$

$$\therefore S(8, 3.6)$$

$$\begin{aligned}\text{Length of } PS &= \sqrt{(5-8)^2+(0-3.6)^2}\\ &= \sqrt{21.96}\\ &= \textbf{4.69 units (3.s.f.)}\end{aligned}$$

(ii)

$$\begin{aligned}\frac{PQ}{PS} &= \frac{\sqrt{61}}{\sqrt{21.96}}\\ &= \frac{5}{3}\end{aligned}$$

$$\frac{PO}{PT} = \frac{5}{3}$$

$$\therefore \frac{PO}{PT} = \frac{PQ}{PS} \quad \text{(SS)}$$

$$\angle OPQ = \angle TPS \text{ (vertically opposite angles) (A)}$$

Since there are 2 sides with the same ratio and one same angle, $\triangle PQO$ and $\triangle PST$ are similar

3. (a)

$$\begin{aligned}\text{Gradient} &= \frac{4-(-8)}{-10-5}\\ &= -\frac{4}{5}\end{aligned}$$

$$\therefore y = -\frac{4}{5}x + c$$

Substitute $(5, -8)$ into the line,

$$\begin{aligned}-8 &= -\frac{4}{5}(5) + c\\ c &= -4\end{aligned}$$

$$\therefore \boldsymbol{y = -\frac{4}{5}x - 4}$$

(b) (i)

$$\begin{aligned}4x + 5y &= 10\\ y &= -\frac{4}{5}x + 2\end{aligned}$$

Both l and AB have the same gradient, implying that they are parallel. Since they do not have the same y-intercept, the lines **do not intersect** at all

(ii) Substitute $x = 10$ into l,

$$\begin{aligned}y &= -\frac{4}{5}(10) + 2\\ &= -6 \neq 10\end{aligned}$$

Hence, the point $C(10, -10)$ **do not lie** on the line l

(iii)

$$\text{Equation 1: } y = \frac{1}{5}x^3 - x^2 - 2$$

$$\text{Equation 2: } y = -\frac{4}{5}x + 2$$

Let Equation (1) = Equation (2),

$$\begin{aligned}\frac{1}{5}x^3 - x^2 - 2 &= -\frac{4}{5}x + 2\\ x^2 - 5x^2 - 10 &= -4x + 10\\ x^3 - 5x^2 + 4x - 20 &= 0\end{aligned}$$

$$\therefore \boldsymbol{p = -5 \qquad q = 4}$$

17 Vectors in 2 Dimensions

17.1 Full Solutions

1. (a) (i)

$$\begin{aligned} |\mathbf{u} - \mathbf{v}| &= \left| \begin{pmatrix} 8 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} \right| \\ &= \left| \begin{pmatrix} 6 \\ -8 \end{pmatrix} \right| \\ &= \sqrt{(6)^2 + (-8)^2} \\ &= \sqrt{100} \\ &= \mathbf{10\ units} \end{aligned}$$

(ii)

$$\begin{aligned} 2\mathbf{v} + \mathbf{u} &= 2\begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{12} \\ \mathbf{10} \end{pmatrix} \end{aligned}$$

(b) By inspection

$$h = \mathbf{10} \qquad \text{or} \qquad h = \mathbf{-4}$$

2. (a) (i)

$$\overrightarrow{AB} = \mathbf{b - 2a}$$

(ii)

$$\overrightarrow{OC} = \mathbf{b + 3a}$$

(iii)

$$\overrightarrow{OX} = \mathbf{\frac{2}{3}\left(b + 3a\right)}$$

(iv)

$$\begin{aligned}\overrightarrow{AX} &= \overrightarrow{AO} + \overrightarrow{OX} \\ &= -2\mathbf{a} + \frac{2}{5}\mathbf{b} + \frac{6}{5}\mathbf{a} \\ &= \mathbf{-\frac{4}{5}a + \frac{2}{5}b}\end{aligned}$$

(b)

$$\begin{aligned}\overrightarrow{AB} &= \frac{5}{2}\left(-\frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\right) \\ &= \frac{5}{2}\overrightarrow{AX}\end{aligned}$$

$\therefore A, X$ and B are collinear

(c) (i)

$$\begin{aligned}\overrightarrow{AZ} &= h\overrightarrow{AY} \\ \overrightarrow{AY} &= \overrightarrow{AB} + \overrightarrow{BY} \\ &= \mathbf{b} - \frac{1}{2}\mathbf{a}\end{aligned}$$

$$\overrightarrow{AZ} = \boldsymbol{h}\mathbf{b} - \mathbf{\frac{1}{2}}\boldsymbol{h}\mathbf{a}$$

(ii)

$$\overrightarrow{AZ} = \mathbf{-2a} + \boldsymbol{k}\mathbf{b}$$

(iii)

$$\mathbf{b}h - \frac{1}{2}h\mathbf{a} = 2\mathbf{a} + k\mathbf{b}$$

Comparing terms,

$$h = \mathbf{4} \qquad \text{and} \qquad k = \mathbf{4}$$

(d) (i)

$$\frac{\text{Area of } \triangle OAX}{\text{Area of } \triangle OAC} = \mathbf{\frac{2}{5}}$$

(ii)

$$\begin{aligned}\frac{\text{Area of } \triangle OBX}{\text{Area of } \triangle ABC} &= \frac{3}{5} \times \frac{2}{3} \\ &= \mathbf{\frac{2}{5}}\end{aligned}$$

3. (a) (i) (a)

$$\begin{aligned}\overrightarrow{WM} &= \overrightarrow{WX} + \overrightarrow{XM}\\ &= 6\mathbf{p} + 3\mathbf{q} + \frac{3}{5}(10\mathbf{p} - 5\mathbf{q})\\ &= \mathbf{12p}\end{aligned}$$

(b)

$$\begin{aligned}\overrightarrow{ZM} &= \overrightarrow{ZY} + \overrightarrow{YM}\\ &= 6\mathbf{p} + 3\mathbf{q} + \frac{2}{5}(-10\mathbf{p} + 5\mathbf{q})\\ &= \mathbf{2p + 5q}\end{aligned}$$

(ii) (a)

$$\begin{aligned}\text{Ratio} &= \frac{\text{Area of } \triangle WMX}{\text{Area of } \triangle WXY} \times \frac{\text{Area of } \triangle WXY}{\text{Area of } WXYZ}\\ &= \frac{3}{5} \times \frac{1}{2}\\ &= \frac{3}{10}\end{aligned}$$

$$\therefore \mathbf{3 : 10}$$

(b)

$$\begin{aligned}\text{Area of } WXYZ &= \frac{10}{3} \times 8\\ &= \mathbf{26\frac{2}{3}\ units^2}\end{aligned}$$

(iii)

$$\begin{aligned}\overrightarrow{WN} &= \frac{5}{2}(6\mathbf{p} + 3\mathbf{q})\\ &= \mathbf{\frac{15}{2}(2p + q)}\end{aligned}$$

(b) (i)

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -13 \end{pmatrix}\\ &= \begin{pmatrix} \mathbf{10} \\ \mathbf{-16} \end{pmatrix}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Acute angle} &= \tan^{-1}\left(\frac{16}{10}\right)\\ &= 57.994...\\ &= \mathbf{58.0^\circ\ (1.d.p.)}\end{aligned}$$

(iii)

$$\overrightarrow{AB} = \boldsymbol{k}\begin{pmatrix} \boldsymbol{n} \\ \boldsymbol{-2m} \end{pmatrix}$$

(iv)

$$\begin{pmatrix} \mathbf{30} \\ \mathbf{-48} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \mathbf{-30} \\ \mathbf{48} \end{pmatrix}$$

4. (a) (i)

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA}\\ &= 8\mathbf{b} - 12\mathbf{a}\end{aligned}$$

$$\overrightarrow{AQ} = \mathbf{4b - 6a}$$

(ii)

$$\begin{aligned}\overrightarrow{BP} &= \overrightarrow{BO} - \overrightarrow{OP}\\ &= \mathbf{-8b + 8a}\end{aligned}$$

(iii)

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{QA} - \overrightarrow{AP}\\ &= -4\mathbf{b} + 6\mathbf{a} - 4\mathbf{a}\\ &= \mathbf{2a - 4b}\end{aligned}$$

(b)

$$\begin{aligned}\overrightarrow{PR} &= 4\overrightarrow{PQ}\\ \overrightarrow{OR} - \overrightarrow{OP} &= 4(4\mathbf{b} - 2\mathbf{a})\\ \overrightarrow{OR} &= \mathbf{16b}\end{aligned}$$

(c)
- $\overrightarrow{OB}$ is parallel to $\overrightarrow{OR}$ and since O is a common point. Hence, O, B and R are **collinear**.
- $\overrightarrow{OR} = 2\overrightarrow{OB}$
- B is the midpoint of $\overrightarrow{OR}$

(d)

$$\begin{aligned}\overrightarrow{QP} &= \overrightarrow{BS}\\ 2\mathbf{a} - 4\mathbf{b} &= \overrightarrow{OS} - 8\mathbf{b}\\ \overrightarrow{OS} &= \mathbf{2a + 4b}\end{aligned}$$

(e)

$$\begin{aligned}\frac{\text{Area of } \triangle OBP}{\text{Area of } \triangle ORA} &= \frac{\text{Area of } \triangle OBP}{\text{Area of } \triangle OBA} \times \frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle ORA}\\ &= \frac{2}{3} \times \frac{1}{2}\\ &= \mathbf{\frac{1}{3}}\end{aligned}$$

(f)

$$\begin{aligned}\overrightarrow{AB} &= 8\begin{pmatrix}-1\\-1\end{pmatrix} - 12\begin{pmatrix}-1\\1\end{pmatrix}\\ &= \begin{pmatrix}4\\-20\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{AB}\right| &= \sqrt{(4)^2 + (-20)^2}\\ &= \textbf{20.4 units (3.s.f.)}\end{aligned}$$

18 Data Analysis & Handling

18.1 Full Solutions

1. (a) (i)

$$\text{Median mass} = \mathbf{55g}$$

(ii)

$$\begin{aligned}\text{Percentage} &= \frac{170-50}{200} \times 100\% \\ &= \mathbf{60\%}\end{aligned}$$

(iii)

$$\begin{aligned}\text{Interquartile range} &= 59\text{g} - 51\text{g} \\ &= \mathbf{8g}\end{aligned}$$

(b) (i) **Agree**. Sunny Farm has a **lower interquartile range** than Happy Farm

(ii) **Disagree**. The box-and-whisker plot **does not provide information on number of eggs which has masses equal to or more than 62g**. From the box-and-whisker plot, we could only tell that Happy Farm's upper quartile is 60g which means that 25% of eggs have masses more than 60g

(c) (i)

$$\text{P(both from Administrative Department)} = \mathbf{\frac{2}{145}}$$

(ii)

$$\begin{aligned}\text{P(at least one from Administrative Department} &= 1 - \left(\frac{26}{30}\right)\left(\frac{25}{29}\right) \\ &= \mathbf{\frac{22}{87}}\end{aligned}$$

(iii)

$$\begin{aligned}\text{P(one male and female from Farming and Outdoors)} &= 2\left(\frac{20}{30}\right)\left(\frac{6}{29}\right) \\ &= \mathbf{\frac{8}{29}}\end{aligned}$$

2. (a) (i)

83 km/h

(ii)

15 km/h

(b)

15%

(c)
- The median speed of the cars at 6:30pm is 45 km/h which is lower than that at 11 am. Therefore, the **cars are travelling slower at 6:30pm**
- The Interquartile Range of the speed of the cars at 6:30pm is 8 km/h which is smaller than that at 11 am. Hence, the **speed of the cars is more consistent at 6:30pm**

(d) **Heavy traffic during peak hours**

3. (a) (i)

28.5°C

(ii)

$$\begin{aligned}\text{Interquartile Range} &= 28.9 - 28.2 \\ &= \mathbf{0.7°C}\end{aligned}$$

(b)

12 days

(c) The temperature at Jurong have a **larger spread/less consistent** than the temperatures at Simei

(d) Based on the given formula, the interquartile range does not get affected by the "+32" as 32 will be added to every temperature. However, the product ×1.8 will affect

$$\begin{aligned}\text{New Interquartile Range} &= 1.8 \times 1.5 \\ &= \mathbf{2.7°F}\end{aligned}$$

4. (a) (i)

$$\begin{aligned}\text{Mean} &= \frac{5(10) + 15(25) + 25(80) + 35(65) + 45(20)}{200} \\ &= \mathbf{28\ m}\end{aligned}$$

(ii)

$$\begin{aligned}\text{Mean} &= \sqrt{\frac{5(10)^2 + 15(25)^2 + 25(80)^2 + 35(65)^2 + 45(20)^2}{200} - (28)^2} \\ &= \mathbf{9.80\ m}\end{aligned}$$

(b)
- The trees in Rainforest A are generally **taller** as their mean height of 28 m is higher than 20.1 m of Tropical Forest B
- The height of trees in Rainforest A are generally **more widespread** as they have a higher standard deviation of 9.80 as compared to Tropical Forest B of 4.53 m

(c) (i)

$$\begin{aligned}\text{P(height} \geq 40\text{ m)} &= \frac{20}{200} \\ &= \mathbf{\frac{1}{10}}\end{aligned}$$

(ii)

$$\begin{aligned}\text{P(height} < 30\text{ m)} &= \frac{80 + 25 + 10}{200} \\ &= \mathbf{\frac{23}{40}}\end{aligned}$$

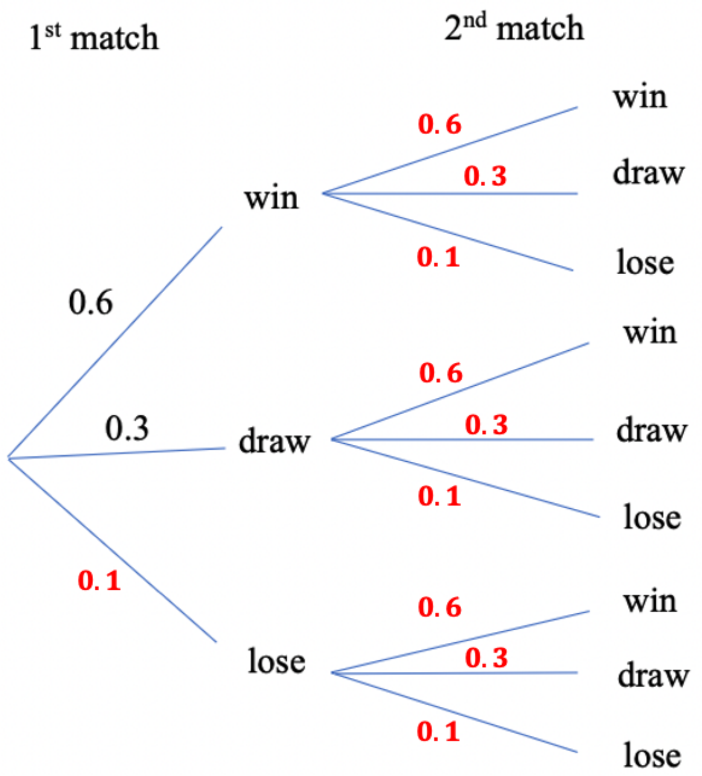
(d)

$$\begin{aligned}\text{P(both tress less than 20m)} &= \left(\frac{35}{200}\right)\left(\frac{34}{199}\right) \\ &= \mathbf{\frac{119}{3980}}\end{aligned}$$

19 Probability

19.1 Full Solutions

1. (a) (i) Tree Diagram



(ii)

$$\begin{aligned}\text{P(qualify for next round)} &= \text{P(WW)} + \text{P(WD)} + \text{P(DW)} \\ &= (0.6)(0.6) + (0.6)(0.3) + (0.3)(0.6) \\ &= \mathbf{0.72}\end{aligned}$$

(b)

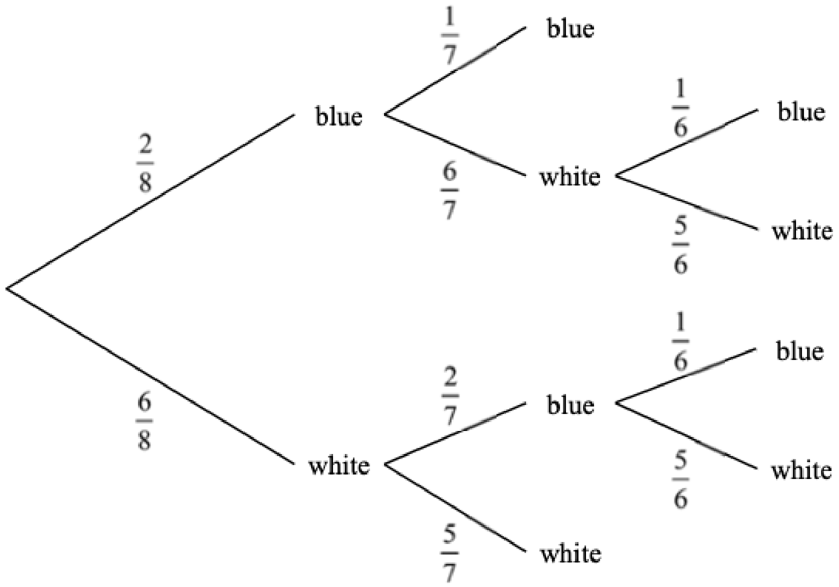
$$\text{P(R)} = \frac{2}{2+x}$$
$$\text{P(RR)} = \left(\frac{2}{2+x}\right)\left(\frac{1}{1+x}\right)$$

Hence,

$$\begin{aligned}\left(\frac{2}{2+x}\right)\left(\frac{1}{1+x}\right) &= \frac{1}{10} \\ (2+x)(1+x) &= 20 \\ x^2+3x-18 &= 0 \\ (x+6)(x-3) &= 0\end{aligned}$$

$$x = \mathbf{3} \quad \text{or} \quad x = -6 \text{ (rej)}$$

2. (a) Tree Diagram



(b) (i)

$$\begin{aligned} P(2 \text{ socks are white}) &= \left(\frac{6}{8}\right)\left(\frac{5}{7}\right) \\ &= \mathbf{\frac{15}{28}} \end{aligned}$$

(ii)

$$\begin{aligned} P(3\text{rd sock same colour as first sock}) &= \left(\frac{2}{8}\right)\left(\frac{6}{7}\right)\left(\frac{1}{6}\right)+\left(\frac{6}{8}\right)\left(\frac{2}{7}\right)\left(\frac{5}{6}\right) \\ &= \mathbf{\frac{3}{14}} \end{aligned}$$

3. (a) Possibility Diagram:

	2	3	6	9
2		(2,3)	(2,6)	(2,9)
3	(3,2)		(3,6)	(3,9)
6	(6,2)	(6,3)		(6,9)
9	(9,2)	(9,3)	(9,6)	

(b) (i)

$$\begin{aligned} \text{P(numbers less than 6)} &= \frac{2}{12} \\ &= \mathbf{\frac{1}{6}} \end{aligned}$$

(ii)

$$\begin{aligned} \text{P(both odd numbers)} &= \frac{2}{12} \\ &= \mathbf{\frac{1}{6}} \end{aligned}$$

(iii)

$$\begin{aligned} \text{P(product more than 10)} &= 1 - \text{P(product less than 10)} \\ &= 1 - \frac{1}{6} \\ &= \mathbf{\frac{5}{6}} \end{aligned}$$

(c) With replacement,

$$\text{P(product more than 10)} = 12\left(\frac{1}{16}\right) = \frac{3}{4} < \frac{5}{6}$$

Hence, the claim is **not true**

4. (a)

$$\begin{aligned} \text{P(1st fail, 2nd pass)} &= \left(\frac{1}{10}\right)\left(\frac{9}{10}\right) \\ &= \mathbf{\frac{9}{100}} \end{aligned}$$

(b)

$$\begin{aligned} \text{P(at least 2 attempts} &= 1 - \text{P(1st attempt pass)} \\ &= 1 - \frac{9}{10} \\ &= \mathbf{\frac{1}{10}} \end{aligned}$$

(c)

$$\begin{aligned} \text{P(1st fail, 2nd fail, ..., nth fail} &= \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right) \times ... \times \left(\frac{1}{10}\right) \\ &= \boldsymbol{\left(\frac{1}{10}\right)^{n}} \end{aligned}$$

(d)

$$\text{P(pass in one of the first nth months} = \boldsymbol{1 - \left(\frac{1}{10}\right)^{n}}$$