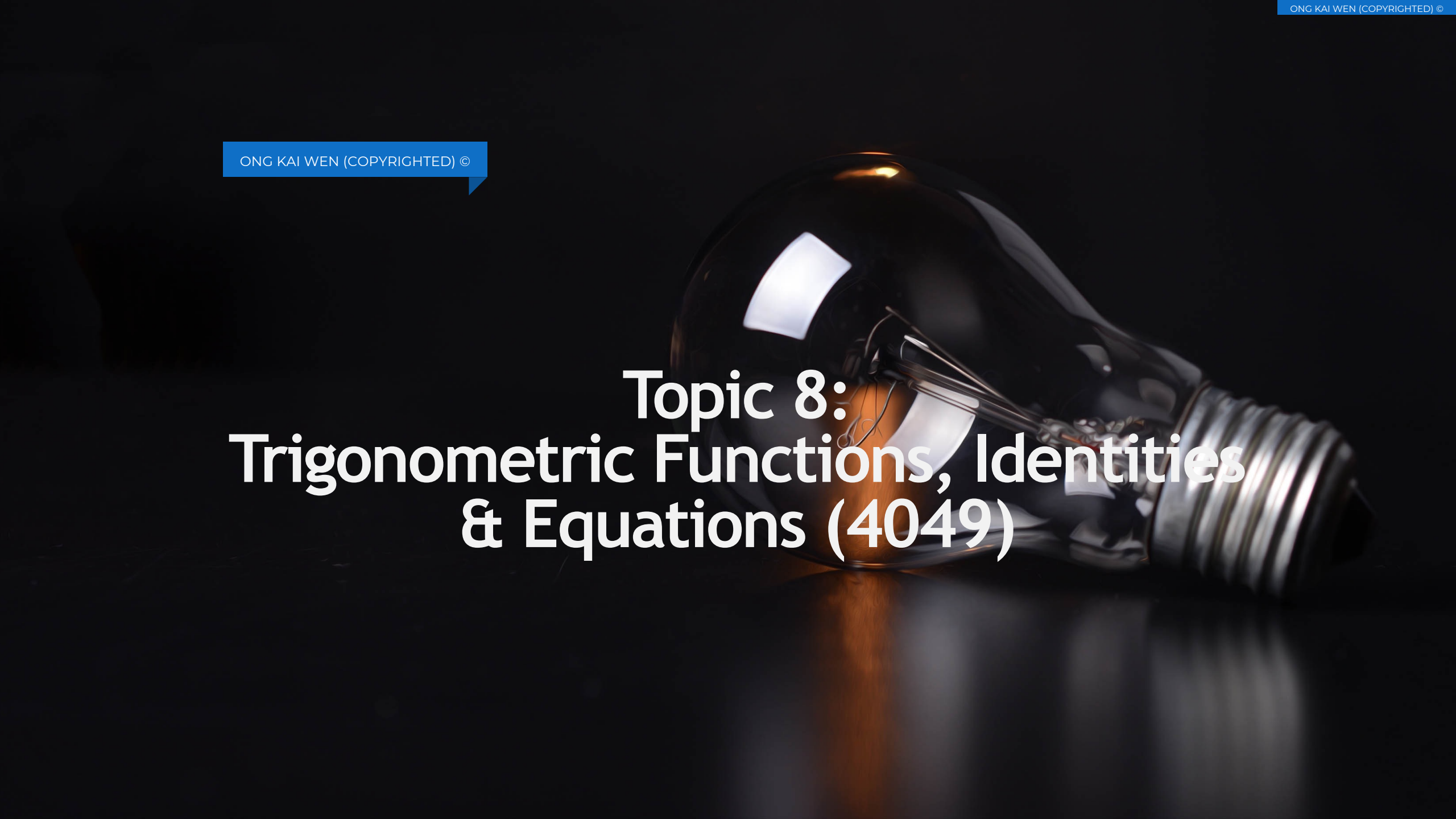

Topic 8: Trigonometric Functions, Identities & Equations (4049)

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THE ABOUT

CHAPTER ANALYSIS

- Trigonometric Functions
- Trigonometric Graphs
- Trigonometric Expressions
- Trigonometric Identities

MASTERY

- Heavy and challenging chapter for students
- 4 **key** concepts

EXAM

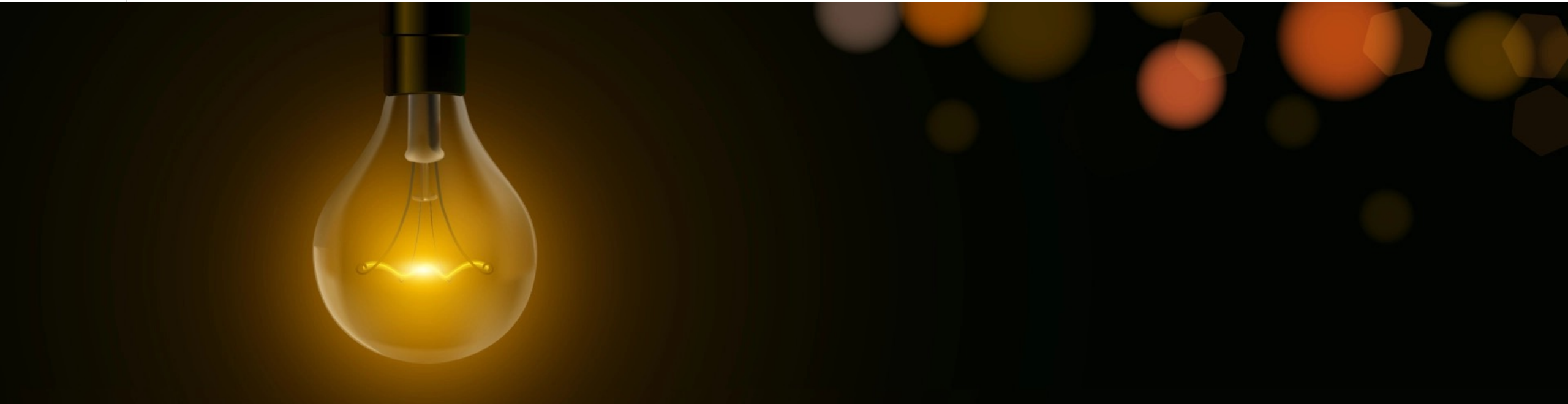
- Concepts usually tested as a stand-alone topic
- Many students struggle with the solving of the questions, not the concepts are the concepts are rather rudimentary

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, an 10%-15% of the overall papers will be testing Trigonometry as there is a wide plethora of concepts

KEY CONCEPT

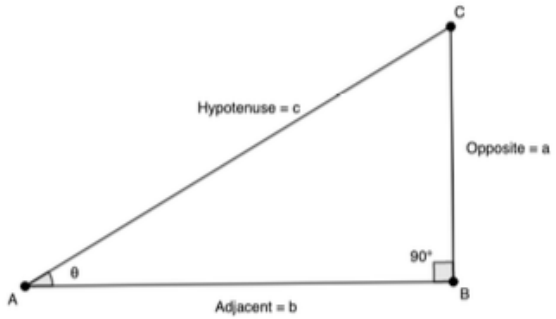
Trigonometric Ratios
Trigonometric Graphs
Trigonometric Identities & Equations



Memorisation Technique

$$\sec\theta = \frac{1}{\cos\theta} \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Look at the **third letter** of new ratios. It will tell you which ratio it corresponds to



- $\sin\theta$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

- $\cos\theta$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

- $\tan\theta$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Memorisation Technique:

TOA CAH SOH

Trigonometric Ratios

List of 6 Trigonometric Ratios

$\sin\theta$	$\operatorname{cosec}\theta$
$\cos\theta$	$\sec\theta$
$\tan\theta$	$\cot\theta$

The principal value of an inverse function is that value of the general value which is numerically least.

- $\sin^{-1}\theta$

Domain: $[-1, 1]$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ or $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

- $\cos^{-1}\theta$

Domain: $[-1, 1]$ Range: $[0, \pi]$ or $0 \le \theta \le \pi$

- $\tan^{-1}\theta$

Domain: $(-\infty, \infty)$ Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ or $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

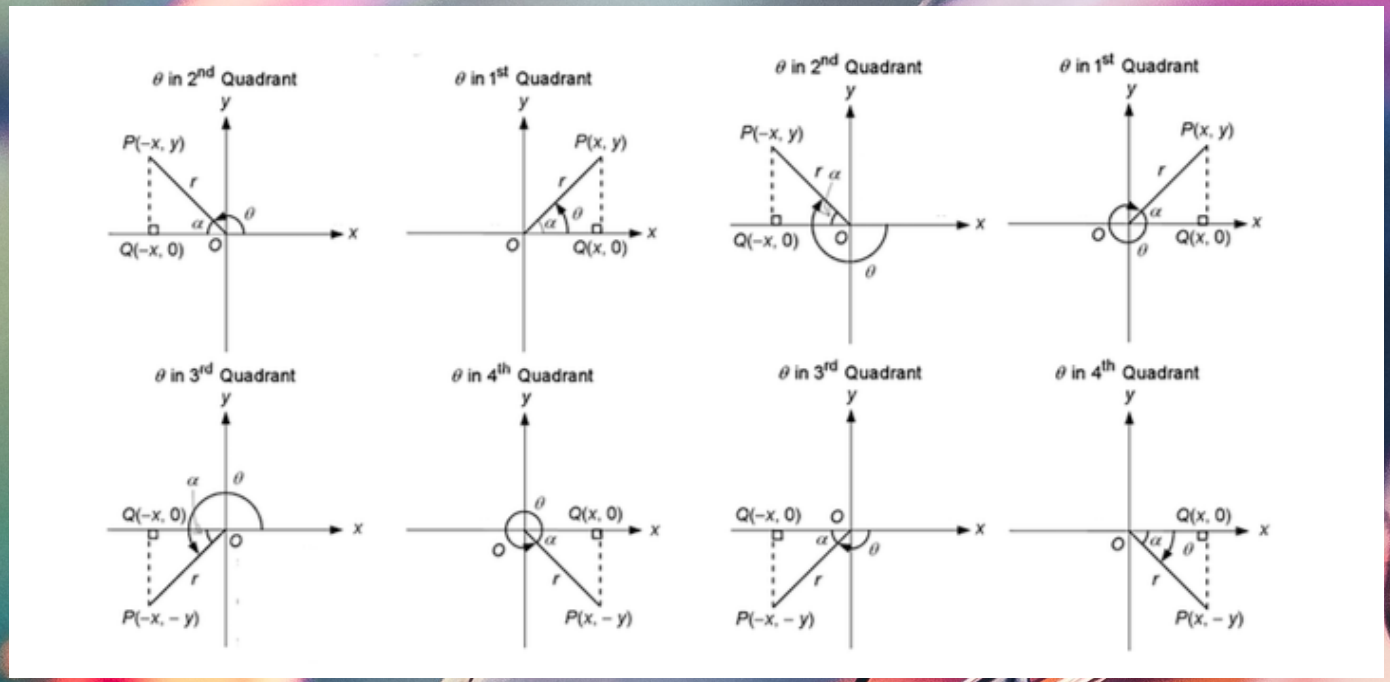
Take Note

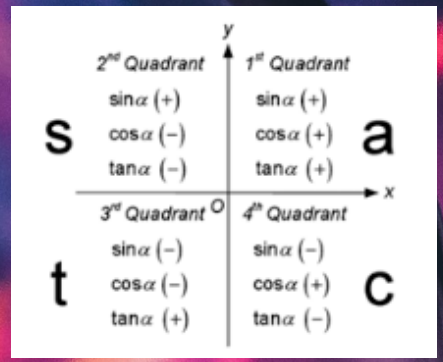
There is a difference between [ ] and ( )

[ ] refers to "inclusive" $\le$ $\ge$

( ) refers to "exclusive" $<$ $>$

Quadrants





Quadrant	Letter	Implication
1st	A	All ratios positive
2nd	S	Only sin positive
3rd	T	Only tan positive
4th	C	Only cos positive

Positive Rotation

Quadrant	Angle
1st	α
2nd	$180^\circ - \alpha$ or $\pi - \alpha$
3rd	$180^\circ + \alpha$ or $\pi + \alpha$
4th	$360^\circ - \alpha$ or $2\pi - \alpha$

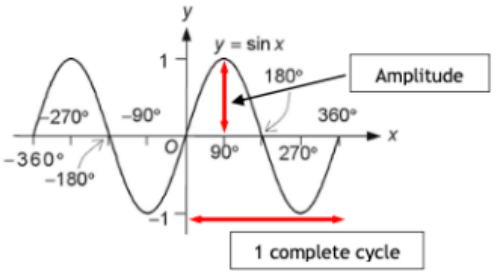
Negative Rotation

Quadrant	Angle
1st	$-(360^\circ - \alpha)$ or $-(2\pi - \alpha)$
2nd	$-(180^\circ + \alpha)$ or $-(\pi + \alpha)$
3rd	$-(180^\circ - \alpha)$ or $-(\pi - \alpha)$
4th	$-\alpha$

Trigonometric Graphs

Given the general form of trigonometric functions

$$y = a \sin bx + c$$



- a is the **amplitude**
  - The maximum displacement from its equilibrium position of the trigonometric graph
  - Note that sometimes there are questions where the a value is negative. This implies that the graph is flipped (reflected about the x-axis)
- b is the **period**
  - Time interval between 2 consecutive points that are in phase (how long does it take to complete 1 cycle)
  - Calculated with the following formula:

$$b = \frac{360°}{\text{period}} \quad \text{or} \quad b = \frac{2\pi}{\text{period}}$$

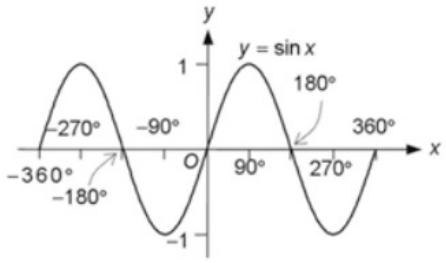
- c is the **vertical translation**
  - How much the graph has moved up or down
  - This value is dependent on how much the graph has shifted up and down dependent on the x axis, and how much has the equilibrium position* shifted from the x-axis

TAKE NOTE

- In the **4049 'O' Level Additional Mathematics** syllabus, graphs will only shift up or down based on the c value. It will **NEVER** shift left and right

Trigonometric graphs

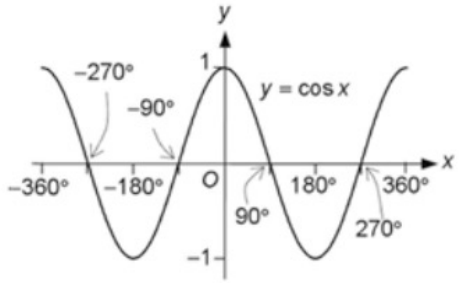
$y = \sin x$



Features:

- Maximum point: 1, Minimum point: −1
- Amplitude: 1
- Equilibrium position: $y = 0$
- Period: 360° or 2π

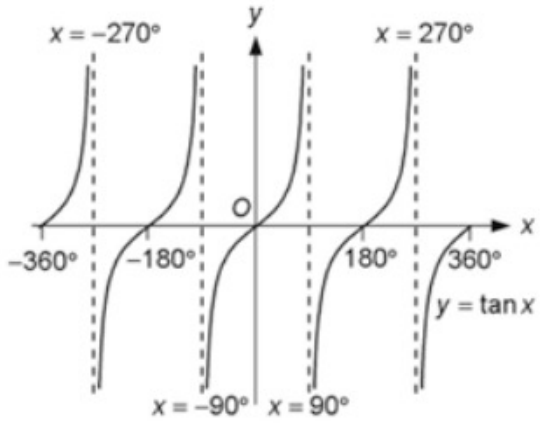
$y = \cos x$



Features:

- Maximum point: 1, Minimum point: −1
- Amplitude: 1
- Equilibrium position: $y = 0$
- Period: 360° or 2π

$y = \tan x$



Features:

- Maximum point: **NIL**, Minimum point: **NIL**
- Amplitude: To be calculated if information is provided in the question
- Equilibrium position: $y = 0$
- Period: 180° or π
- The graph approaches the lines $x = \pm 90°$ or $x = \pm\frac{\pi}{2}$ and $x = \pm 270°$ or $x = \pm\frac{3\pi}{2}$, but does not touch them at all. These are **vertical asymptotes**. Dotted lines must be drawn to show the asymptotes

Trigonometric Identities

Identity	Formulae
Quotient	$\tan A = \frac{\sin A}{\cos A}$
	$\cot A = \frac{\cos A}{\sin A}$

Identity	Formulae
Reciprocal	$\sec A = \frac{1}{\cos A}$
	$\operatorname{cosec} A = \frac{1}{\sin A}$
	$\cot A = \frac{1}{\tan A}$

Identity	Formulae
Pythagorean	$\sin^2 A + \cos^2 A = 1^*$
	$\sec^2 A = 1 + \tan^2 A^*$
	$\operatorname{cosec}^2 A = 1 + \cot^2 A^*$

Identity	Formulae
Addition	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B^*$
	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B^*$
	$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}^*$

Identity	Formulae
Double Angle	$\sin 2A = 2 \sin A \cos A^*$
	$\cos 2A = \cos^2 A - \sin^2 A^*$ $= 2\cos^2 A - 1^*$ $= 1 - 2\sin^2 A^*$
	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}^*$

Identity	Formulae
R-formula	$a \cos A \pm b \sin A = R \cos(A \mp \alpha)$
	$a \sin A \pm b \cos A = R \sin(A \pm \alpha)$
$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$	$R = \sqrt{a^2 + b^2}$

Equations with the * are in the Formulae List provided for every examination

Trigonometric Identities

Identity	Formulae
Negative Angles	$\sin(-A) = -\sin A$
	$\cos(-A) = \cos A$
	$\tan(-A) = -\tan A$

Identity	Formulae
Complementary Angles	$\sin(90° - A) = \cos A$
	$\cos(90° - A) = \sin A$
	$\tan(90° - A) = \cot A$

Identity	Formulae
Supplementary Angles	$\sin(360° - A) = -\sin A$
	$\cos(360° - A) = \cos A$
	$\tan(360° - A) = -\tan A$

Identity	Formulae
Half-Angle	$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$
	$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$
	$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$

TAKE NOTE

- **Section 3.6, 3.7 & 3.8** Pythagorean Identities, Addition Formulae and Double Angle Formulae can be found in the formula sheet
- **Section 3.9** Half-Angle Formulae is not taught explicitly in the **4049 'O' Level Additional Mathematics Syllabus** but knowing how to solve such questions is necessary as we can derive the formulae using standard conventions

Proof of the sine/cosine half-angle formulae

$$\cos 2A = 1 - 2\sin^2 A$$
$$\cos A = 1 - 2\sin^2\frac{A}{2}$$
$$2\sin^2\frac{A}{2} = 1 - \cos A$$
$$\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$$
$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

□

Proof of the tangent half-angle formulae

Recall that

$$\tan A = \frac{\sin A}{\cos A}$$

Take Note

1. Always check your equations and see if they can be solved quadratically

2. Before solving for the angle, always find the basic angle by taking the inverse of the trigonometric function
   - If the value is negative, always take the positive of that value

3. Always draw out the ASTC diagram, to help you visualize which quadrants are needed in the calculations

4. Always check for the range of the question, and include all possible angles
   - Includes extra positive rotation
   - Includes negative rotations (if necessary)

COMMON MISTAKE

- Refer to the following example

Example workings:

$$4\sin x\cos x = \cos x$$

Incorrect next step:

$$4\sin x = 1$$

Correct next steps:

$$4\sin x\cos x - \cos x = 0$$
$$\cos x(4\sin x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \sin x = \frac{1}{4}$$

- Do not divide the equation / cancel by $\cos x$ throughout as this will result in the loss of the solution for $\cos x = 0$. Its the same mistake as cancelling x for $4x^2 = x$
- We should always factorise out the common terms and solve them separately

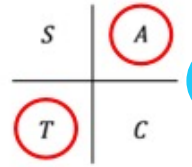
Trigonometric Equations

Standard Algorithm to solving Trigonometric Equations

$$12\tan^2 y - 5\tan y - 3 = 0$$
$$(4\tan y - 3)(3\tan y + 1) = 0 \quad (1)$$
$$\tan y = \frac{3}{4} \quad \text{or} \quad \tan y = -\frac{1}{3}$$

$$\tan y = \frac{3}{4}$$

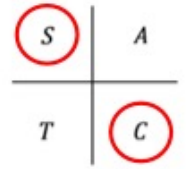
(2) Basic angle $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$



(3)

$$y = \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 36.869\ldots°$$
$$= 36.9°\ (1.\text{d.p.})$$

$$\tan y = -\frac{1}{3}$$



Basic angle $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$

$$y = 180° - \tan^{-1}\left(\frac{1}{3}\right)$$
$$= 161.565\ldots°$$
$$= 161.6°\ (1.\text{d.p.})$$

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