Wave Motion

Overmugged

CONTENT

- Progressive waves
- Transverse and longitudinal waves
- Energy and intensity of waves
- Polarisation
- Determination of frequency and wavelength of sound waves

Progressive Waves

Wave

- is a propagation of a disturbance which transfers energy from one point in space to another without the physical transfer of matter.
- the source of any wave is a vibration or an oscillation.

Progressive Wave

- a.k.a. **travelling wave**

- a wave which results in a net transfer of energy from one place to another.
- energy is transferred from the source outwards, along the direction of propagation of the wave

Stationary Wave

- a.k.a. **standing wave**

- has a waveform that does not move.
- it is formed from superposition of two similar progressive waves travelling in opposite directions.

Types of Waves

Waves can be classified in different ways according to

1. Mechanical or Non-mechanical
2. Transverse or Longitudinal

Mechanical

- requires a medium for their propagation.
  e.g., water or sound waves

Non-mechanical

- does not require a medium for their propagation.
  e.g., electromagnetic, gravitational, and quantum mechanical waves

Types of Waves

Waves can be classified in different ways according to

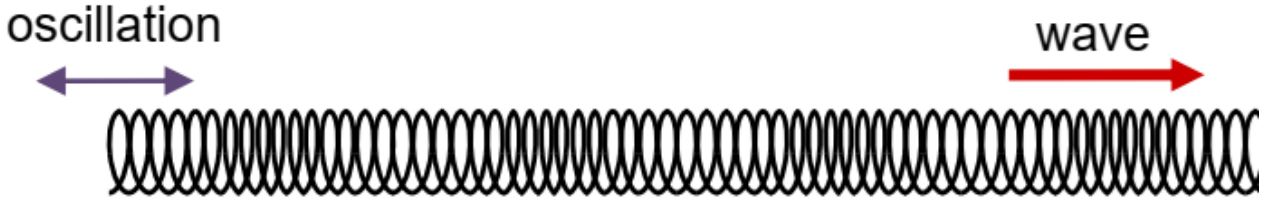
1. Mechanical or Non-mechanical
2. Transverse or Longitudinal

Transverse

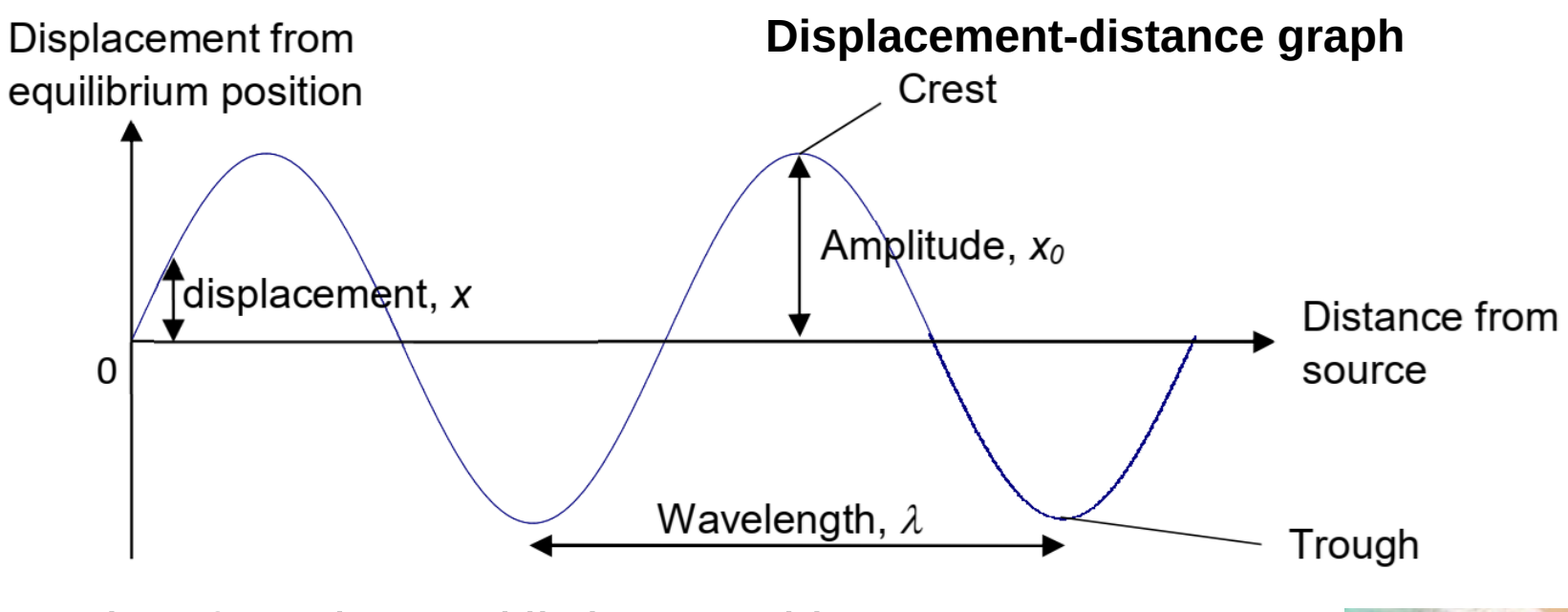
- has a direction of oscillation **perpendicular** to its direction of propagation.
  e.g., waves on a rope, electromagnetic waves

Longitudinal

- has a direction of oscillation **parallel** to its direction of propagation.
  e.g, slinky spring and sound waves



Terminologies



Displacement *x/y*

- Position of an oscillating particle from its equilibrium position

Amplitude *A*

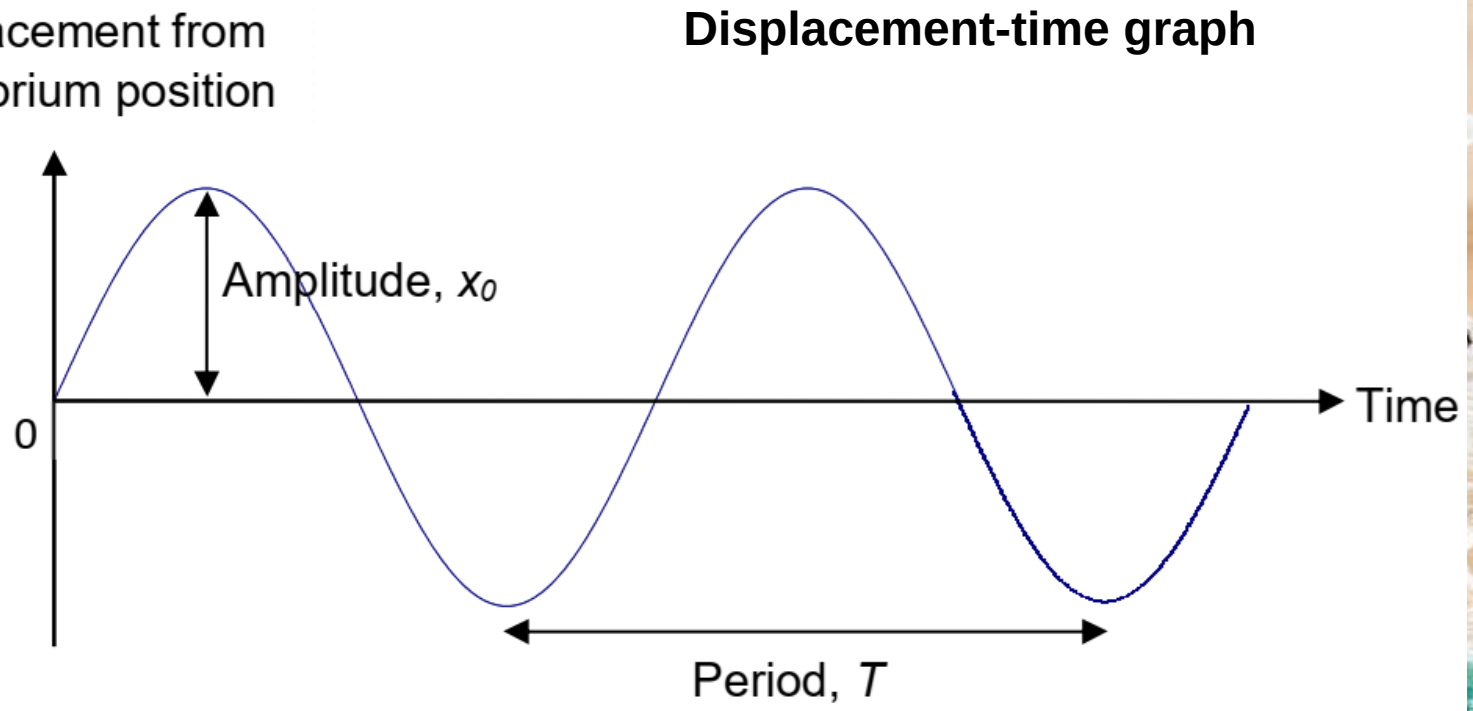
- The maximum distance (magnitude of displacement) of an oscillating particle from its equilibrium position

Wavelength λ

- For a progressive wave, it is the distance between any two successive particles that are in phase. (e.g. the distance between 2 adjacent maximum displacements)

Terminologies

Displacement-time graph



Period *T*

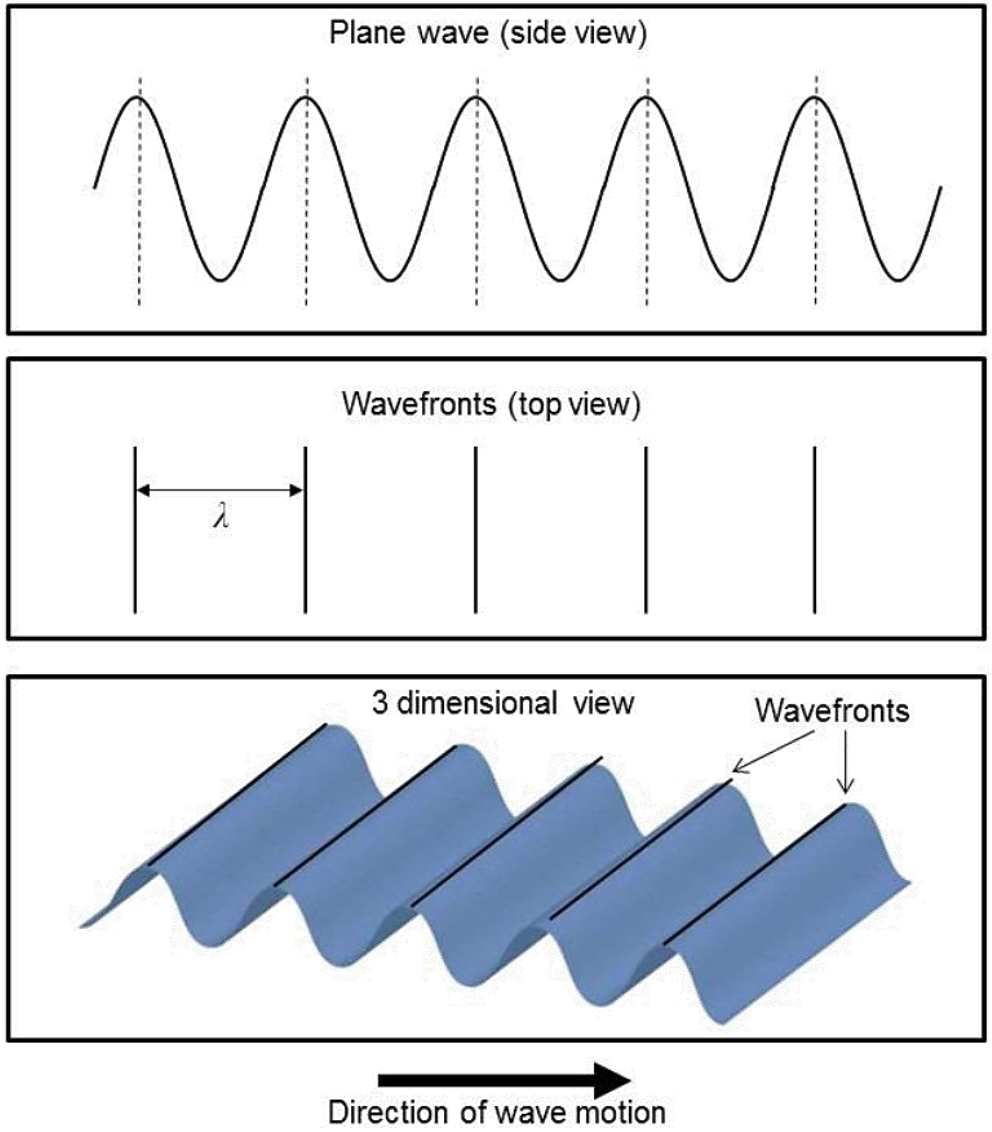
- Time taken for a particle to undergo one complete cycle of oscillation
- Time for the wave to travel through one wavelength.

Frequency *f*

$$f = \frac{1}{T}$$

- Is the number of complete cycle performed by a particle per unit time
- Number of wavelengths that pass a given point per unit time.

Terminologies



Wave speed *v*

- Distance travelled by the waveform/wave profile per unit time

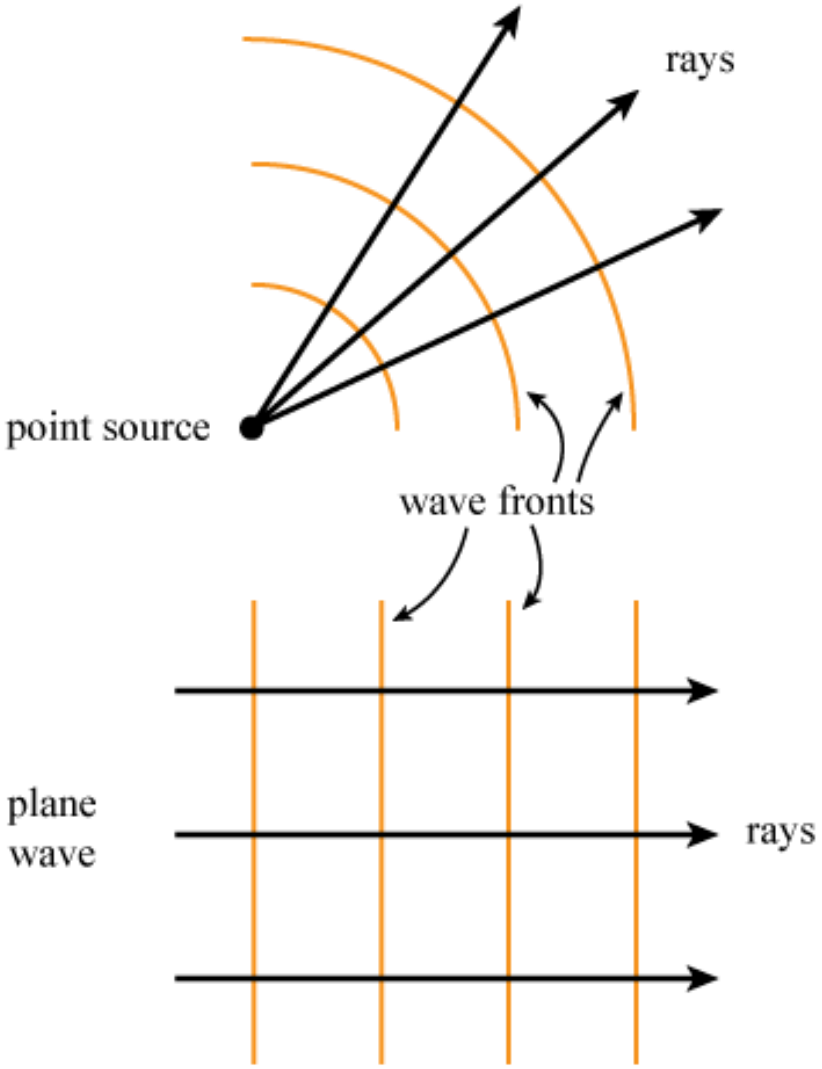
$$v = f\lambda$$

Wavefront

- is a locus or imaginary line joining all the points of the wave that have the same phase.
- It is useful to draw the wavefront by joining all the crests of a wave, and then seeing it from a bird's eye view.

- 1 wavelength = distance between 2 successive wavefronts

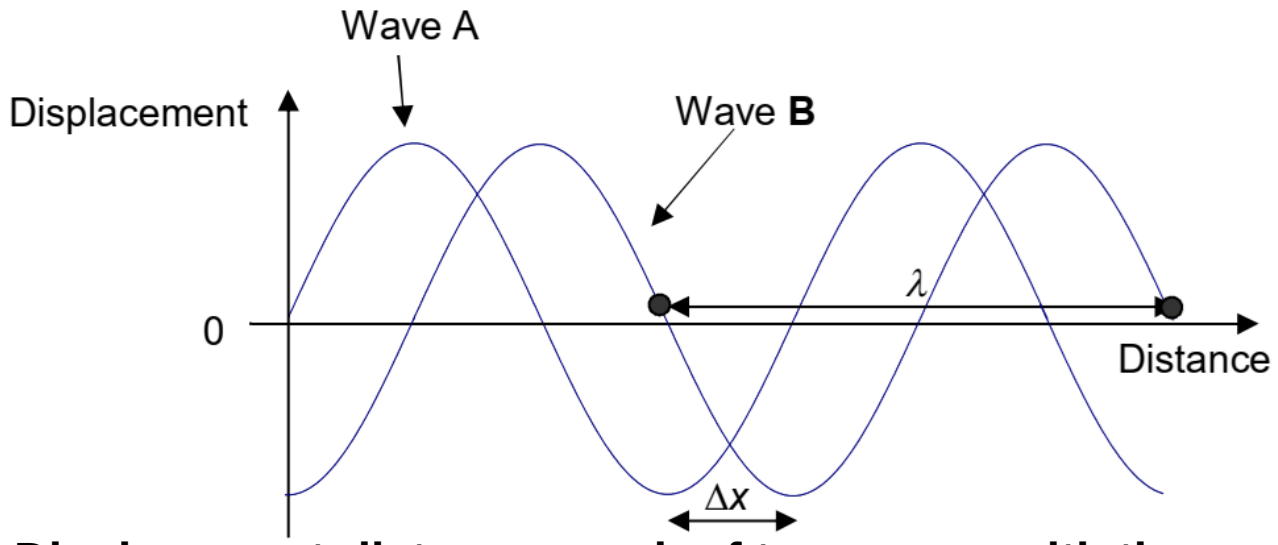
Terminologies



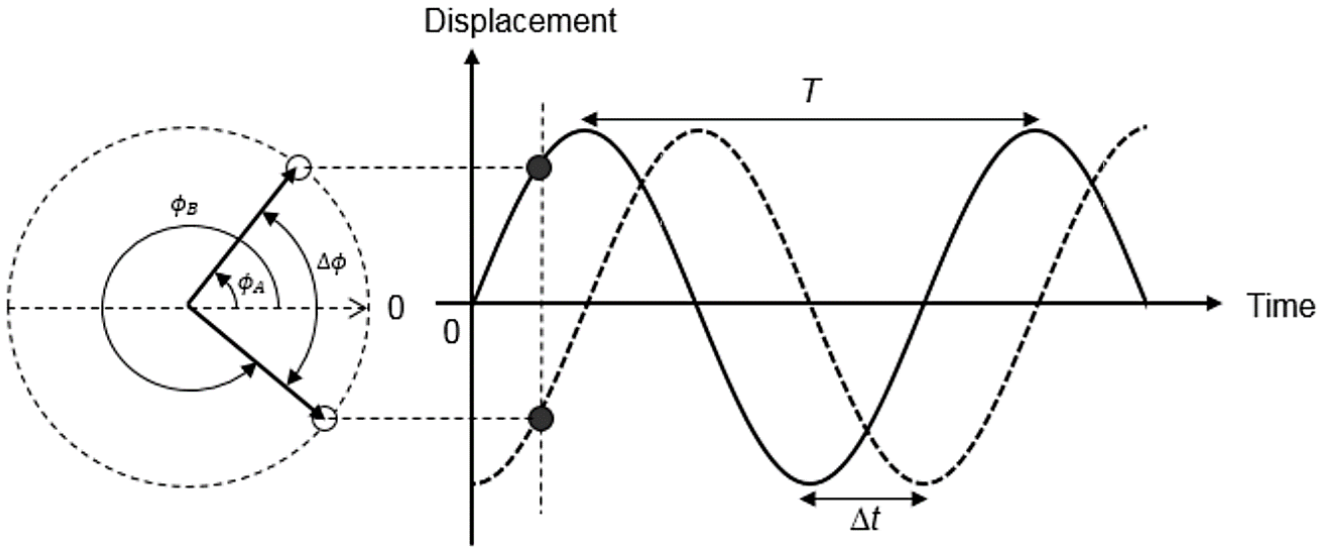
Ray

- Indicates the path taken by the wave
- Always perpendicular to the wavefronts

Terminologies



Displacement-distance graph of two waves with the same λ



Displacement-time graph of two waves with the same *T*

Phase ϕ

- of a particle gives a measure of the fraction of a cycle that has been completed by an oscillating particle.
- One cycle corresponds to 360° or 2π rad.

Terminologies

Phase Differrence

- is a measure of how much one wave is out of step with another or one particle in a wave is out of step with another particle in the same wave.

$$\Delta\phi = \frac{\Delta x}{\lambda}(2\pi) \qquad \Delta\phi = \frac{\Delta t}{T}(2\pi)$$

where Δx = distance between maximum/minimum distance
Δt = time interval between maximum/minimum distance
λ = wavelength
T = period

In phase: $\Delta\phi = 0$
Out of phase: $\Delta\phi \neq 0$
Antiphase: $\Delta\phi$ =180° or π rads

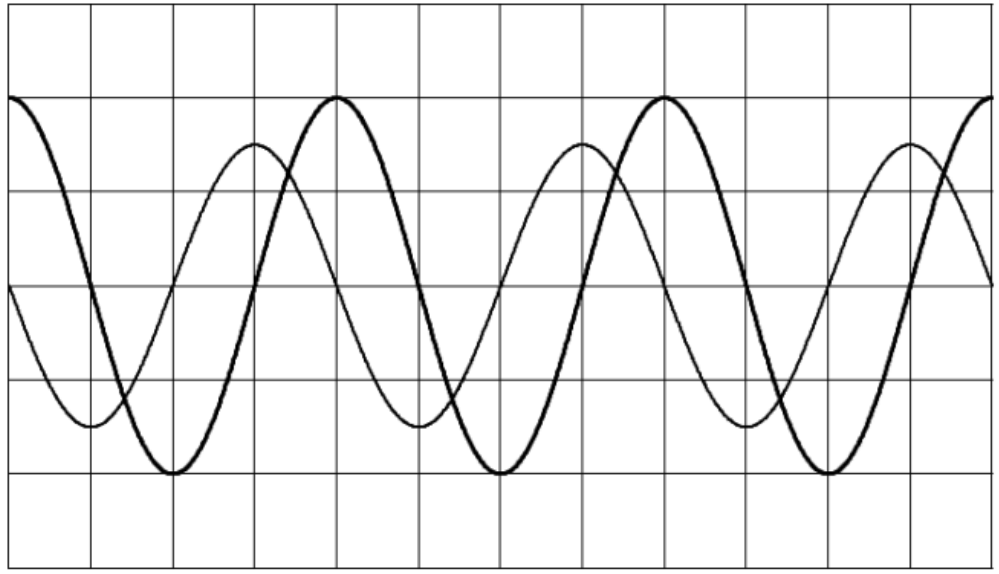
Note

1. In order to compare phase or finding phase difference, **amplitudes** of oscillating particles need not be the **same** but they must have the same **frequency** and **wavelength**
2. Two particles or two waves are said to be in phase when their $\Delta\phi = 0$.
   - Waves that are $n\lambda$ apart (where *n* is a positive integer) are also **in phase**, e.g. λ, 2λ, 3λ, etc.
3. Two particles or two waves are said to be in **anti-phase** when their $\Delta\phi = \pi$ radian.
   - Waves that are $\left(n + \frac{1}{2}\right)\lambda$ apart (where *n* is a positive integer) are also **exactly out of phase**, e.g. $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.

Practice Example 1

With a frequency of 440 Hz, a sound wave is travelling with a speed of 330 m/s. What is the phase difference between two points on the wave 0.25 m apart in the direction of travel?

Practice Example 2



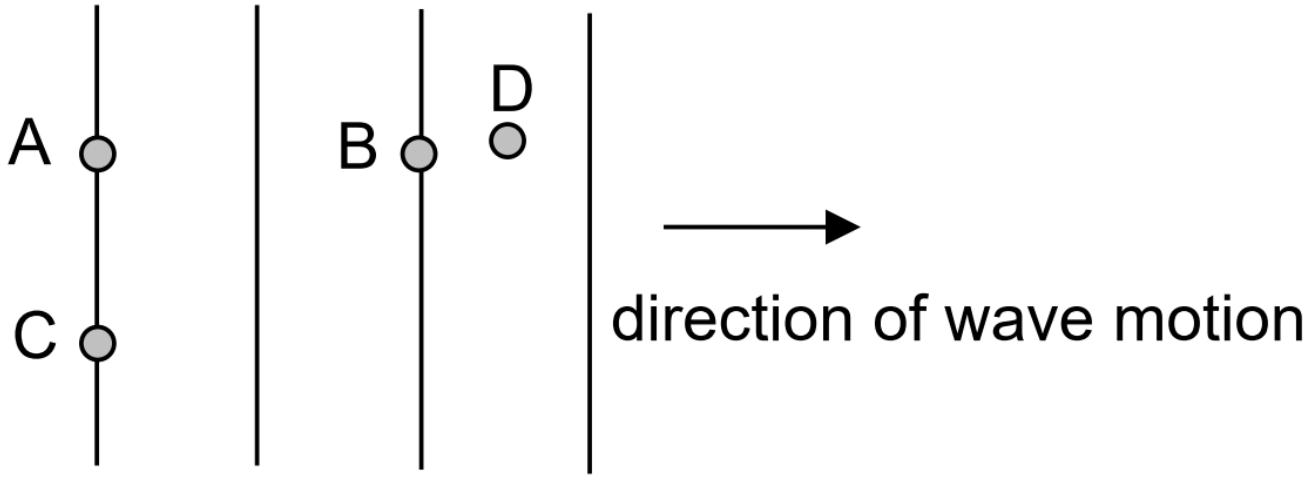
Consider a displacement-time graph of two waves detected by a sensor shown on the left. The waves have the same frequency. What is the phase difference between the two waves?

Practice Example 3

Shown below are the wavefronts of a wave travelling to the right. It has a speed of 2.25 m/s and a frequency of 0.80 Hz.

Determine the following:

a. Distance between points A and B
b. Phase difference between points B and C
c. Phase difference between points B and D

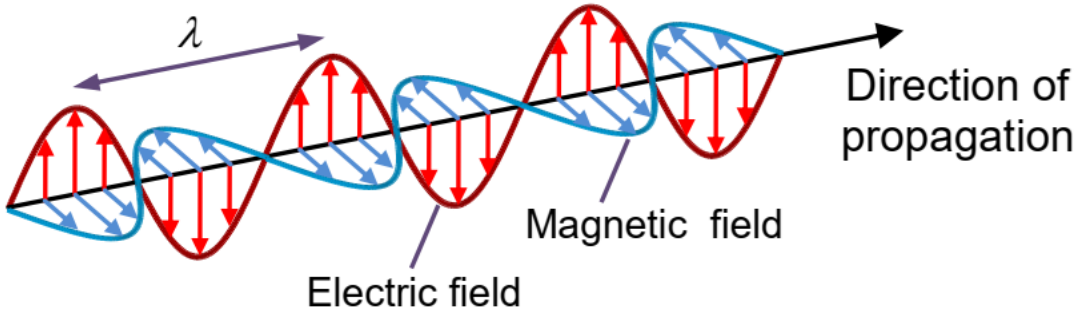


Transverse Waves

- is a wave in which its particles oscillates in a direction **perpendicular** to its direction of propagation.

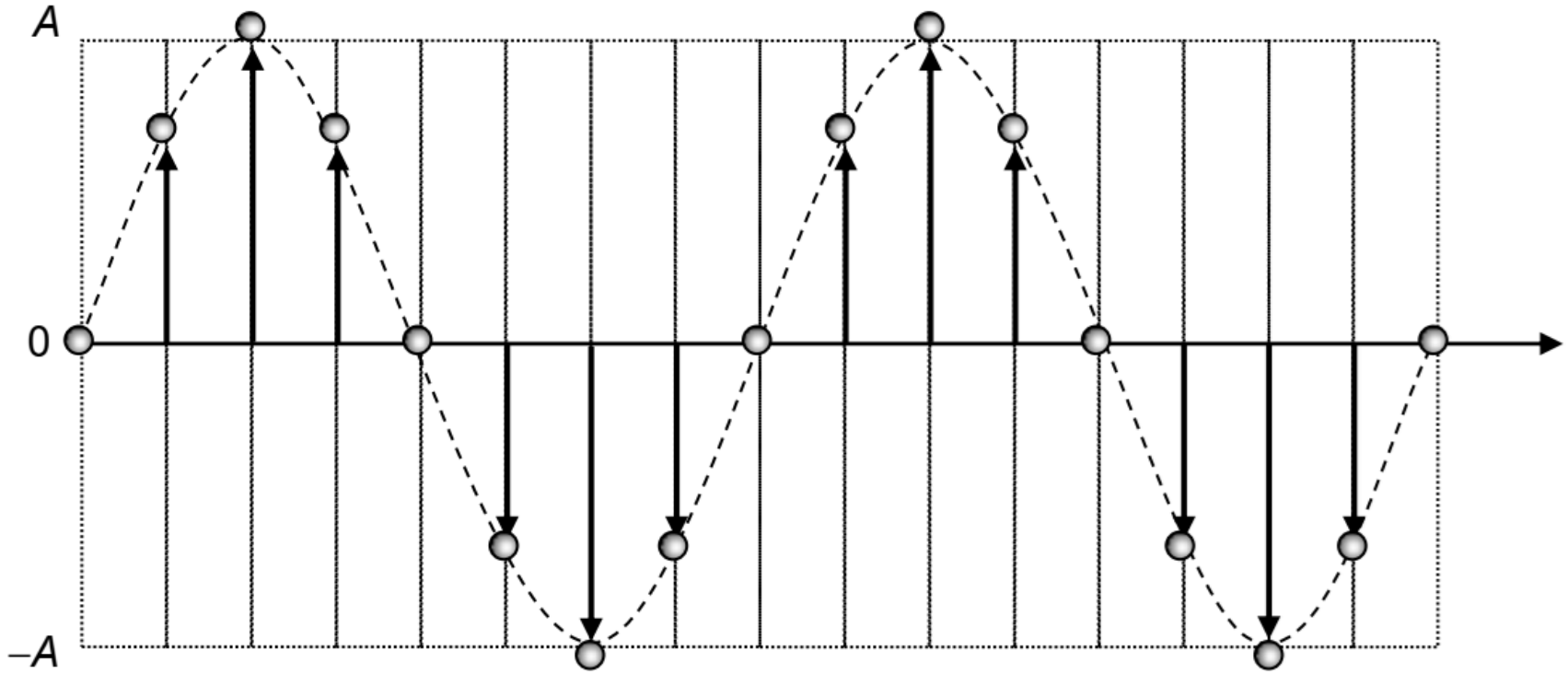
  e.g., waves on a rope, electromagnetic waves

Electromagnetic Waves



- the EM wave spectrum is divided into radio waves, microwaves, infra-red, visible light, ultraviolet, X-rays, and gamma rays.
- the frequency of an EM wave does not change when the waves go from one medium to another.
- They travel at the speed of $c = 3.00 \times 10^8\ m/s$ in vacuum

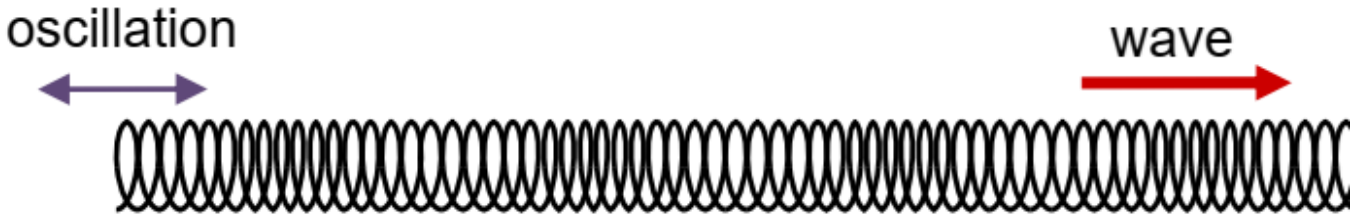
Graphical Representation of Transverse Wave



Longitudinal Waves

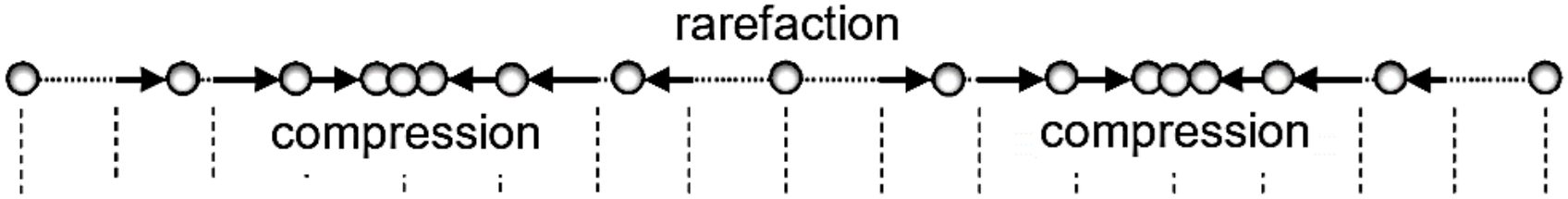
- is a wave in which its particles oscillate **parallel** to its direction of propagation.

  e.g, slinky spring and sound waves

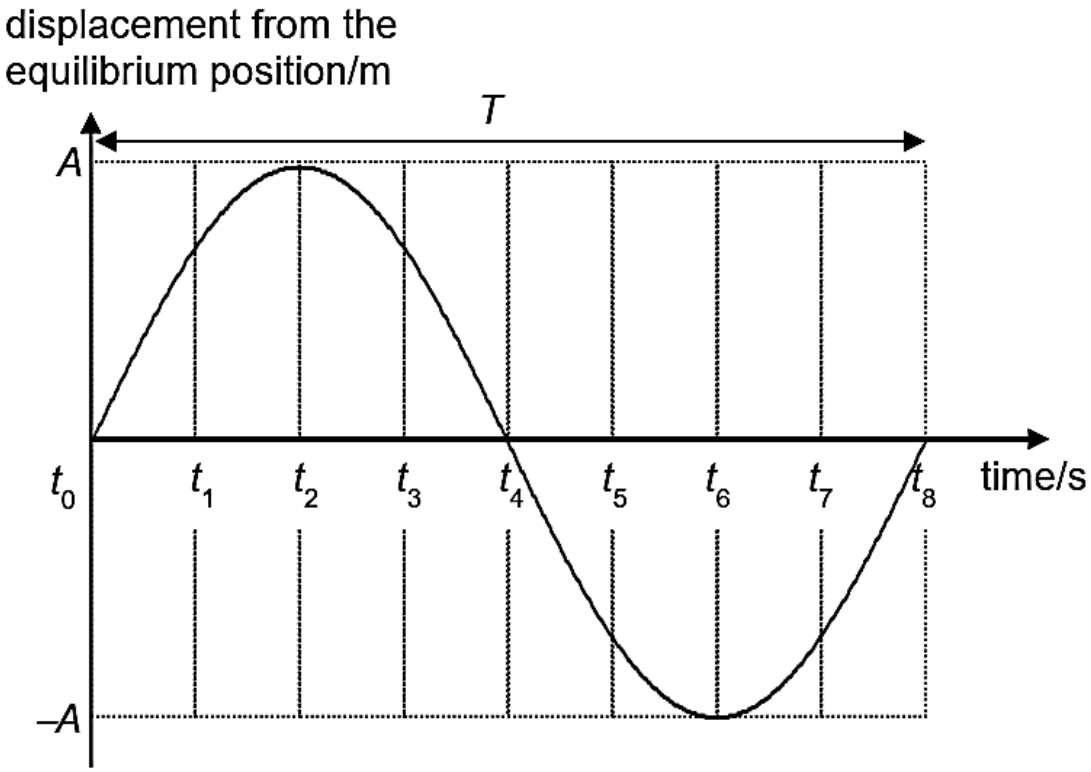


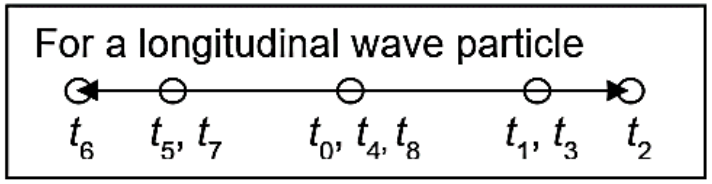
- **Compression** occurs where the air molecules are closest together.
- **Rarefaction** occurs where the air molecules are furthest apart from each other.
  - The distance between successive compressions or successive rarefactions is equal to the wavelength.
  - The compressions and rarefactions occur at points of zero displacement.

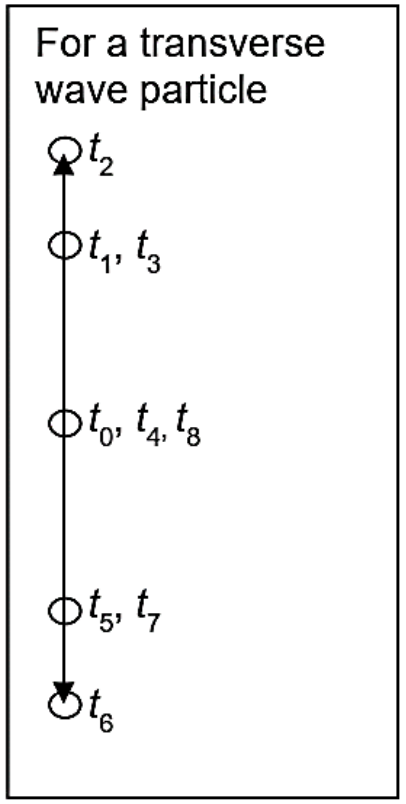
Graphical Representation



Displacement-time graph of transverse and longitudinal wave







All the particles move in a similar manner with the **same amplitude** and **frequency** as the wave. That is,

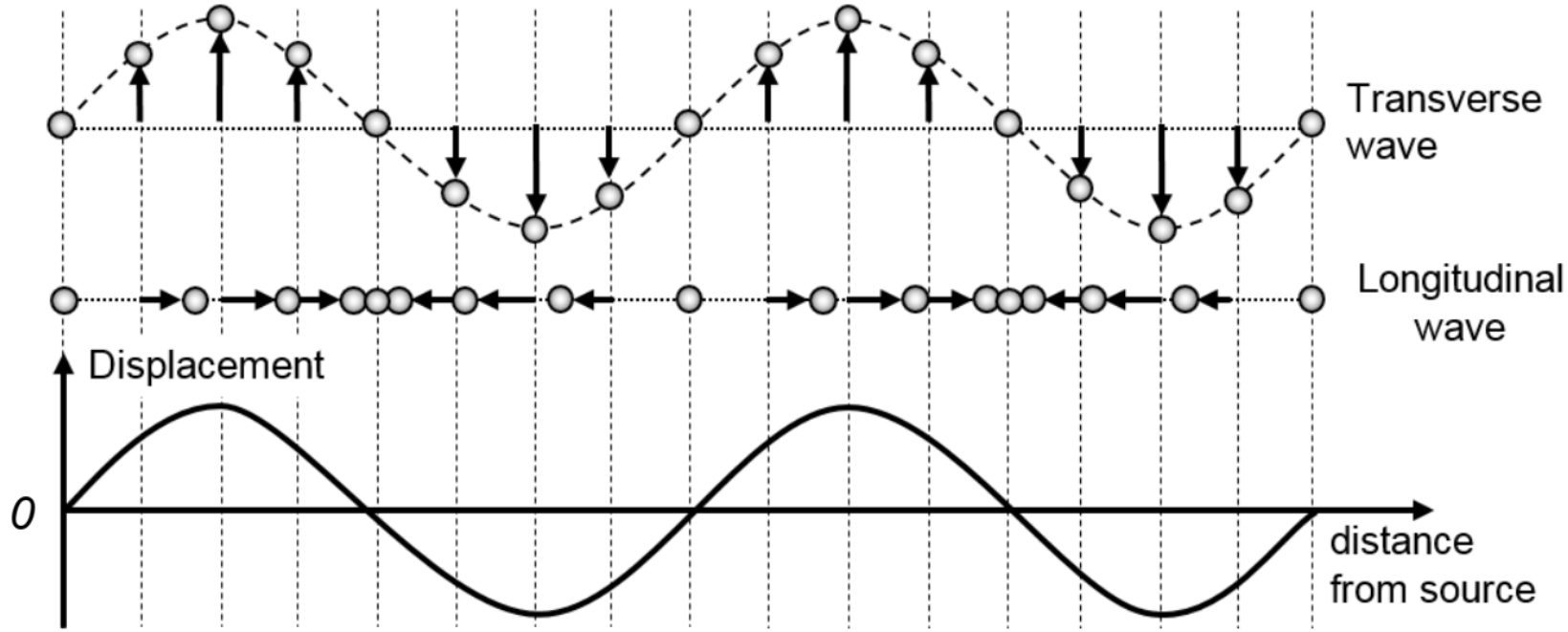
- frequency of particle = frequency of the wave
- amplitude of particle = amplitude of the wave

From the graph, t_0 to t_8 signifies the completion of one oscillation, and is the **period** of the wave.

Displacement-distance graph of transverse and longitudinal wave

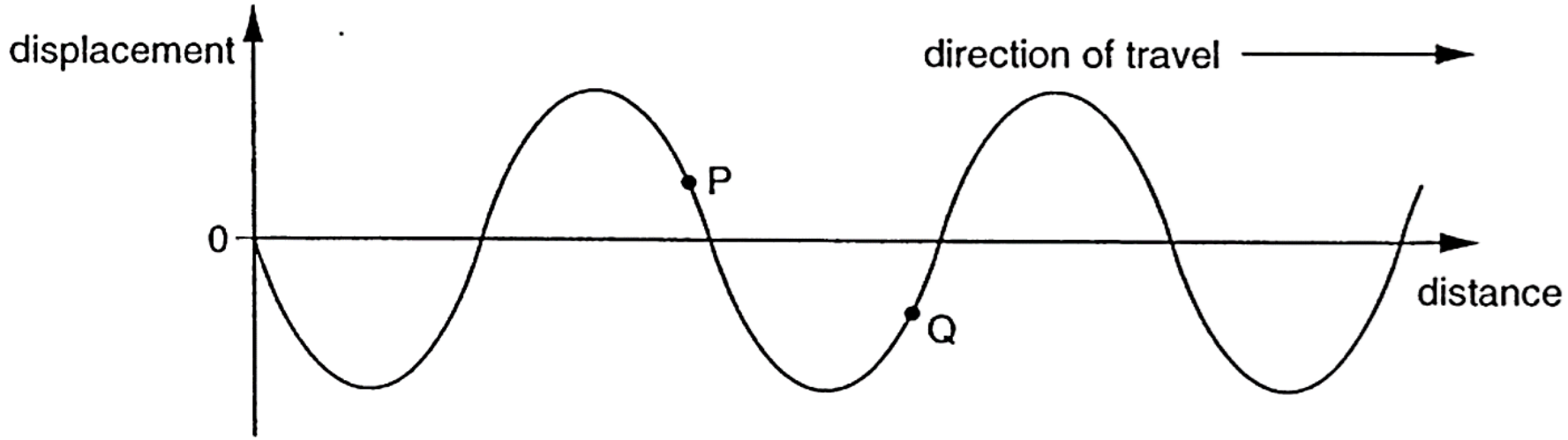
The displacement-distance graph shows how the displacements of the particles (from their individual equilibrium position) vary with the distance from the source at **a particular instant in time.**

- For transverse waves, this is similar to a snapshot of the actual wave travelling through the medium.
- For longitudinal waves, however, unlike transverse waves, the displacement-distance graph is not a snapshot of the actual wave travelling through the medium and has to be found by finding the displacement of individual particles.



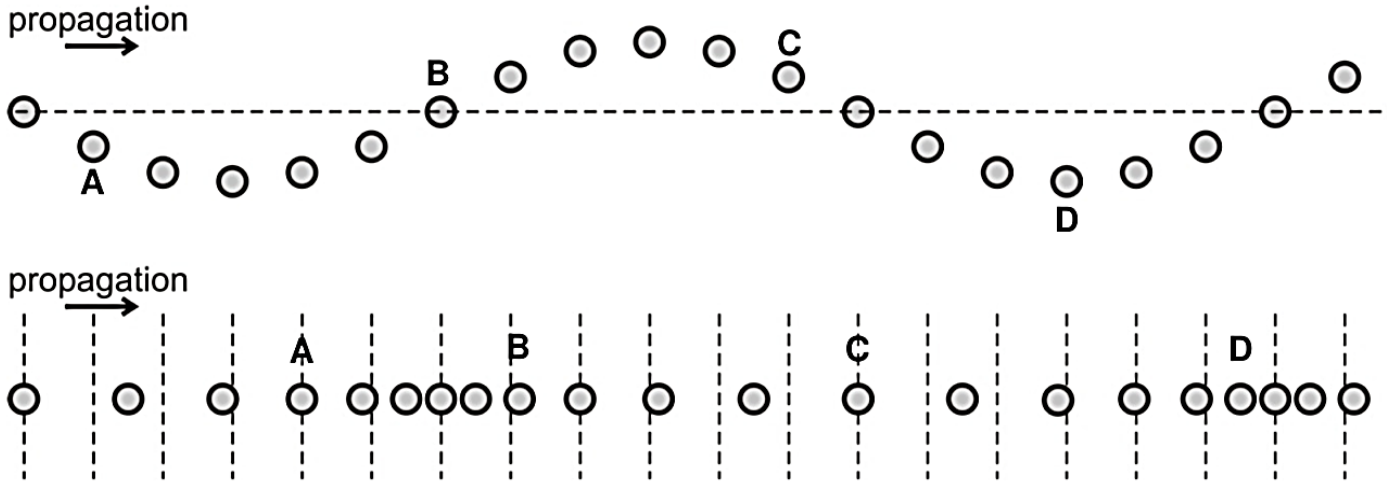
Practice Example 4

A displacement-distance graph of a transverse progressive wave is shown below. It travels to the right along a rope. In which direction are P and Q moving?



Practice Example 5

The figures below show a progressive transverse and a progressive longitudinal wave respectively,



On each wave, identify a point or points at which

a. the velocity is zero
b. the acceleration is zero
c. the velocity is in the same direction as the displacement
d. the acceleration is in the same direction as velocity

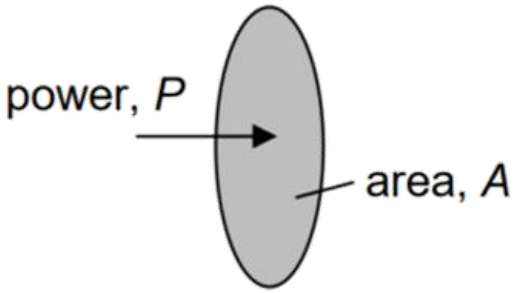
Energy and Intensity of Waves

- Wave motion involves the transportation of energy from one place to another.

Intensity

Symbol: I
SI unit: Watts per meter squared [W/m^2]

- is the energy delivered per unit area per unit time



$$I = \frac{E}{At} = \frac{P}{A}$$

where E = energy
P = power
A = area

Sinusoidal Waves

- wave vibrates in simple harmonic motion

$$E = \frac{1}{2} m\omega^2 A^2$$
$$I \propto A^2$$

Energy and Intensity of Waves

Spherical Waves

- waves coming from a point source have spherical wavefronts
- Surface area: $= 4\pi r^2$

$$I = \frac{P}{4\pi r^2}$$

$$I \propto \frac{1}{r^2}$$

$$\text{Amplitude} \propto \frac{1}{r}$$

Practice Example 6

A person is initially 5.0 m from a point source which emits energy uniformly in all directions at a constant rate. If the power of the source is to be doubled but the sound is to be as loud as before, at what distance should the person be from the source?

Practice Example 7

Suppose a loudspeaker operating at 35 W is producing sound waves in all directions. Calculate the following:

a) the intensity of sound at a distance of 12 m away
b) the power received by a square microphone of length 4.0 cm placed at a distance 8.0 m away from the loudspeaker
c) the amplitude of the vibrations at 8.0 m, given that at 4.0 m, the amplitude of the vibrations is 3.0 cm.

Polarisation

- is a phenomenon whereby the oscillation of **transverse waves** are restricted to a single plane.

*does not apply to longitudinal waves

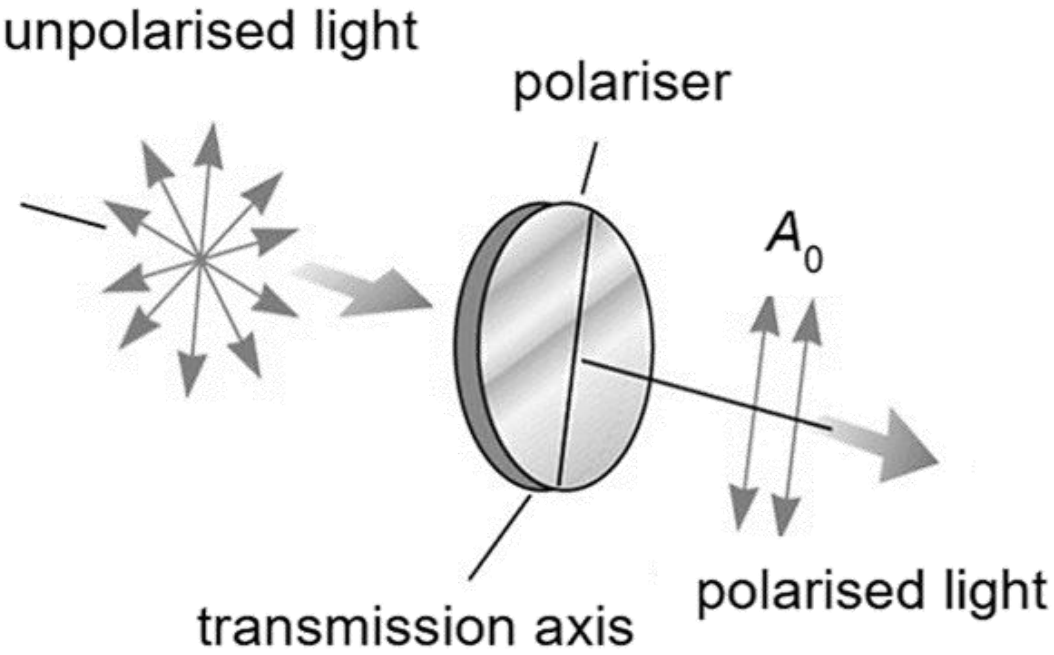
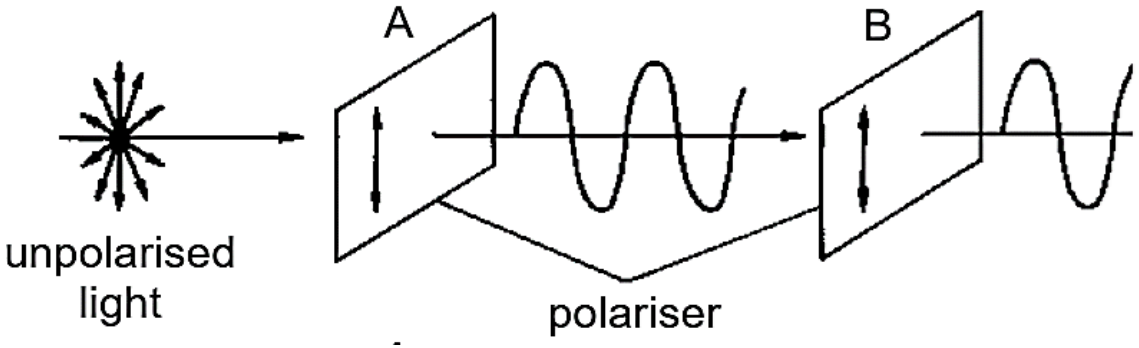
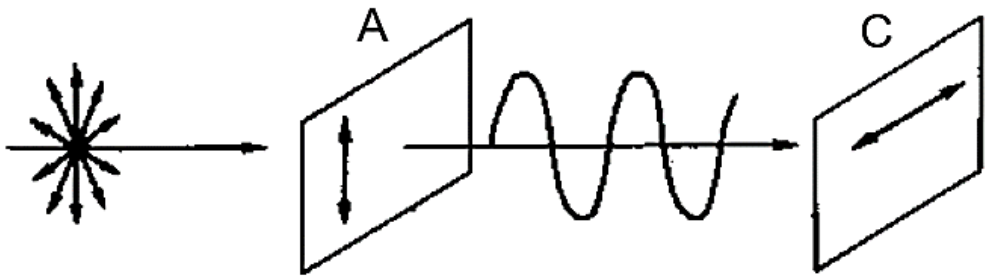


Illustration of polarization of light wave using a polariser

Polarisation of transverse wave



- the transmitted wave through polarizer A is said to be **plane-polarised** or polarized in the vertical plane.
- This vertically polarized wave is able to pass through polariser B and the polarizer B has the same transmission (polarization axis) as polariser A.



(no light)

- The polarized wave is completely blocked by polarizer C which has a transmission axis to A and no light is able to pass through

Applications (polarization)

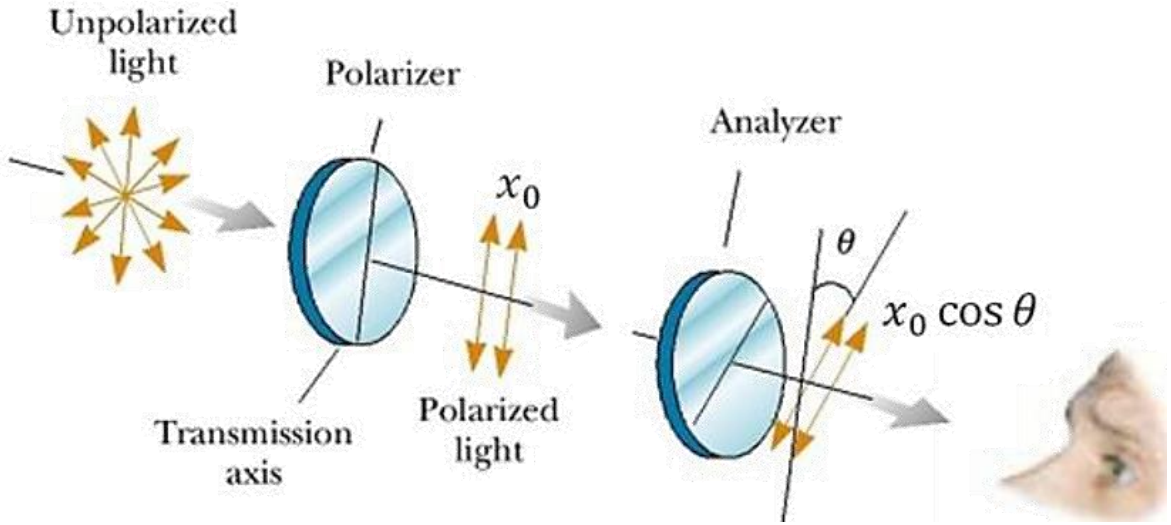
- Polaroid used in sunglasses to reduce glare
- 3D glasses used to watch 3D movies
- Reduction of haziness of pictures

Malus Law

- states that the intensity of a beam of plane-polarised light after passing through a polariser varies with the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity.

$$I = I_0 \cos^2 \theta$$

where I_0 = intensity of unpolarized light



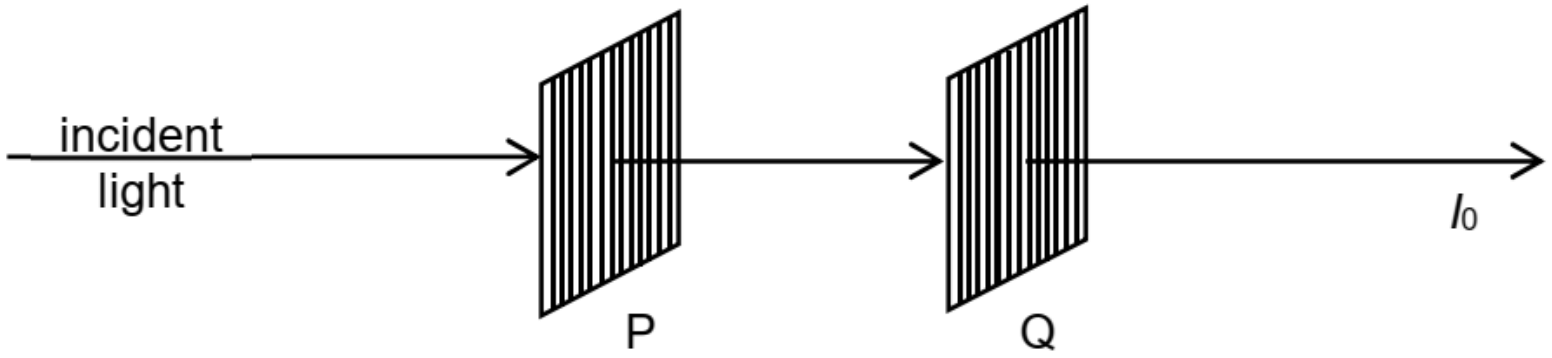
When polarised visible light is now incident on a second polariser (usually called an analyser) placed with its polarising axis at an angle θ to the polarising axis of the first polariser, only the electric field component parallel to the polarising axis of the analyser will be transmitted, while the component perpendicular to the polarising axis will be absorbed by analyser.

Practice Example 8

Explain why it would not be possible to polarize sound waves.

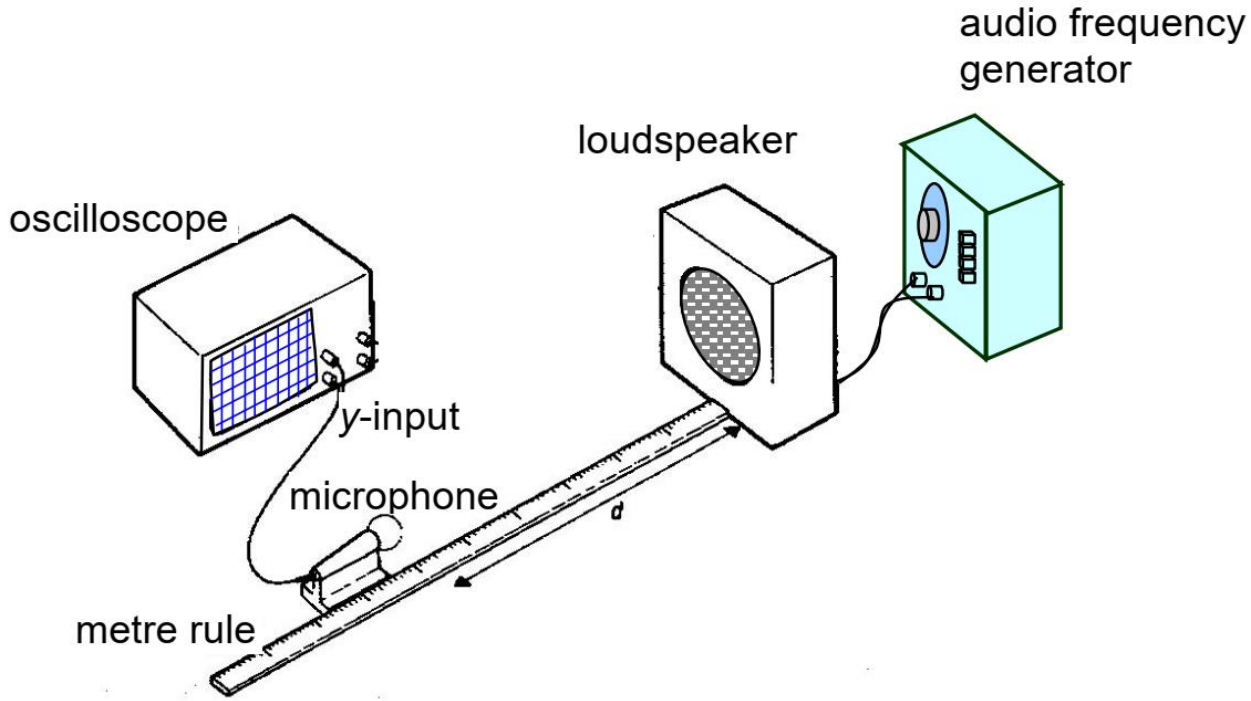
Practice Example 9

Two polarisers P and Q are placed next to each other such that their polarization axes are parallel and vertical, as shown below. If intensity of the emergent light is I_0, through what angle must Polaroid Q be rotated so that the intensity of the emergent light decreases to $\frac{1}{4}I_0$?



Determination of frequency and wavelength of sound waves

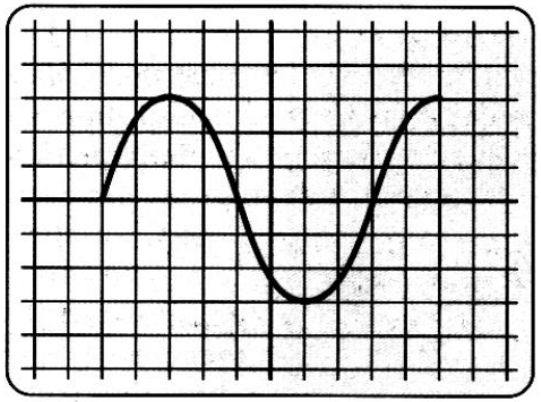
- As discussed earlier, soundwaves produce regions of compressions and rarefactions as they travel trough air. This gives rise to pressure variations as the wave travels.
- Hence, by placing a microphone in front of a loudspeaker connected to a signal generator, the microphone will detect a continuous series of compressions and rarefactions over time.
- If the microphone is connected to a cathode ray (CRO), the CRO will be able to display a variation of the pressure experienced by the microphone with respect to time.A



Determination of frequency and wavelength of sound waves

When a signal is viewed on the CRO display, the period of the signal can be measured by counting the number of horizontal divisions a complete waveform covers and multiplying it by the scale of each division. This is known as the **time base.**

An interpretation of a CRO signal is shown below



time base set to
50 ms/div

Y-gain set to 0.5 V/div

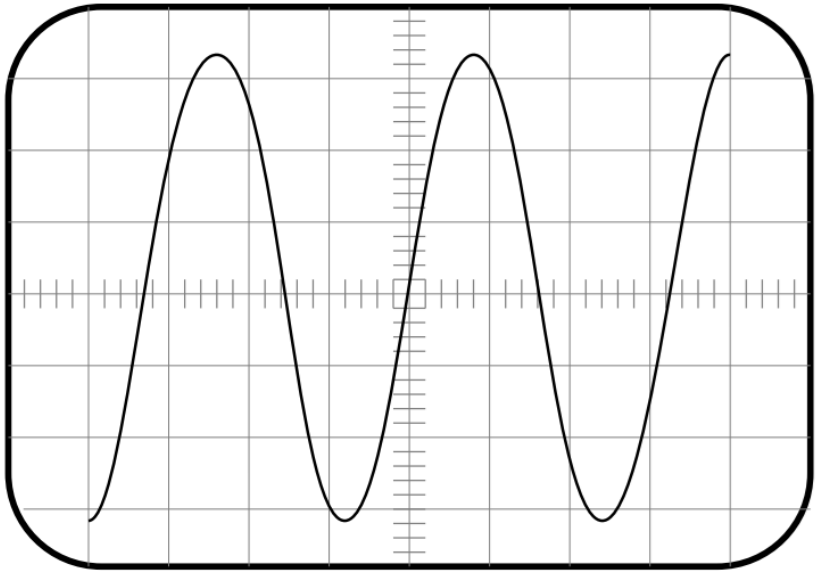
The horizontal distance from peak to peak (1 wave) is 8 divisions. As time-base is set to 50 ms/div, the period, *T*, is therefore $T = 8 \times 50 = 400\ ms$

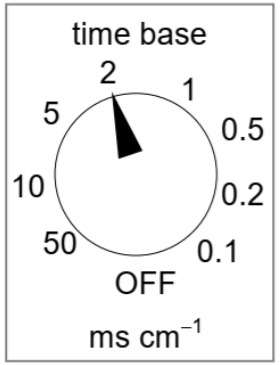
Using frequency, $f = \frac{1}{T}$

$$f = \frac{1}{400 \times 10^{-3}} = 2.5\ Hz$$

Practice Example 10

A sinusoidal sound wave of unknown frequency is fed into a C.R.O. and the waveform on C.R.O is shown below. The length of each division for the time-base is 1 cm. Find the frequency of the sound.





Suggested Solutions to Practice Examples

Practice Example 1

With a frequency of 440 Hz, a sound wave is travelling with a speed of 330 m/s. What is the phase difference between two points on the wave 0.25 m apart in the direction of travel?

Solution:

$$v = f\lambda \rightarrow \lambda = \frac{v}{f} = \frac{330\ m/s}{440\ Hz}$$

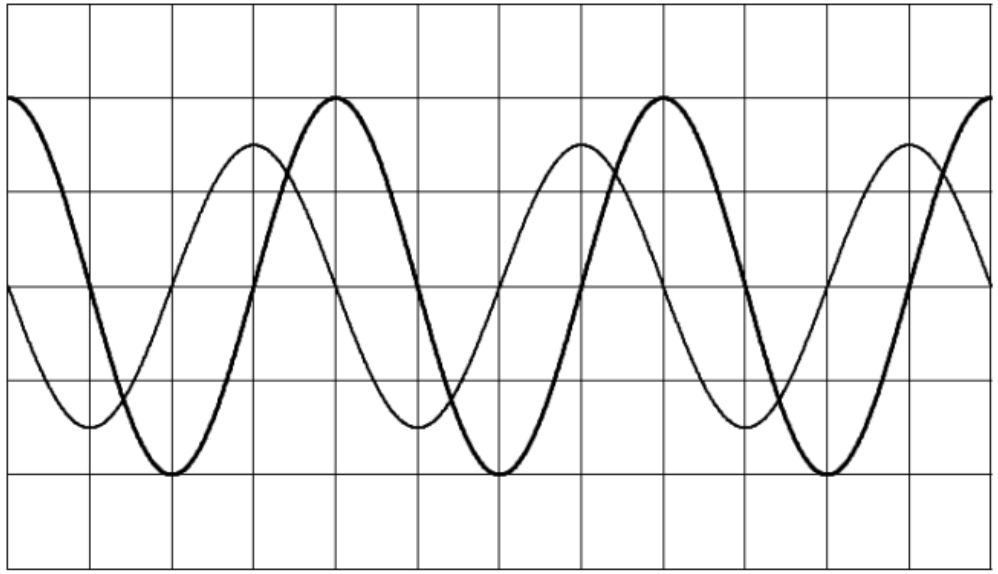
$$\lambda = 0.75\ m$$

$$\Delta\phi = \frac{\Delta x}{\lambda} 2\pi = \frac{0.25}{0.75} 2\pi$$

$$\Delta\phi = \frac{2\pi}{3}$$

Practice Example 2

Consider a displacement-time graph of two waves detected by a sensor shown on the left. The waves have the same frequency. What is the phase difference between the two waves



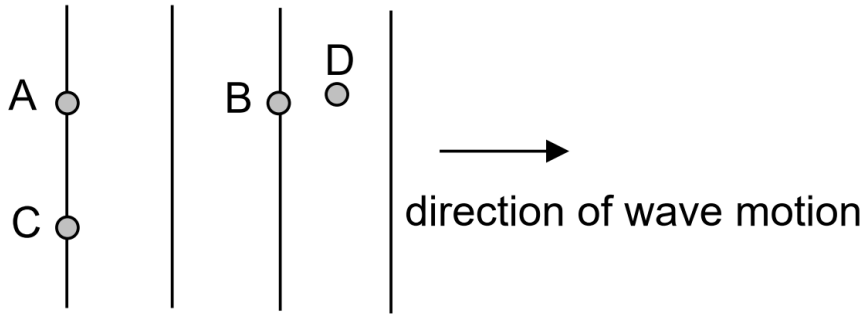
Solution:

$$\Delta\phi = \frac{\Delta t}{T}(2\pi)$$

$$= \frac{\left(\frac{1}{4}T\right)}{T}(2\pi)$$

$$= \frac{\pi}{2}\ rad$$

Practice Example 3



Shown below are the wavefronts of a wave travelling to the right. It has a speed of 2.25 m/s and a frequency of 0.80 Hz.

Determine the following:

a. Distance between points A and B
b. Phase difference between points B and C
c. Phase difference between points B and D

a. distance between A and B $= 2\lambda$

$$v = f\lambda \rightarrow \lambda = \frac{v}{f}$$

$$\lambda = \frac{2.25 m/s}{0.8\ Hz} = 2.81\ m$$

distance between A and B $= 2\lambda = 2(2.81m) = 5.62\ m$

b. Since B and C are 2λ apart, they are in phase. Hence their phase difference is 0 rad.

Alternatively,

$$\Delta\phi = \frac{\Delta x}{\lambda}(2\pi) = \left(\frac{2\lambda}{\lambda}\right)(2\pi)$$

$$\Delta\phi = 4\pi = 0\ rad$$

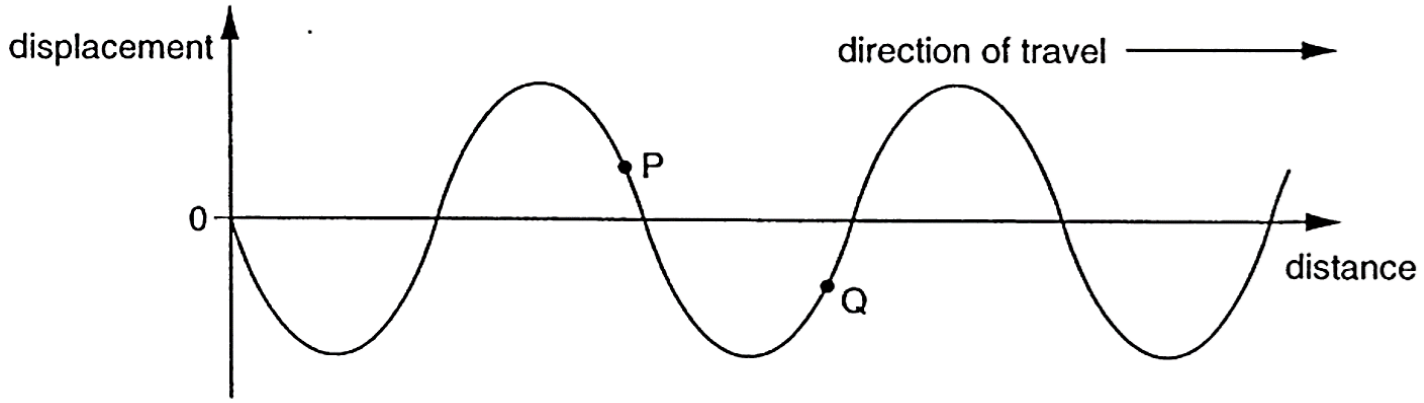
c. Since B and D are $\frac{1}{2}\lambda$ apart, they are exactly out of phase. Hence their phase difference is $\pi\ rad$. Alternatively,

$$\Delta\phi = \frac{\Delta x}{\lambda}(2\pi) = \left(\frac{\frac{1}{2}\lambda}{\lambda}\right)(2\pi)$$

$$\Delta\phi = \pi\ rad$$

Practice Example 4

A displacement-distance graph of a transverse progressive wave is shown below. It travels to the right along a rope. In which direction are P and Q moving?

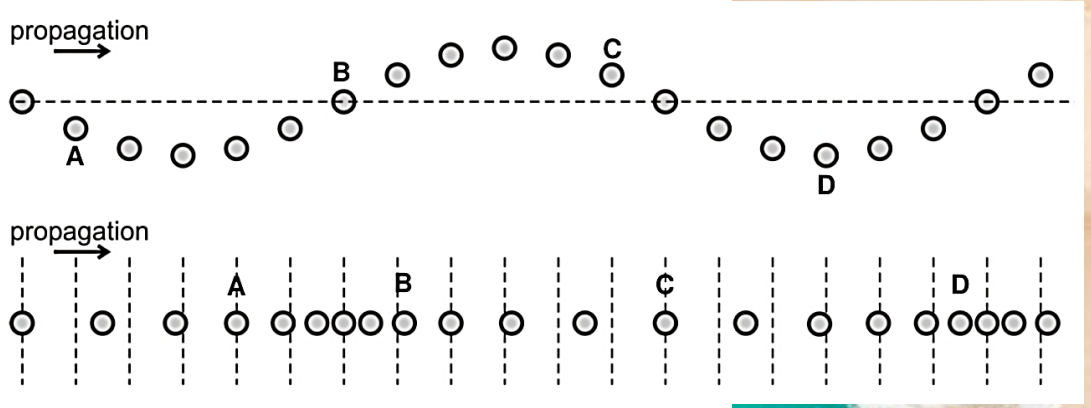


Answer:

Movement of P is upwards while the movement of Q is downwards.

Practice Example 5

The figures below show a progressive transverse and a progressive longitudinal wave respectively,



On each wave, identify a point or points at which

a. the velocity is zero
b. the acceleration is zero
c. the velocity is in the same direction as the displacement
d. the acceleration is in the same direction as velocity

Answer:

	Transverse	Longitudinal
a.	**D**	**A**
a.	**B**	**C**
a.	**C**	**D**
a.	**A**	**B**

Practice Example 6

A person is initially 5.0 m from a point source which emits energy uniformly in all directions at a constant rate. If the power of the source is to be doubled but the sound is to be as loud as before, at what distance should the person be from the source?

Solution:

Use $I = \frac{P}{4\pi r^2}$ and note that intensity is the same. Let the primed variables be the variables taken at a distance from the source.

$$I = I'$$

$$\frac{P}{4\pi r^2} = \frac{P'}{4\pi r'^2} \rightarrow \frac{P}{r^2} = \frac{P'}{r'^2}$$

$$\left(\frac{r'}{r}\right)^2 = \frac{P'}{P} \rightarrow r' = \sqrt{\frac{P'}{P}}\, r$$

Power of the source is doubled

$$r' = \sqrt{\frac{(2P)}{P}}\, r = \sqrt{2} r = \sqrt{2}\,(5)$$

$$r' = 7.07\ \text{m}$$

Practice Example 7

Suppose a loudspeaker operating at 35 W is producing sound waves in all directions. Calculate

a) the intensity of sound at a distance of 12 m away
b) the power received by a square microphone of length 4.0 cm placed at a distance 8.0 m away from the loudspeaker
c) the amplitude of the vibrations at 8.0 m, given that at 4.0 m, the amplitude of the vibrations is 3.0 cm.

Solution (a):

Use $I = P/A$

$$I = \frac{P}{4\pi r^2} = \frac{35}{4\pi(12)^2}$$

$$I = 0.019\ W/m^2$$

Solution (b):

Use $I = P/A$

$$P = IA$$

$$P = (0.019)(4 \times 10^{-2})^2$$

$$P = 3.04 \times 10^{-5}\ W$$

Solution (c):

Let Am = amplitude. Primed variables = measured at 8.0 m

$$Am \propto \frac{1}{r}$$

$$\Rightarrow \frac{Am'}{Am} = \frac{r}{r'}$$

$$Am' = \frac{r}{r'} Am$$

$$Am' = \frac{4}{8}(3 \times 10^{-2})$$

$$Am' = 0.015\ m$$

Practice Example 8

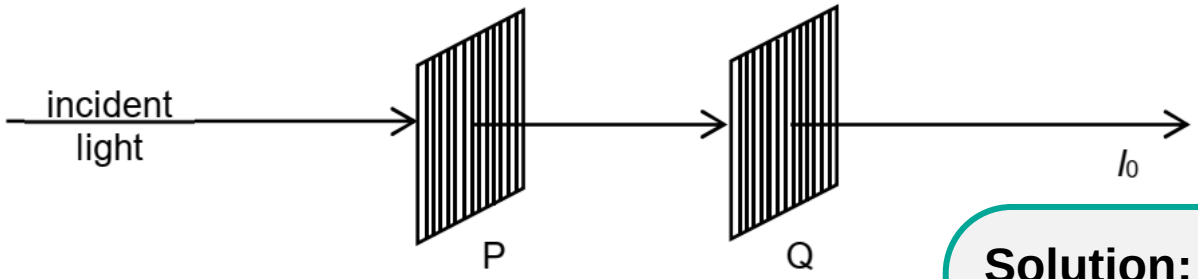
Explain why it would not be possible to polarize sound waves.

Answer:

Sound waves in a gas or liquid do not have polarization because the medium vibrates only along the direction in which the waves are travelling.

Practice Example 9

Two polarisers P and Q are placed next to each other such that their polarization axes are parallel and vertical, as shown below. If intensity of the emergent light is I_0, through what angle must Polaroid Q be rotated so that the intensity of the emergent light decreases to $\frac{1}{4}I_0$?



Solution:

$$I \propto x_0^2$$

$$\left(\frac{I'}{I}\right) = \left(\frac{x_0'}{x_0}\right)^2$$

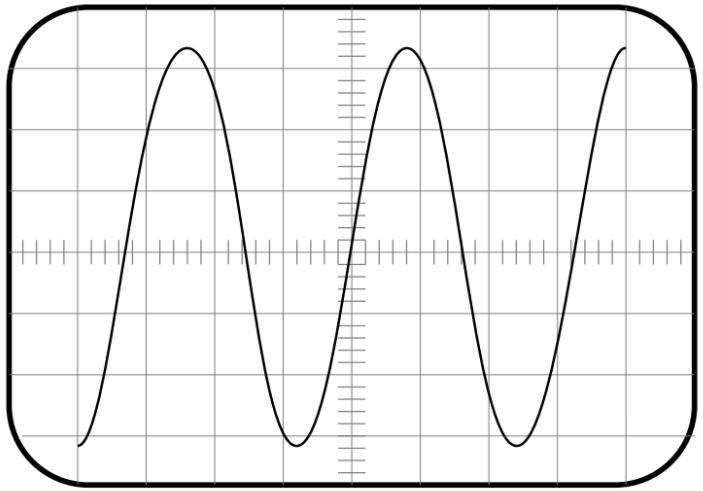
$$\left(\frac{\frac{1}{4}I_0}{I_0}\right) = \left(\frac{x_0 \cos\theta}{x_0}\right)^2$$

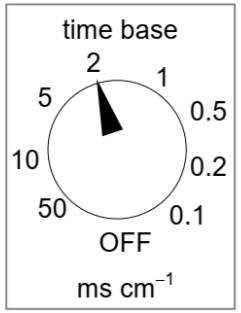
$$\frac{1}{4} = \cos^2\theta$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60° = \frac{\pi}{3}\ rad$$

Practice Example 10

A sinusoidal sound wave of unknown frequency is fed into a C.R.O. and the waveform on C.R.O is shown below. The length of each division for the time-base is 1 cm. Find the frequency of the sound.





Solution:

From C.R.O., 8 divisions correspond to 3.5 wavelengths,

$$2.5T = 8(2 \times 10^{-3})$$

$$T = 0.0064\ s$$

$$f = \frac{1}{T} = \frac{1}{0.0064} = 156.25$$

$$f = 160\ Hz$$

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