A LEVEL
H2 MATHEMATICS
COMPLEX NUMBERS

CHAPTER ANALYSIS

MASTERY

- Imaginary Number i
- Complex Numbers in Cartesian and Polar form
- Complex Conjugates
- Complex Roots of Polynomial Equations
- Argand Diagrams, Modulus and Argument
- Effect of Multiplying 2 Complexes

EXAM

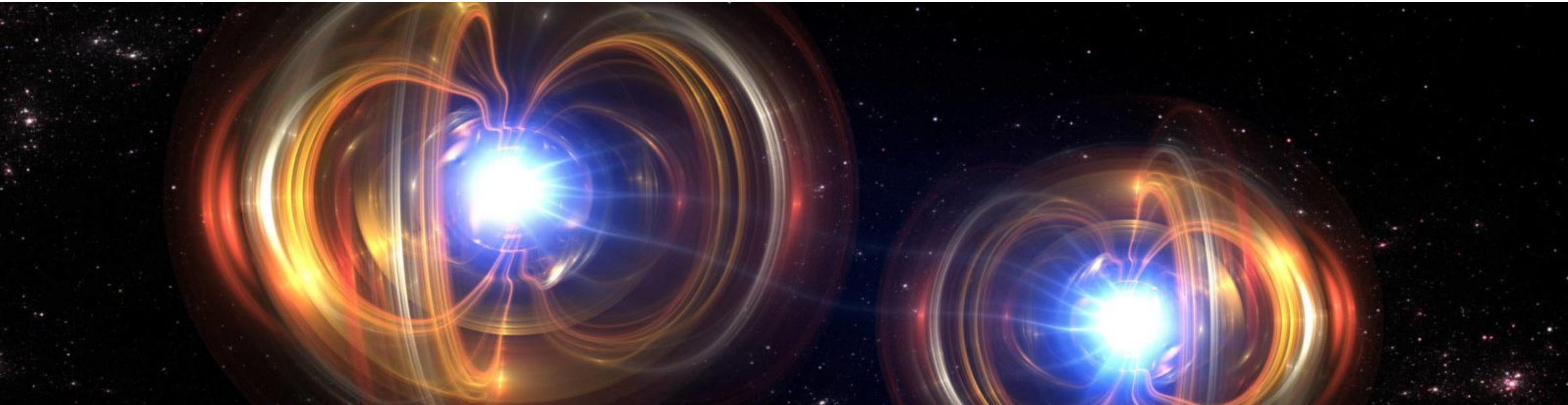
- Practice question types, there are only a few
- Understand the chapter from an algebraic point of view
- The new syllabus does not test drawing on argand diagrams

WEIGHTAGE

- Appears every year, typically 1 big question, or 2 small questions
- Constitutes approximately 3-7% of final grade

COMPLEX NUMBERS I

COMPLEX NUMBERS & IMAGINARY NUMBER i
COMPLEX NUMBER OPERATIONS
COMPLEX CONJUGATES
COMPLEX ROOTS OF POLYNOMIAL EQUATIONS



Imaginary Number *i*

$i = \sqrt{-1}$	$i^{4k+1} = \sqrt{-1} = i$
$i^2 = -1$	$i^{4k+2} = -1$
$i^3 = -i$	$i^{4k+3} = -i$
$i^4 = 1$	$i^{4k} = 1$

Complex Numbers

A **complex number** is of the form:

Cartesian Form

$$z = x + iy$$

where x and y are real numbers and $i = \sqrt{-1}$

x is the real part of z, Re(z)
$x = 0 \Longrightarrow z = iy$ is a purely imaginary number

y is the imaginary part of z, Im(z)
Note that Im(z) **does not include *i***
$y = 0 \Longrightarrow z = x$ is a real number

The set of complex numbers is denoted by $\mathbb{C}$

Division of Complex Numbers

$$\frac{x+iy}{a+ib} = \frac{x+iy}{a+ib} \times \frac{a-ib}{a-ib}$$

$$\frac{3+5i}{2-i}$$

$$= \frac{3+5i}{2-i} \times \frac{2+i}{2+i}$$

$$= \frac{6+3i+10i+5i^2}{4-i^2}$$

Let $i^2 = -1$

$$= \frac{6+13i-5}{4+1}$$

$$= \frac{1+13i}{5}$$

$$= \frac{1}{5} + \frac{13}{5}i$$

Complex Number Operations

Equality of 2 Complex Numbers

$$x + iy = a + ib \iff x = a \text{ and } y = b$$

Addition of Complex Numbers

$$(x+iy) + (a+ib) = (x+a) + i\,(y+b)$$

Subtraction of Complex Numbers

$$(x+iy) - (a+ib) = (x-a) + i\,(y-b)$$

Multiplication of Complex Numbers

$$\begin{aligned} &(x+iy)(a+ib) \\ &= xa + ixb + iya + i^2 yb \\ &= xa + ixb + iya + (-1)\,yb \\ &= (xa - yb) + i\,(xb + ya) \end{aligned}$$

Let $i^2 = -1$

The **complex conjugate** of $z = x + iy$ is denoted by z^* and defined as:

Cartesian Form

$$z^* = x - iy$$

where x and y are real numbers and $i = \sqrt{-1}$

z and z^* are conjugates of each other and known as conjugate pairs

Observe that $\mathrm{Re}(z)$ = x = $\mathrm{Re}(z^*)$
While $\mathrm{Im}(z^*)$ = $-y$ = $-\mathrm{Im}(z)$

Complex Conjugates

Useful Properties:

1. $(z^*)^* = z$
2. $z + z^* = 2Re(z)$
3. $z - z^* = 2i\ Im(z)$
4. $zz^* = x^2 + y^2$
5. $z = z^* \Leftrightarrow z\ is\ real$
6. $(z + w)^* = z^* + w^*$
7. $(zw)^* = z^*\ w^*$

*****Important Result: Complex Roots of Polynomial Equations**

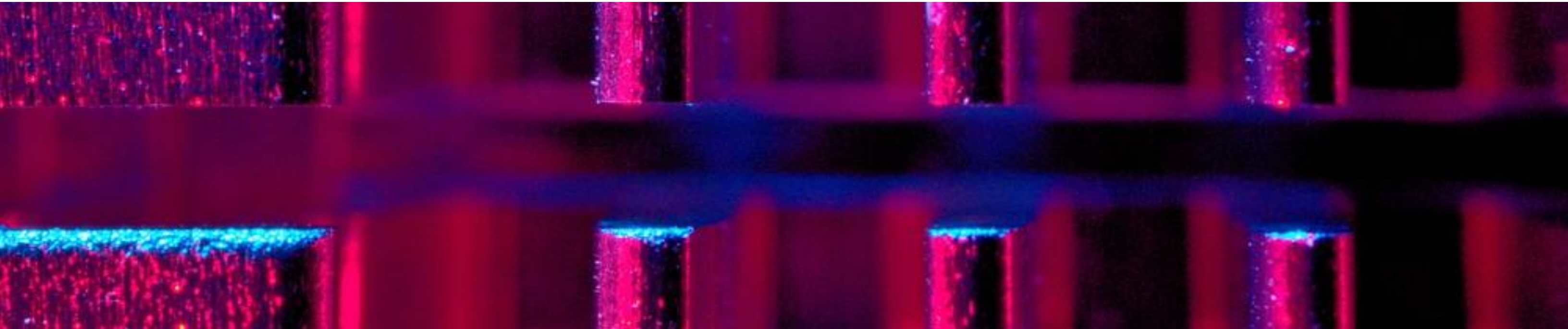
Non-real roots of a polynomial equation with <u>real coefficients</u> occur in conjugate pairs

$x^2 - 2x + 2 = 0$ has REAL coefficients to which $x = 1 \pm i$ are conjugate pair solutions

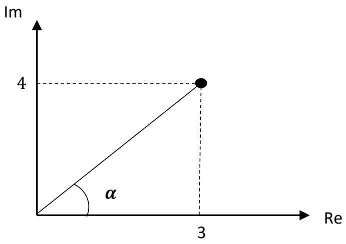
$z^2 + (-2 + 3i)\,z + (5 - i) = 0$ has IMAGINARY coefficients therefore the solution does not contain conjugate pairs

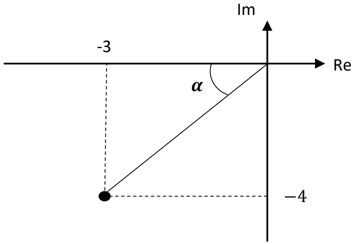
COMPLEX NUMBERS II

ARGAND DIAGRAMS
MODULUS & ARGUMENTS
POLAR FORM OF COMPLEX NUMBERS
MOD & ARG RELATIONSHIP WITH CONJUGATES
GEOMETRICAL EFFECT OF MULTIPLYING 2 COMPLEXES



Example





$|3+4i| = \sqrt{3^2+4^2} = 5$

$\tan\alpha = \frac{4}{3} \Longrightarrow \alpha = 0.927\ rad$

$\arg(3+4i) = 0.927\ rad$

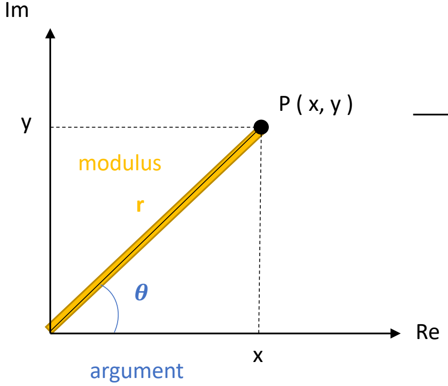
$|-3-4i| = \sqrt{3^2+4^2} = 5$

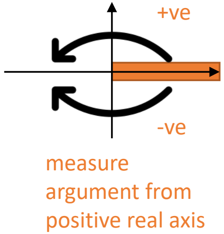
$\tan\alpha = \frac{4}{3} \Longrightarrow \alpha = 0.927\ rad$

$\arg(3+4i) = -\pi + 0.927\ rad$

Geometrical Representation of Complex Numbers

Argand Diagram





r is the modulus of the complex number z, denoted by $|z|$

$$|\boldsymbol{z}| = \boldsymbol{r} = \sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2}$$

θ is the argument of the complex number z, denoted by arg(z)
where $-\boldsymbol{\pi} < \boldsymbol{arg}(\boldsymbol{z}) \leq \boldsymbol{\pi}$ and arg(z) should be given in **radians**

Complex Number addition and subtraction follow the vector parallelogram law of addition and subtraction

Multiplication & Division of Complex Numbers in Polar Form

$$z_1 z_2 = r_1 r_2 \, e^{i(\theta_1+\theta_2)}$$

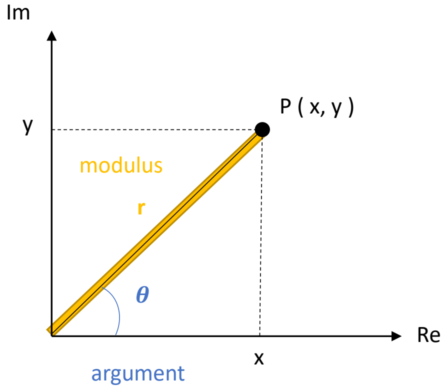
$$\frac{z_1}{z_2} = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$$

Other Useful Properties

1. $|z_1 z_2| = |z_1||z_2|$
2. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
3. $|z^n| = |z|^n$

1. $arg(z_1 z_2) = arg(z_1) + arg(z_2) = \theta_1 + \theta_2$
2. $arg\left(\frac{z_1}{z_2}\right) = arg(z_1) - arg(z_2) = \theta_1 - \theta_2$
3. $arg(z^n) = n \, arg(z) = n \, \theta_1$

Complex Numbers Polar Form



Any complex number can be written as:

Cartesian Form

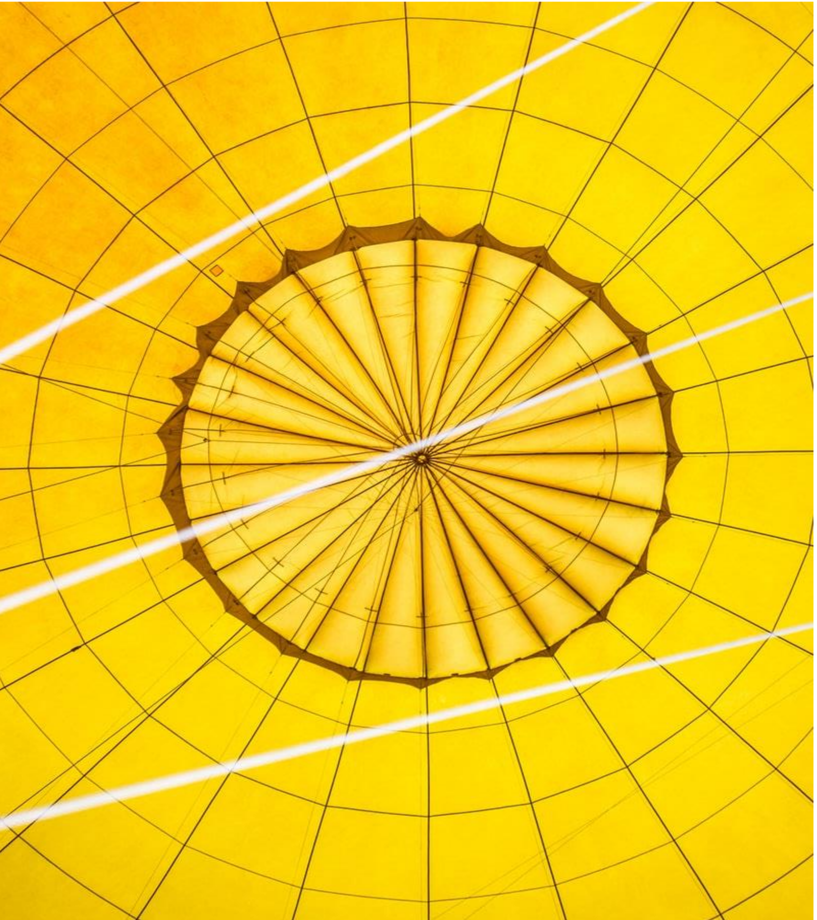
$z = x + iy$

Polar Form

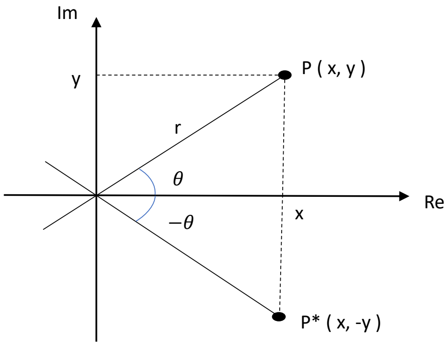
$r\,(\cos\theta + i\sin\theta)$

OR

$r\,e^{i\theta}$



Mod & Arg Relationship With Conjugates



$$|z^*| = |z|$$

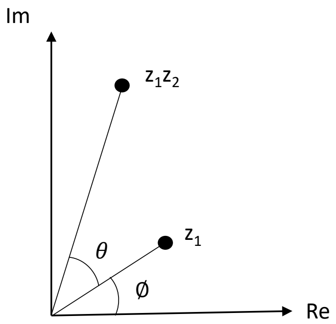
$$arg(z^*) = -\,arg(z)$$

$$zz^* = x^2 + y^2 = |z|^2$$

$$z = r\,e^{i\theta}$$

$$z^* = r\,e^{-i\theta}$$

Multiplying 2 Complexes

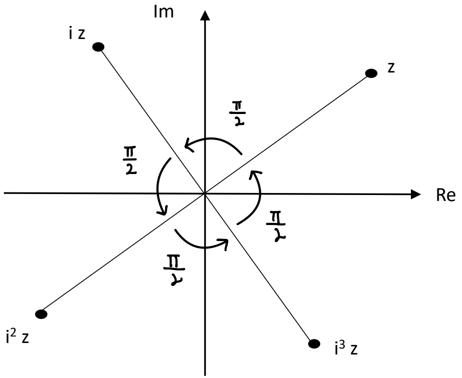


$$z_1z_2 = r_1r_2\, e^{i(\theta_1+\theta_2)}$$

Scale a factor r of the length Oz_1 , followed by an **anti-clockwise rotation through an angle of θ_2 radians about O**

Geometrical Effect Of Multiplying 2 Complexes

Multiplying a Complex by i



When complex number z is multiplied by i, the point represented by z on the argand diagram is **rotated anti-clockwise through an angle of $\frac{\pi}{2}$ radians about O**

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