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Topic 10:
Coordinate Geometry (4049)

THE ABOUT

CHAPTER ANALYSIS

- Conditions for 2 lines to be parallel or perpendicular
- Midpoint of line segment
- Area of rectilinear figure

[Note that E-Math Coordinate Geometry is a pre-requisite]

MASTERY

- Relatively straight forward chapter
- 3 **key** concepts

EXAM

- Concepts usually tested as a stand-alone topic
- Questions are repetitive, just need to follow the same algorithm to solve the same type of questions

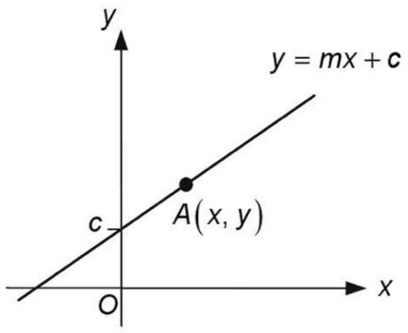
WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, an 8-9m question, 1 question in one of the papers

KEY CONCEPT

Parallel/Perpendicular lines
Midpoint of a line segment
Area of rectilinear figure

Equation of a straight line



$$\boxed{y = mx + c}$$

Term	Name	Definition
c	y-intercept	Represents the y-value where the line cuts the y-axis
m	Gradient	Represents the change in the y-value arising from a per unit change in x

Alternative to the equation of a straight line

There is another equation that can be used to find the equation of a straight line. This equation is more powerful (and useful) than the standard equation as it only requires 1 gradient and 1 point while its latter requires 1 gradient and 2 points minimum

$$y - y_1 = m(x - x_1)$$

- (x_1, y_1) is the coordinate needed
- m is the gradient of the line

Take Note

This formula is actually derived from the gradient formula

$$m = \frac{y - y_1}{x - x_1} \quad \Rightarrow \quad y - y_1 = m(x - x_1)$$

Common Mistake

The coefficient of y must be 1 when reading off the gradient and y-intercept. Many students will forget about this fact and carry on the question without checking

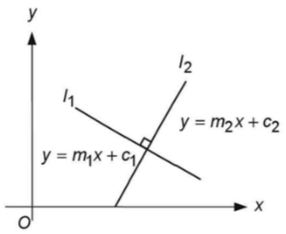
$$2y = 4x + 8 \quad \Rightarrow \quad y = 2x + 4$$

The gradient of the line is 2 and the y-intercept is 4. This is because the whole equation has to be divided by 2 first as the coefficient of y must be 1

Perpendicular Lines

For 2 lines to be perpendicular, the product of their gradients is -1

$$m_1 \times m_2 = -1$$

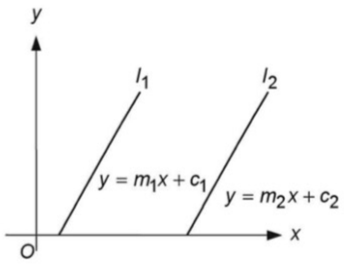


These lines intersect each other at 90°

Parallel Lines

The condition for parallel lines is that both lines have the same gradient, but different y-intercepts

$$m_1 = m_2$$



There are questions where students are asked to determine if there are any intersection points between 2 lines. A very easy way to check is to check the gradient and y-intercept values. There will be possible 3 cases:

- Gradient and y-intercept same
  - The lines are identical, they have infinitely many intersection points
- Gradient same, y-intercept different
  - The 2 lines are parallel, they have no intersection points
- Gradient and y-intercept different
  - The 2 lines have no unique relationships between them, they have 1 point of intersection

Gradient of a straight line

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

m value	Indication
$m > 0$	Positive gradient, upwards sloping
$m < 0$	Negative gradient, downwards sloping
$m = 0$	Parallel to the x-axis, horizontal line
m undefined	Parallel to the y-axis, vertical line

Take Note

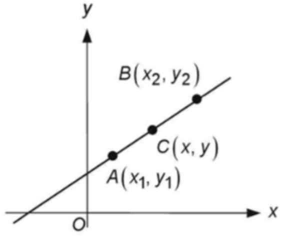
Do note that from the value of the gradient, we can tell how steep a line is. The smaller the gradient value, the shallower the gradient is going to be. The greater the value, the steeper the gradient is going to be

Collinearity with 3 points

Students are not allowed to assume that if 3 points lie on the same line that the line is straight **UNLESS** it is explicitly stated in the question of this line is part of a standard geometric figure

To test for collinearity:

$$m_{AC} = m_{CB} = m_{AB}$$

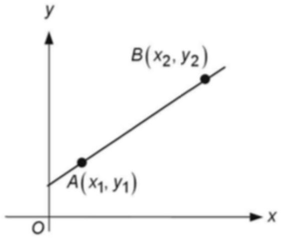


All 3 line segments, AB, BC and AC must have the **same gradient** and there exist a shared common point B

Distance between 2 points

The formula for calculating the distance between 2 points on a straight line is given as such

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Take Note

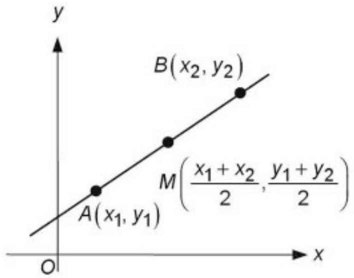
This formula is linked to Pythagoras' Theorem

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad AB = \sqrt{x^2 + y^2}$$

During the examinations, if students forget the distance formula, they can opt to draw a right-triangle and compute the length using Pythagoras' Theorem instead

Midpoint of a Line Segment

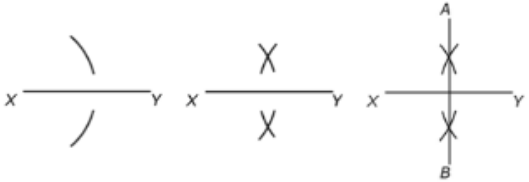
Think of calculating the average of the x and y coordinates



$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Perpendicular Bisectors

A line that is perpendicular to the segment and divides it into 2 congruent segments



To find the equation of the perpendicular bisector,

$$y - y_1 = m(x - x_1)$$

we need

- 1 point: **Midpoint of the line**
- 1 gradient: $\dfrac{-1}{\textbf{Gradient of line}}$

Area of rectilinear figures

Method to use is the Shoelace method. Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, … and $M(x_m, y_m)$ be the vertices of a rectilinear figure and the points are arranged in an anti-clockwise direction

$$\text{Area} = \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_m & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_m & y_1 \end{matrix}\right|$$

To be very honest, the direction of how the points are arranged does not really matter [due to the modulus signs], but the ordering does. Always ensure that you follow one specific direction when calculating

Take Note

Always remember to repeat the very first point that you choose

$$\text{Area} = \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_m & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_m & y_1 \end{matrix}\right|$$

TAKE NOTE

- Do note that the first coordinate you choose is **repeated**. So if you have 3 vertices, your shoelace should have 4 points, 4 vertices, shoelace should have 5 points etc..
- Do note that the bars on the side of the formula represent the modulus sign.

$$\text{Area} = \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_m & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_m & y_1 \end{matrix}\right|$$

This forces anything within them to be positive. So let's say you get a negative value, these bars will cause the value to turn positive. Also note that the reason for this is that areas are strictly positive

- How to tabulate:

$$\frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_m & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_m & y_1 \end{matrix}\right|$$

Downward arrows are +, upward arrows are −

Hence, this evaluates to

$$\frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & \cdots & x_m & x_1 \\ y_1 & y_2 & y_3 & \cdots & y_m & y_1 \end{matrix}\right| = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + \ldots + x_my_1) - (y_1x_2 + y_2x_3 + \ldots + y_mx_1)\right|$$

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