



**GCSE Preliminary Examination Mock Paper 2022
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC**

CANDIDATE
NAME

CENTRE

ADDITIONAL MATHEMATICS

4049/01

Mock Paper 1

August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name and centre in the spaces at the top of this page.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is **90**.

Grade Tables: For Examiner's Use

Total Score	Deductions			Grade
	Rounding	Units	Presentation	

Setter: **Kaiwen** :)

This question paper consists of 27 printed pages including the cover page

Grade Tables: For Examiner's Use

Question	Points	Score	Question	Points	Score
1	8		7	6	
2	10		8	3	
3	7		9	10	
4	10		10	8	
5	9		11	10	
6	9		Total:	90	

Examiner's Comments

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Answer all questions: 90 marks

1. (a) Given that

$$\frac{d}{dx} [F(x)] = \frac{9}{2} \sqrt{3x-1} - \frac{3}{\sqrt{3x-1}}$$

it is given that

$$F(3) - F(1) = k\sqrt{2}, \quad k \in \mathbb{Z}$$

Find the value of k *Answer* _____ [4]

(b) The equation of a curve is given by

$$y = \operatorname{cosec}^2\left(\frac{x}{2} - \frac{\pi}{6}\right), \quad 0 < x < \frac{\pi}{2}$$

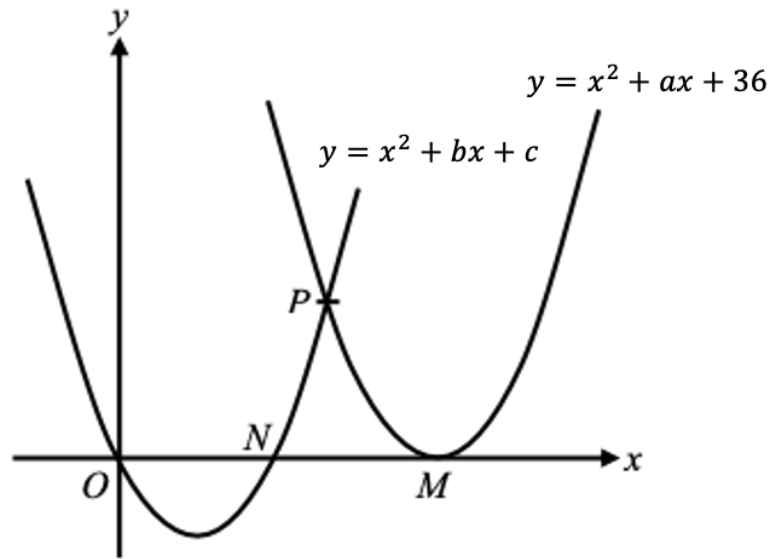
Given that x is increasing at 0.3 radians per second, find the rate of change of y with respect to time when $x = \frac{5\pi}{6}$

Answer _____ [4]

2. The diagram shows 2 curves

$$A : y = x^2 + ax + 36$$

$$B : y = x^2 + bx + c$$



Graph A touches the x -axis at M . N is the midpoint of OM , where O is the origin. Graph B passes through the points O and N , and intersects Graph A at P

(a) Find the values of a , b and c where $a, b, c \in \mathbb{Z}$

Answer _____ [4]

(b) Find the coordinates of P

Answer _____ [2]

(c) Another new graph has the equation

$$C : y = mx^2 + nx + r$$

Graph C has its vertex (turning point) at P and passes through M , find the values of m , n and r where $m, n, r \in \mathbb{Z}$

Answer _____ [4]

3. (a) Prove that

$$\operatorname{cosec}(60^\circ - \theta) = \frac{2}{\cos \theta (\sqrt{3} - \tan \theta)}$$

Answer

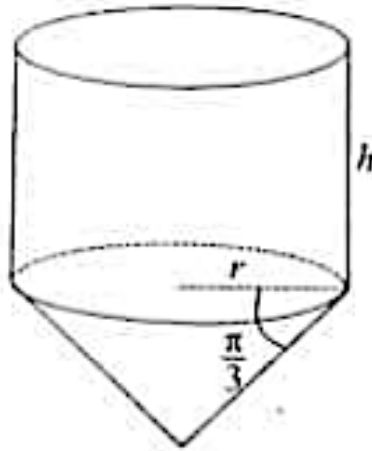
[3]

(b) **Hence**, find, in surds form, the value of

$$\operatorname{cosec}15^\circ$$

Answer _____ [4]

4. The diagram shows a solid machine part that is made up of a closed cylinder joined to an inverted right circular cone



The height of the cylinder is h m and the slant height of the cone makes an angle of $\frac{\pi}{3}$ radians to its base radius, r m

- (a) Given that the total volume of the machine part is 50π m³, express h in terms of r

Answer _____ [2]

(b) Show that the total surface area of the machine part is given by

$$A = \frac{\pi r^2}{3} (9 - 2\sqrt{3}) + \frac{100\pi}{r}$$

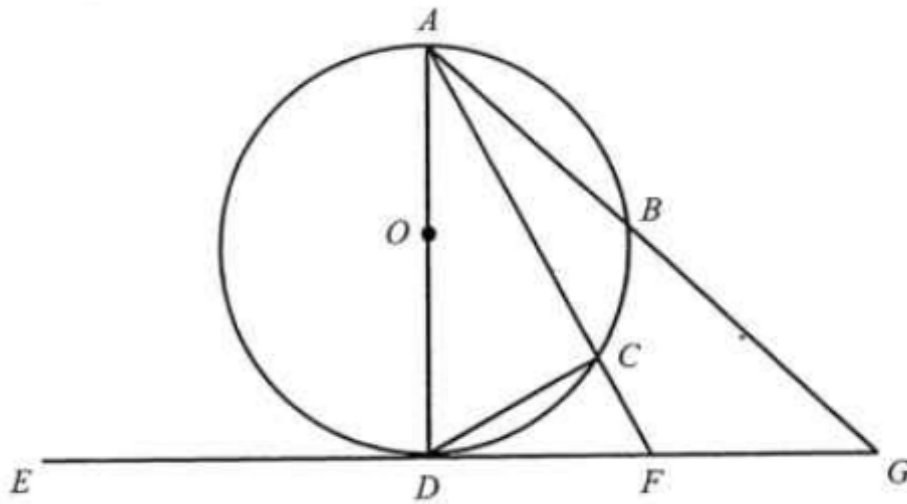
Answer

[3]

- (c) Given that r can vary, find the value of r for which the total surface area of the machine part is stationary and determine its nature

Answer _____ [5]

5. In the diagram, A , B , C and D are points on the circumference of the circle with centre O



$EDFG$ is a tangent to the circle at D . Given that $AB = BG$ and $DF = FG$, prove that

(a) ABD is an isosceles triangle

Answer

[3]

(b) $DB^2 - DF^2 = \frac{1}{4}AD^2$
Answer

[2]

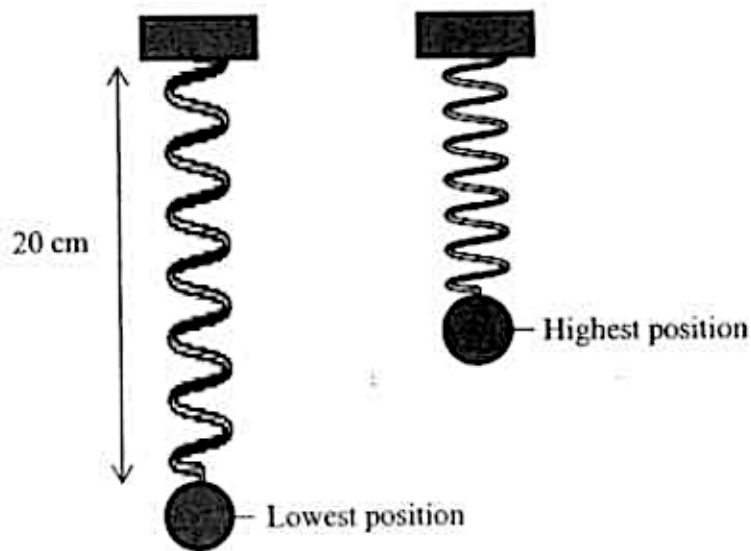
(c) $\triangle ADF$ is similar to $\triangle DCF$
Answer

[2]

(d) $GF^2 = AF \times CF$
Answer

[2]

6. The diagram shows an experimental setup where a weighted spring is released from a stretched position and follows a periodic up and down motion



The length of the spring, l cm, during the experiment is modelled by the equation

$$l = a \cos kt + 16$$

a , k are constants, and t is the time in seconds after releasing the weight from the lowest position. The length of the spring is 20 cm when the weight is at its lowest position and it takes 2 seconds for the weight to move from the lowest to the highest position

- (a) Find the value of a

Answer _____ [1]

- (b) Show that the value of k is $\frac{\pi}{2}$

Answer

[2]

(c) Find the length of the spring when the weight is at its highest position

Answer _____ [1]

(d) Sketch the graph of $l = a \cos kt + 16$ for $0 \leq t \leq 4$

Answer

[2]

(e) Find the time interval which the length of the spring will be longer than 18 cm for $0 \leq t \leq 4$

[3]

7. DO NOT USE A CALCULATOR FOR THIS WHOLE QUESTION

(a) If a is a root of the equation of

$$x^2 - 3x + 1 = 0$$

without solving for a , find the numerical value of

$$\frac{a^3}{a^6 + 1}$$

[Hint: Cubic Identities]

Answer _____ [3]

(b) Solve for the value of x

$$x^3 = 303^3 + 404^3 + 505^3$$

Answer _____ [3]

8. Given that

$$\lg\left(\frac{x+y}{3}\right) = \frac{\lg x + \lg y}{2}$$

find the numerical value of

$$\frac{x}{y} + \frac{y}{x}$$

Answer _____ [3]

9. AB is a chord of the circle and $M\left(\frac{4}{5}, 2\frac{2}{5}\right)$ is the midpoint of chord AB

$$x^2 + y^2 - 8x - 2y - 3 = 0$$

(a) Find the radius and the coordinates of the centre of the circle

Answer _____ [3]

(b) Find the equation of chord AB

Answer _____ [3]

If P is a variable point (point can move) on the circle,

(c) find the maximum area of $\triangle ABP$

Answer _____ [4]

10. (a) Prove that

$$\frac{\sec \theta + \operatorname{cosec} \theta}{\tan \theta + \cot \theta} = \sin \theta + \cos \theta$$

Answer

[3]

(b) Given that

$$\left(\frac{\sec \theta + \operatorname{cosec} \theta}{\tan \theta + \cot \theta}\right)^2 = 3 \sin 2\theta$$

deduce that

$$\sin 2\theta = \frac{1}{2}$$

Answer

[3]

(c) **Hence**, find the value of $\cos 2\theta$ where $\cos 2\theta > 0$. Leave your answer in exact form

Answer _____ [2]

11. (a) The independent term in the expansion of $(2 + x)^n$ and the independent term in the expansion of $(2 - ax)^{2n+1}$ are in the ratio of 1 : 8
- (i) Show that $n = 2$

[2]

- (ii) Find the value(s) of a given that

$$(1 + x)(2 - ax)^{2n+1} = \dots + 60x^2 + \dots$$

Answer _____ [3]

(b) Find the value of n given that

$$\left(\frac{2}{x} - x\right)^6 - \left(1 + \frac{2}{x}\right)^n = \dots + \frac{128}{x^2} + \dots$$

Answer _____ [5]

END OF PAPER

Question Source

Question	Credit	Remarks
1	S4 P2/ACS(I) 2015 PRELIM Qn 8	Modified
2	S4 P1/BPGHS 2015 PRELIM Qn 10	Modified
3	S4 P1/CHIJ SJC 2015 PRELIM Qn 2	-
4	S4 P2/CCHS(Y) 2015 PRELIM Qn 7	Modified
5	S4 P2/PLMGS 2015 PRELIM Qn 10	-
6	S4 P2/SCSS 2015 PRELIM Qn 7	-
7	TikTok & Instagram	Modified
8	Instagram	Modified
9	S4 P2/VS 2015 PRELIM Qn 8	Modified
10	S4 P1/YISS 2015 PRELIM Qn 6	-
11	S4 P2/CCHS(M) 2014 PRELIM Qn 2	Modified