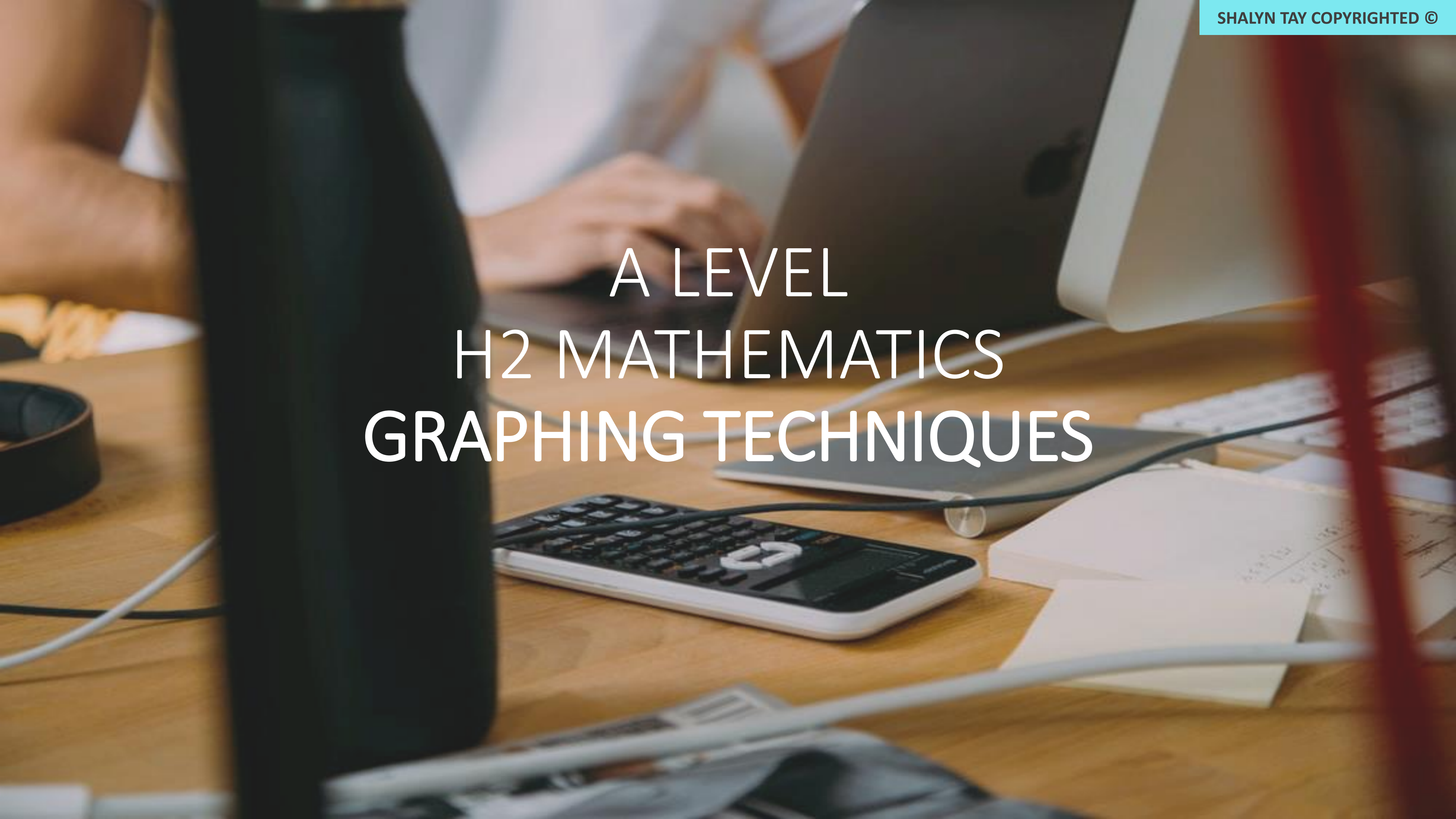


A LEVEL
H2 MATHEMATICS
GRAPHING TECHNIQUES

CHAPTER ANALYSIS

MASTERY

- Sketch graphs with and without G.C.
- Conic graphs
- Parametric curves
- Transformation of graphs

EXAM

- Graphical sketching is critical
- Understanding is useful in functions, differentiation, integration and their applications

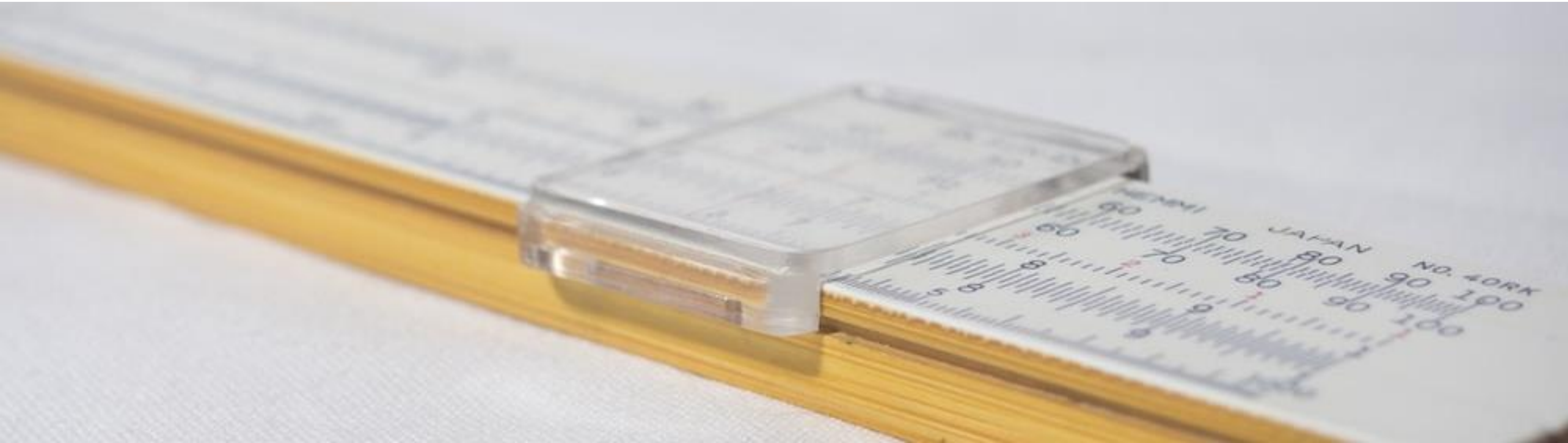
WEIGHTAGE

- Appears every year, at least 1 question
- Typically constitutes less than 5% of final grade alone, but is commonly integrated in other chapters

GRAPHING TECHNIQUES PART I

STANDARD GRAPHS

CONICS



Characteristics of Graphs

When sketching a graph, it is necessary to indicate the following:

1. X- and Y- Axes
2. Origin
3. Co-ordinates of Stationary Points
4. Co-ordinates of Intercepts
5. Equations of Asymptotes
6. Co-ordinates of End-Points
7. Equation of Graph

You may use the following acronym to help you with curve sketching:

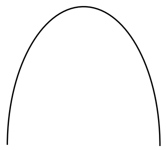
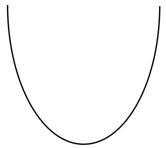
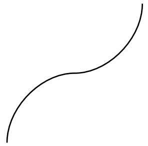
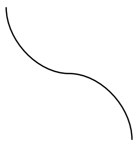
S	I	A
Shape / Stationary Point	Intercepts	Asymptotes

Standard Graphs

Intercepts

To find y-intercept: sub x = 0 and solve for y
To find x-intercept: sub y = 0 and solve for x

Stationary Points

Stationary Point	Turning Point		Point of Inflection	
	Maximum	Minimum		
Shape of Curve				
$\frac{dy}{dx}$	$+ \quad 0 \quad -$	$- \quad 0 \quad +$	$+ \quad 0 \quad +$	$- \quad 0 \quad -$
$\frac{d^2y}{dx^2}$	≤ 0	≥ 0	0	0

Asymptotes

For proper function $f(x) = \frac{A(x)}{B(x)}$, vertical asymptote can be found by letting $Q(x) = 0$
For improper function $f(x) = \frac{A(x)}{B(x)}$, carry out long division to express it as $f(x) = C(x) + \frac{D(x)}{B(x)}$ where $\frac{D(x)}{B(x)}$ is a proper function. If $C(x) = c$ where c is a constant, then $C(x)$ is the horizontal asymptote. If $C(x) = cx + d$, then $C(x)$ is the oblique asymptote.

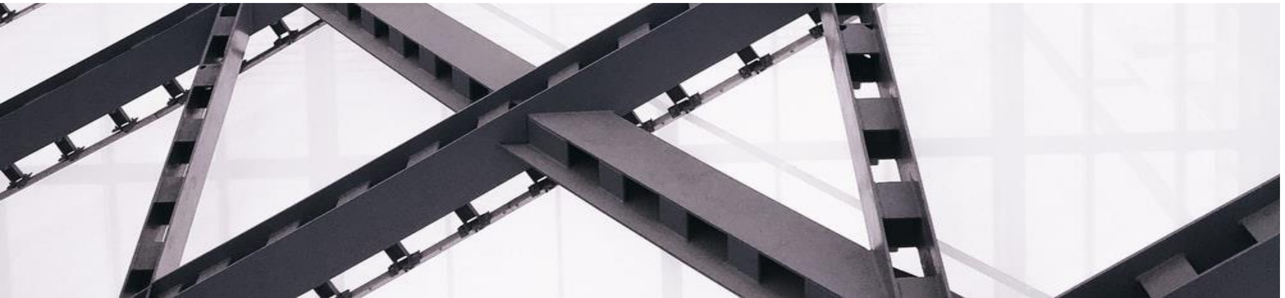
Cartesian Equations
Expresses the relationship between x and y directly.

Parametric Equations
Expresses the relationship between x and y in terms of a parameter, t.
You should be able to directly plot the graph from your G.C. by changing the mode to '*parametric*'.

Conversion from Parametric to Cartesian
To convert equations from parametric to cartesian, we must eliminate the parameter using substitution.

i.e. for $x = t^2 + 2$ and $y = 3t$

Cartesian equation: $x = (\frac{y}{3})^2 + 2$

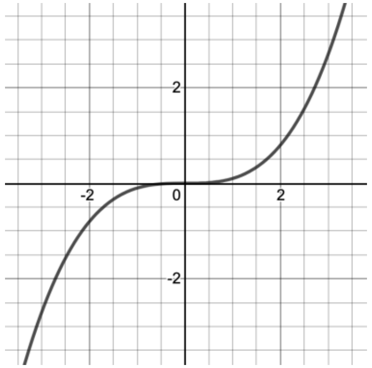


Points & Lines of Symmetry

A graph can be symmetric to the:

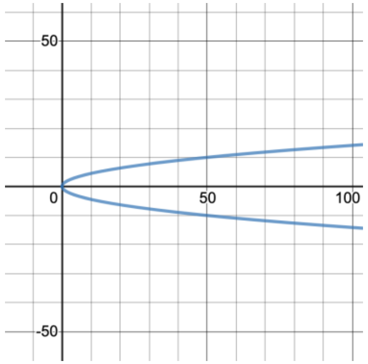
Origin

When rotated 180° about the origin, the same graph is obtained



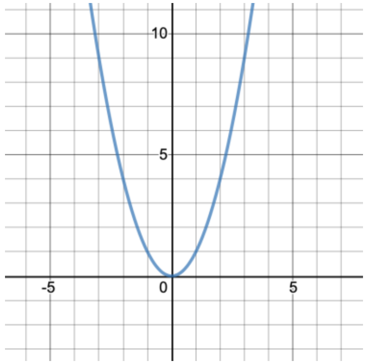
X-axis

Equation stays the same when y is replaced with -y



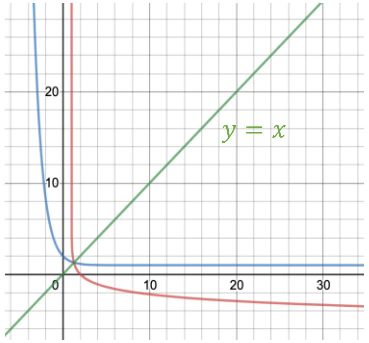
Y-axis

Equation stays the same when x is replaced with -x



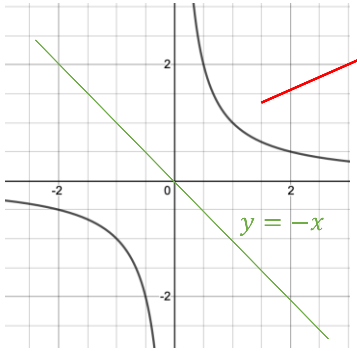
Line $y = x$

If (x,y) is on the graph, (y,x) is also on the graph.



Line $y = -x$

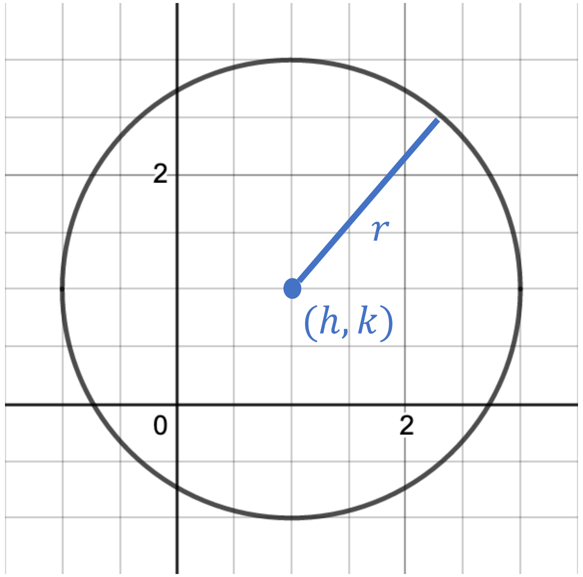
If (x,y) is on the graph, (y,x) is also on the graph.



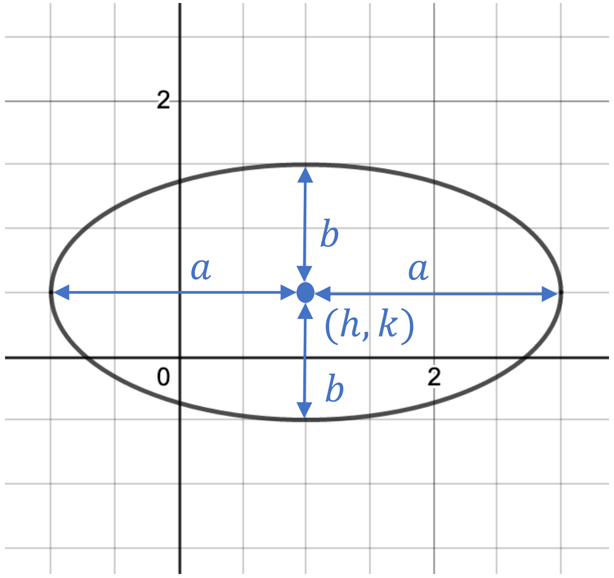
Graph is also symmetrical about the origin

Conics – Circles & Ellipses

Circle



Ellipse



No Asymptotes

Circles

General Form

$$x^2 + y^2 + ax + by + c = 0$$

Standard Form

$(x - h)^2 + (y - k)^2 = r^2$, where $r \neq 0$

Ellipses

General Form

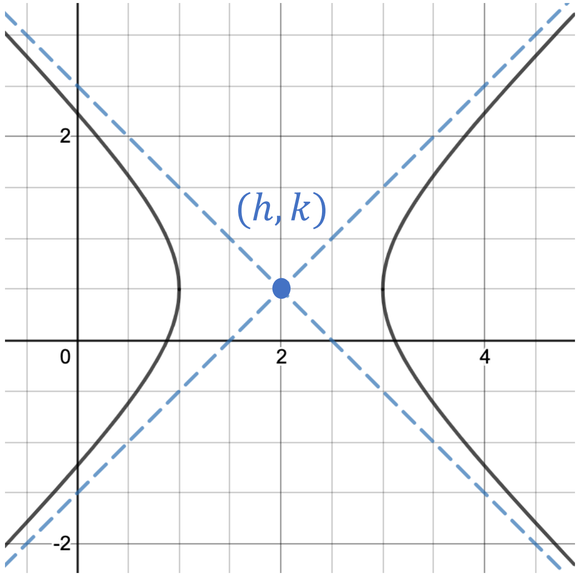
$b^2x^2 + a^2y^2 + cx + dy + e = 0$, where $a \neq 0, b \neq 0, a \neq b$

Standard Form

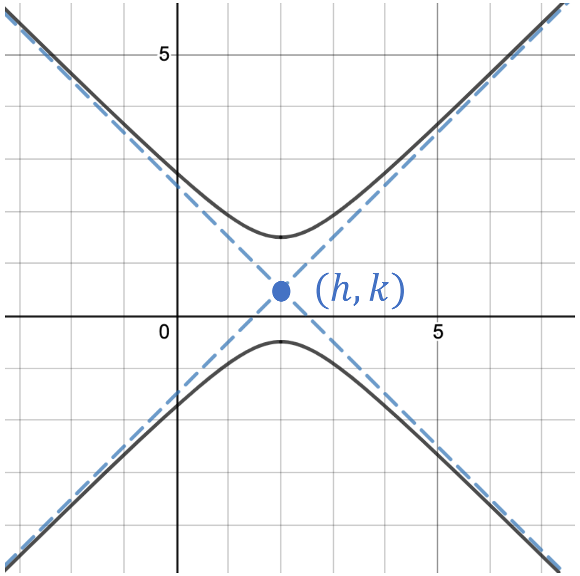
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Conics – Hyperbolas

Left-Right Hyperbola



Top-Bottom Hyperbola



Equation of Asymptotes:

$$y = \pm\frac{b}{a}(x - h) + k$$

Left-Right Hyperbola

General Form

$$b^2x^2 - a^2y^2 + cx + dy + e = 0, \text{ where } a \neq 0, b \neq 0$$

Standard Form

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Top-Bottom Hyperbola

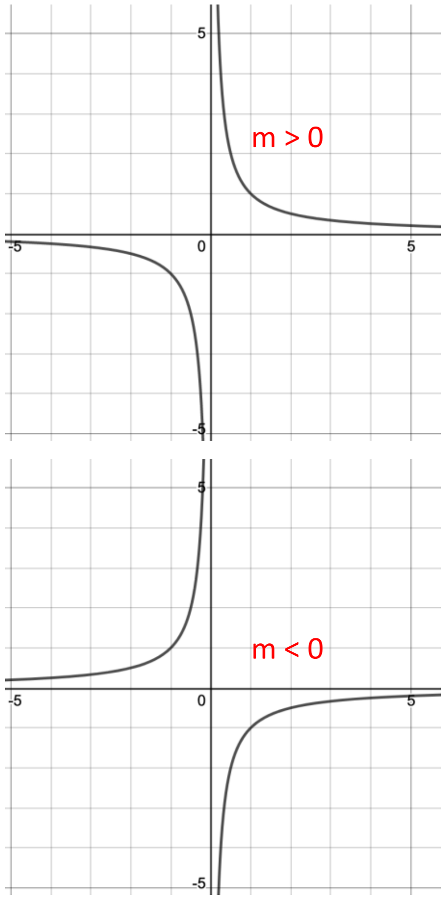
General Form

$$a^2y^2 - b^2x^2 + cx + dy + e = 0, \text{ where } a \neq 0, b \neq 0$$

Standard Form

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

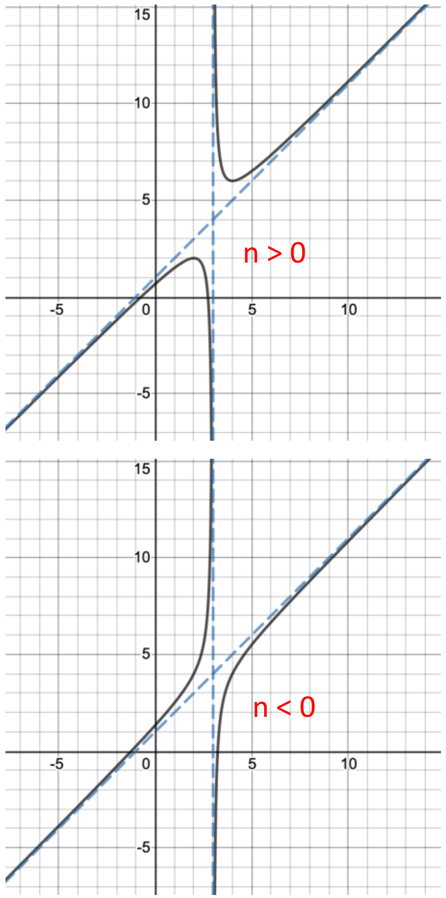
Rectangular Hyperbola



Horizontal Asymptote: $y = k = \frac{a}{c}$

Vertical Asymptote: $\mathrm{x} = h = -\frac{d}{c}$

Oblique-Vertical Asymptote Hyperbola



Oblique Asymptote: $y = sx + t$

Vertical Asymptote: $\mathrm{x} = h = -\frac{e}{d}$

Conics – Special Hyperbolas

Rectangular Hyperbola

Improper Form

$$y = \frac{ax + b}{cx + d} = \frac{linear}{linear}$$

Proper Form

$y = k + \frac{m}{x-h}$, where $k = \frac{a}{c}$ and $\mathrm{h} = -\frac{d}{c}$

Hyperbola with an Oblique and Vertical Asymptote

Improper Form

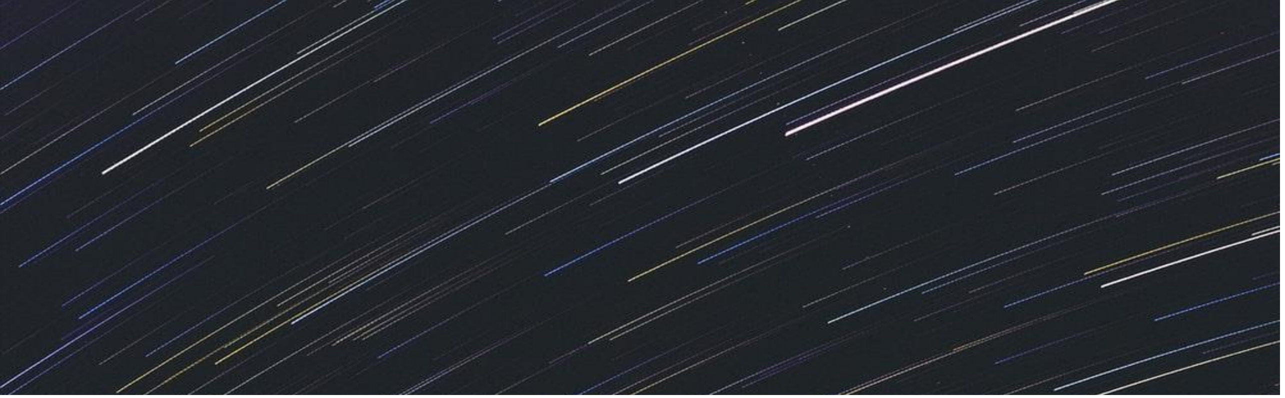
$$y = \frac{ax^2 + bx + c}{dx + e} = \frac{quadratic}{linear}$$

Proper Form

$y = sx + t + \frac{n}{x-h}$, where $\mathrm{h} = -\frac{e}{d}$

GRAPHING TECHNIQUES PART II

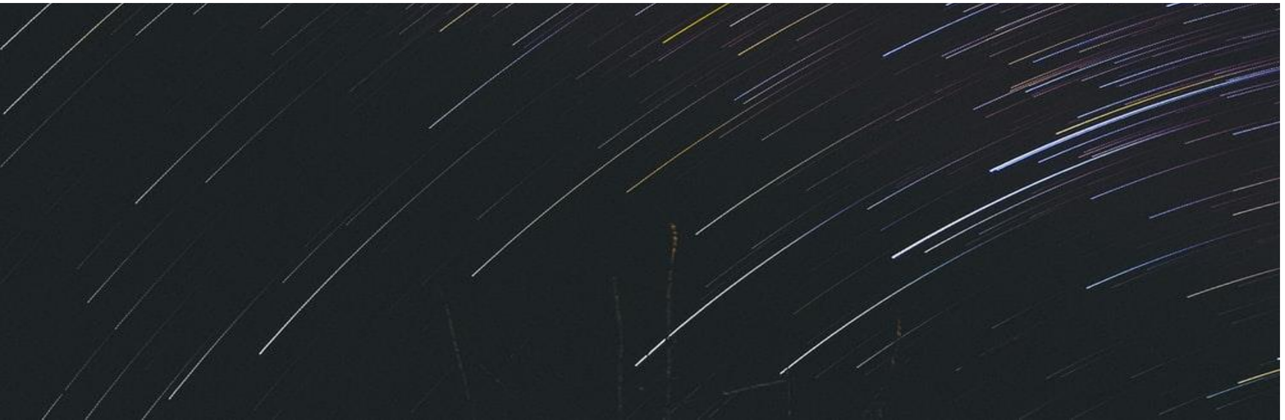
TRANSFORMATION OF GRAPHS: TRANSLATION SCALING & REFLECTION
MODULUS FUNCTIONS
DERIVATIVE FUNCTIONS
RECIPROCAL FUNCTIONS



Order of Transformations

You may find the following acronym useful in helping you remember the order of transformations:
Riley The Salesman Roasts Some Turkey

x			y		
R Reflect	T Translate	S Scale	R Reflect	S Scale	T Translate



Transformation of Graphs

Translation in the direction of an axis is to move the graph in the direction of the axis (i.e. up, down, left or right) without changing its shape or size.

Replace x with $(x - a)$ $y = f(x - a)$	Translation of a units in the positive x direction	Graph moves right
Replace x with $(x + a)$ $y = f(x + a)$	Translation of a units in the negative x direction	Graph moves left
Replace y with $(y - a)$ $y = f(x) + a$	Translation of a units in the positive y direction	Graph moves up
Replace y with $(y + a)$ $y = f(x) - a$	Translation of a units in the negative y direction	Graph moves down

Scaling parallel to an axis changes the size of the curve, stretching it.

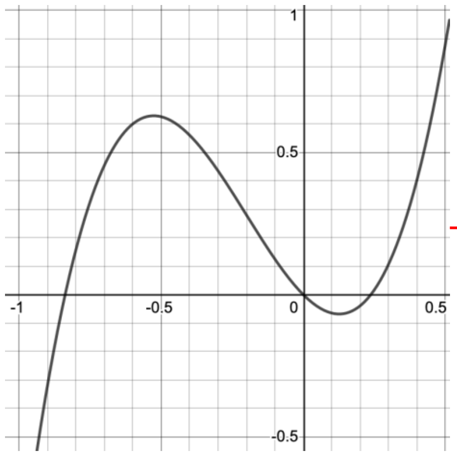
Replace x with $\frac{x}{a}$ $y = f(\frac{x}{a})$	Scaling parallel to the x-axis by a factor of a	Remember to change the coordinates and asymptotes according to the new equation
Replace y with $\frac{y}{a}$ $y = a\,f(x)$	Scaling parallel to the y-axis by a factor of a	

Reflection

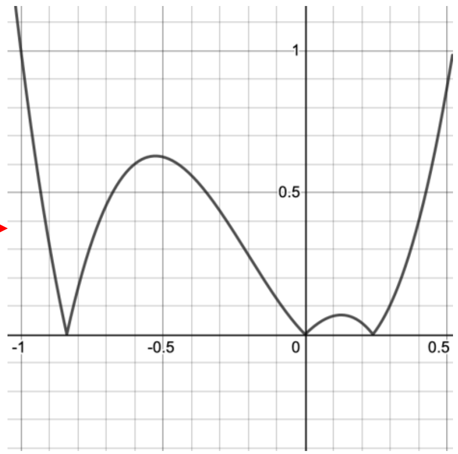
Replace x with $-x$ $y = f(-x)$	Reflection in the y-axis	Flip left/right
Replace y with $-y$ $y = -f(x)$	Reflection in the x-axis	Flip up/down

From $y = f(x)$ to $\boldsymbol{y = |f(x)|}$

$y = f(x)$

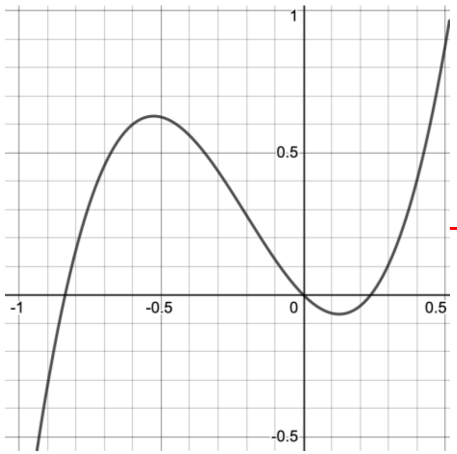


$\boldsymbol{y = |f(x)|}$

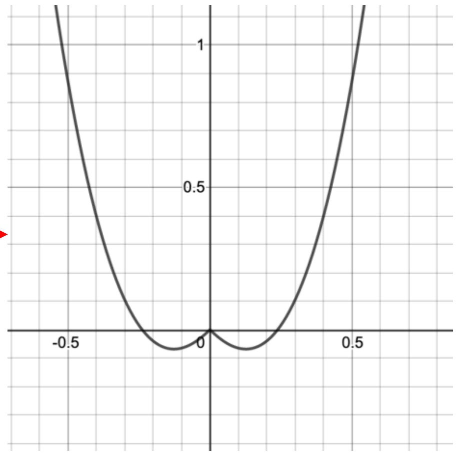


From $y = f(x)$ to $\boldsymbol{y = f(|x|)}$

$y = f(x)$



$\boldsymbol{y = f(|x|)}$



Modulus Functions

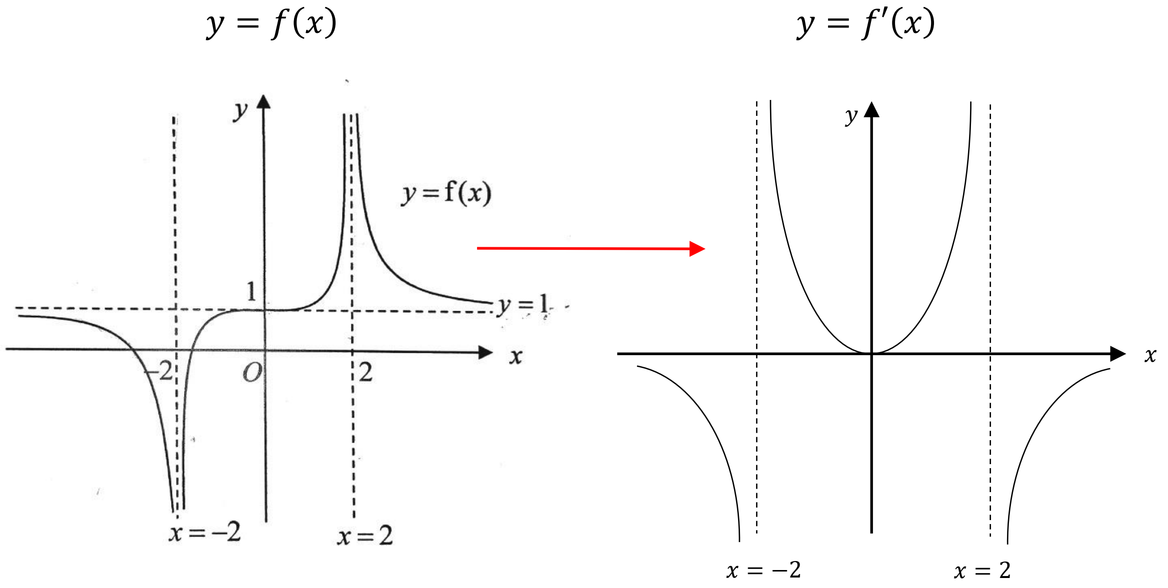
To obtain graph of $\boldsymbol{y = |f(x)|}$ from $y = f(x)$:

1. Keep part of graph above the x-axis
2. Flip up (or reflect in the x-axis) the part of the graph below the x-axis

To obtain graph of $\boldsymbol{y = f(|x|)}$ from $y = f(x)$:

1. Keep part of the graph where $x \geq 0$
2. Reflect part from (1) in the y-axis so the graph is symmetrical about the y-axis

From $y = f(x)$ to $\boldsymbol{y = f'(x)}$:



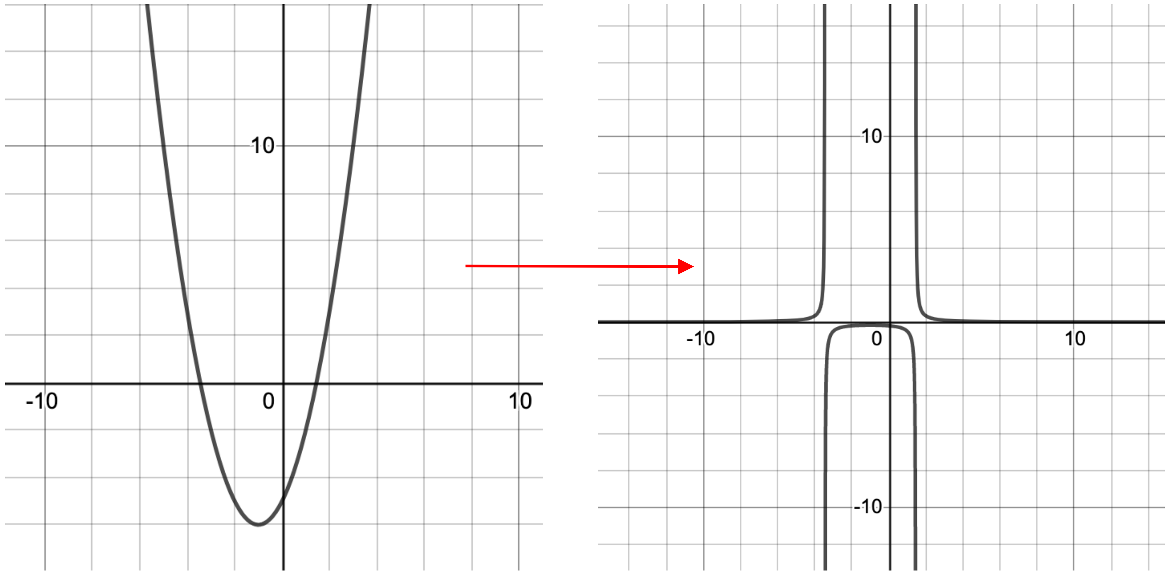
Derivative Functions

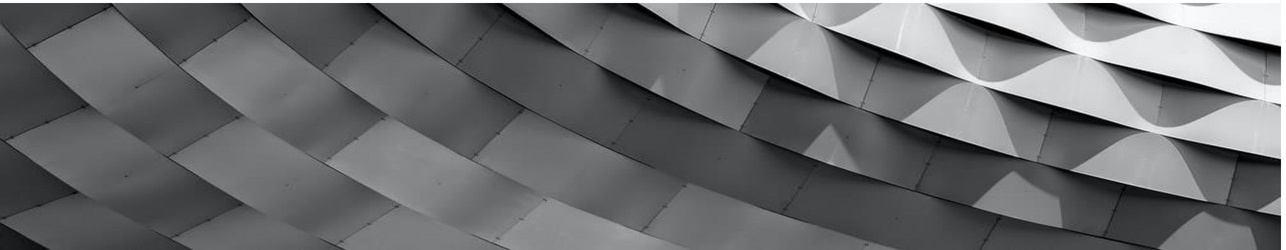
From $y = f(x)$ to $\boldsymbol{y = f'(x)}$:

$\boldsymbol{y = f(x)}$	$\boldsymbol{y = f'(x)}$
Stationary Point at *x = a*	x-intercept (a, 0)
Strictly increasing ($\frac{dy}{dx} > 0$)	Above x-axis
Strictly decreasing ($\frac{dy}{dx} < 0$)	Below x-axis
Gradient or $\frac{dy}{dx}$ getting steeper	Graph is increasing
Gradient or $\frac{dy}{dx}$ getting less steep	Graph is decreasing
Vertical Asymptote *x = a*	Vertical Asymptote *x = a*
Horizontal Asymptote *y = a*	Horizontal Asymptote *y = 0*
Oblique Asymptote *y = ax + b*	Horizontal Asymptote *y = a*

From $y = f(x)$ to $\boldsymbol{y = f'(x)}$:

$y = f(x)$ $\qquad\qquad$ $y = \dfrac{1}{f(x)}$





Reciprocal Functions

From $y = f(x)$ to $\boldsymbol{y = \frac{1}{f(x)}}$:

$\boldsymbol{y = f(x)}$	$\boldsymbol{y = \frac{1}{f(x)}}$
$f(x) > 0$	$\frac{1}{f(x)} > 0$
$f(x) < 0$	$\frac{1}{f(x)} < 0$
f (x) increases	$\frac{1}{f(x)}$ decreases
f (x) decreases	$\frac{1}{f(x)}$ increases
x-intercept (a, 0)	Vertical Asymptote at *x = a*
Vertical Asymptote *x = a*	x-intercept (a, 0)
Horizontal Asymptote *y = a* , $a \neq 0$	Horizontal Asymptote y $= \frac{1}{a}$, $a \neq 0$
Oblique Asymptote *y = ax + b*	Horizontal Asymptote *y = 0*
Maximum Point (a, b), b $\neq 0$	Minimum Point (a, $\frac{1}{b}$), b $\neq 0$
Minimum Point (a, b), b $\neq 0$	Maximum Point (a, $\frac{1}{b}$), b $\neq 0$
$f(x) \to \infty$	$\frac{1}{f(x)} \to 0$
$f(x) \to 0$	$\frac{1}{f(x)} \to \infty$