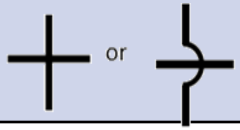
Overmugged

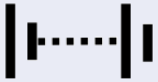
Direct-Current Circuits

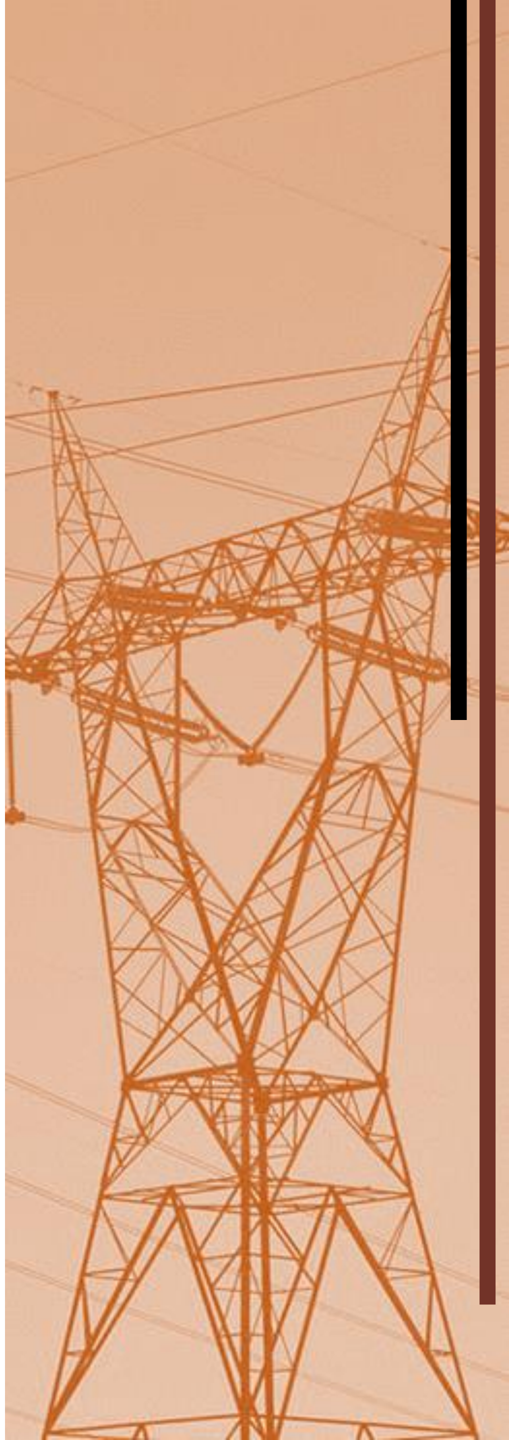
Contents

Circuit Symbols

The following are some basic circuit symbol:

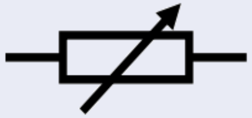
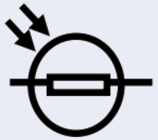
Wires and Connections		
Wire		To pass through current
Wire joined		To show wires that are connected
Wire not joined		To show unconnected wires

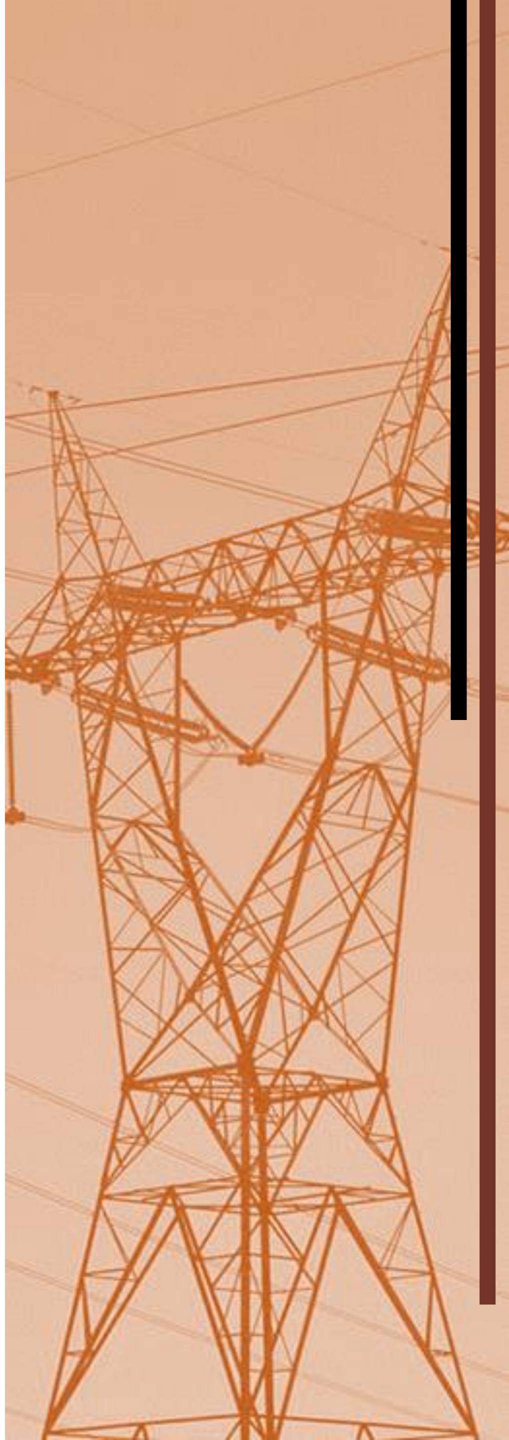
Sources of emf		
Cell		Supplies electrical energy. Source of direct current. A battery comprises of more than one cell
Battery		
AC power supply		Source of alternating current.



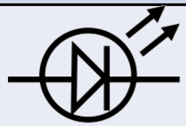
Circuit Symbols

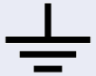
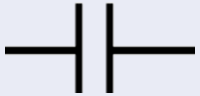
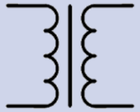
Measuring Instruments		
Voltmeter		Measures potential difference
Ammeter		Measures current
Galvanometer		Detects tiny currents, usually <1mA

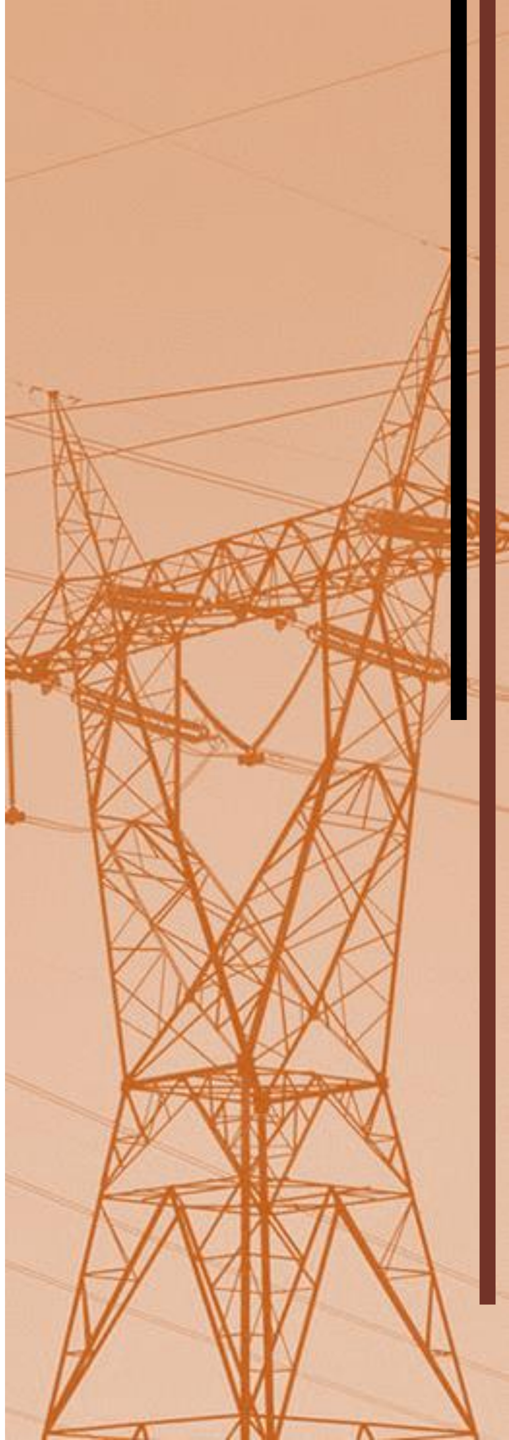
Resistors		
Resistor		Opposes current
Variable Resistor		Its resistance can be varied
Thermistor		A semiconductor. Its resistance is dependent on its temperature
Light-Dependent Resistor (LDR		A semiconductor. Its resistance is sensitive to light



Circuit Symbols

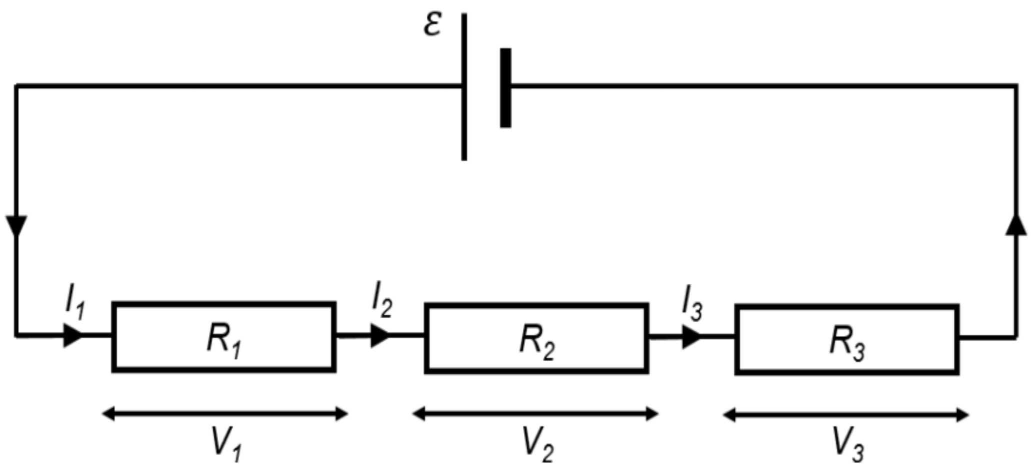
Diodes		
Diode		Allows current to flow in one direction
Light-Emitting Diode (LED)		A diode that converts electrical energy to light

Others		
Switch		Allows current to flow if it is in closed position
Ground		The zero-volt (0V) reference point in the circuit
Lamp		A transducer which converts electrical energy to light
Capacitor		Stores electrical energy
Transformer		Transfers electrical energy from one circuit to another



Series Circuit

- is a closed circuit in which the current follows one path, and the same current flows through every part of the circuit



Current *

$$I = I_1 = I_2 = I_3$$

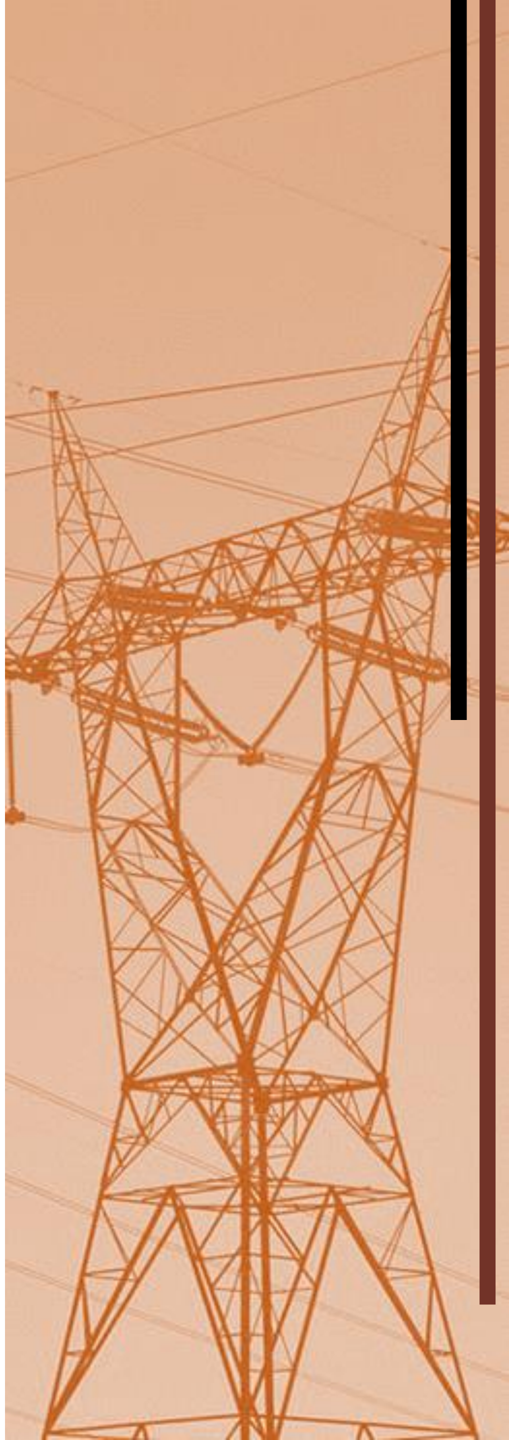
Voltage **

$$\varepsilon = V_1 + V_2 + V_3$$

Effective Resistance

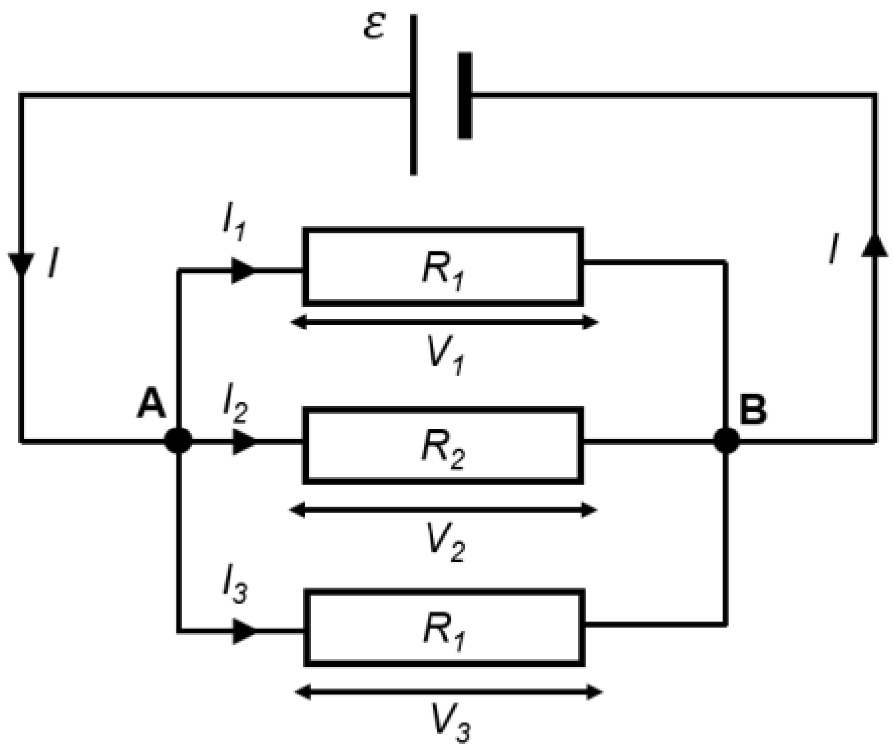
$$R_{eff} = R_1 + R_2 + R_3$$

* By **conservation of charge** (no charge is lost from the circuit)
** By **conservation of energy**, the algebraic sum of emf is equal to of p.d. of all individual component



Parallel Circuit

- is a closed circuit in which circuit components share common 2 points



Current *

$$I = I_1 + I_2 + I_3$$

Voltage **

$$\varepsilon = V_1 = V_2 = V_3$$

Effective Resistance

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

* By conservation of charge (current is divided in junction A),

** By conservation of energy (all resistors are directly connected to battery),

Practice Example 1

Consider a battery with an emf of $9.0V$ and three resistors $(R_1 = 5\Omega, R_2 = 10\Omega, R_3 = 15\Omega)$ connected in series. Calculate the

a) the effective resistance of the circuit,
b) current through each resistor
c) potential difference across each resistor
d) power dissipated by each resistor

a. $R_{eff} = R_1 + R_2 + R_3 = (5 + 10 + 15)\Omega = 30\Omega$

b. The same current through each resistor

$$I = \frac{\varepsilon}{R_{eff}} = \frac{9.0V}{30\Omega} = 0.30A$$

$$I_1 = I_2 = I_3 = 0.30A$$

c. $V_1 = IR_1 = 1.5V$, $V_2 = IR_2 = 3.0V$, $V_3 = IR_3 = 4.5V$
To check, the total voltage must be equal to the emf. $V = 1.5V + 3V + 4.5V = 9V$

d. $P_1 = IV_1 = 0.45W$, $P_2 = IV_2 = 0.90W$, $P_3 = IV_3 = 1.35W$

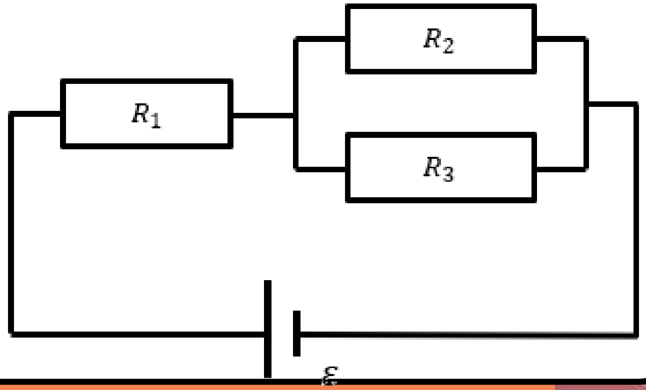
Practice Example 2

Consider same components as in *Practice example 1* but connected as shown on the right. Calculate

a. the effective resistance of the circuit,

b. current through entire circuit

c. current through each resistor.



a. Effective resistance of the parallel connection

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \rightarrow R_{23} = \left(\frac{1}{10\Omega} + \frac{1}{15\Omega}\right)^{-1} = 6\Omega$$

Effective resistance $R_{eff} = R_1 + R_{23} = (5+6)\Omega = 11\Omega$

b. $I = \frac{\varepsilon}{R_{eff}} = 0.82A$

c. The current I is the same as the current through R_1 and R_{23}

$$I = I_1 = 0.82A$$

$$I_{23} = 0.82A = \frac{V_{23}}{R_{23}} \rightarrow V_{23} = 0.82A(6\Omega) = 4.92V$$

For R_2 and R_3 parallel connection, the voltage across these two resistors is equal.

$$V_{23} = V_2 = V_3 = 4.92V$$

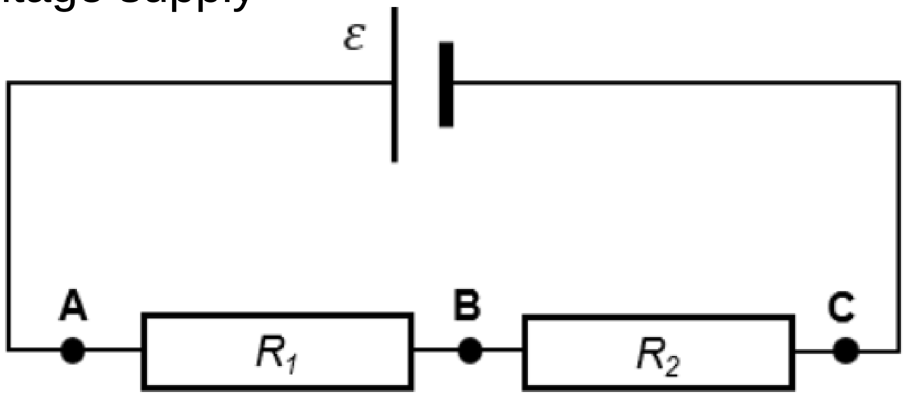
Current through R_2 and R_3:

$$I_2 = \frac{V_2}{R_2} = 0.492A$$

$$I_3 = \frac{V_3}{R_3} = 0.328A$$

Potential Divider

- is a chain of two or more resistors connected in series, across which a p.d. is applied, with the output taken from the junction(s) of the resistors.
- is used to produce an output p.d. that is a *fraction* of the voltage supply



$$V_{out} = \frac{R}{R_{out}} V\_in$$

V_{out} = output p.d. across the resistor R
R_{tot} = total resistance (effective)
$V_{in} = \varepsilon$ = input p.d. across all resistors provided by the voltage supply

Derivation

As the circuit is series, the ffg statements are valid:

a. The current I at any point in the circuit is the same.
b. $\varepsilon = V_{AC} = V_{AB} + V_{BC}$ (V_{xy} =p.d. across points x and y)

By Ohm's law (V=IR),

$$\varepsilon = V_{AC} = IR_{tot} = I(R_1 + R_2)$$

Solving for the current I

$$I = \frac{\epsilon}{R_1 + R_2} = \frac{V_{AC}}{R_1 + R_2}$$

In determining V_{AB} and V_{BC}, we need to solve for the p.d. across individual resistors using Ohm's law.

Output p.d. across R_1: $V_{AB} = IR_1 = \left(\frac{V_{AC}}{R_1+R_2}\right)R_1 = \left(\frac{R_1}{R_1+R_2}\right)V_{AC} = \left(\frac{R_1}{R_{tot}}\right)V_{AC}$

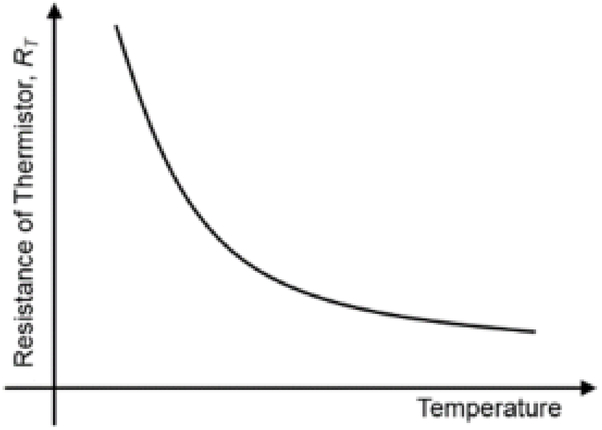
Output p.d. across R_2: $V_{BC} = IR_2 = \left(\frac{V_{AC}}{R_1+R_2}\right)R_2 = \left(\frac{R_2}{R_1+R_2}\right)V_{AC} = \left(\frac{R_2}{R_{tot}}\right)V_{AC}$

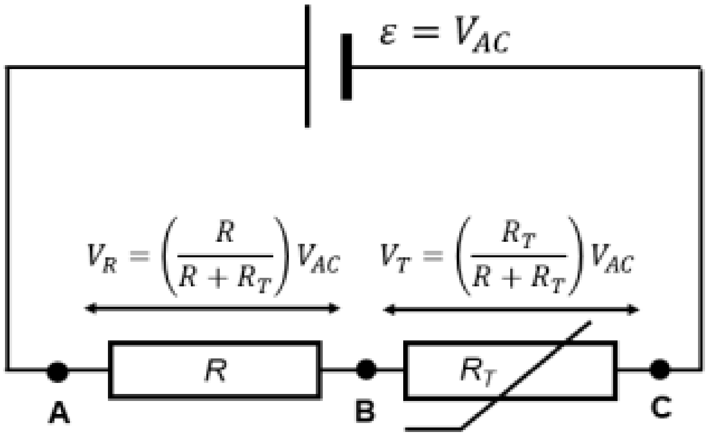
From the above results, the general formula for the ouput p.d V_{0ut} across a resistor R is given by the highlighted formula above.

Ex: For V_{out} across R_1, the equation is $V_{out} = \left(\frac{R_1}{R_{tot}}\right)\varepsilon$

Thermistors in Potential Dividers

- Resistance R_T of the thermistor decreases as temperature rises





Use:

- Commonly used as fire detection alarm system or air conditioner temperature regulating system

Condition	R_T	p.d. across thermistor V_T	p.d. across resistor V_R
Temp increase	↓	↓	↑
Temp decrease	↑	↑	↓

LDR in Potential Dividers

- Resistance R_L of the light-dependent resistor (LDR) decreases with the intensity of light hitting them

Use:

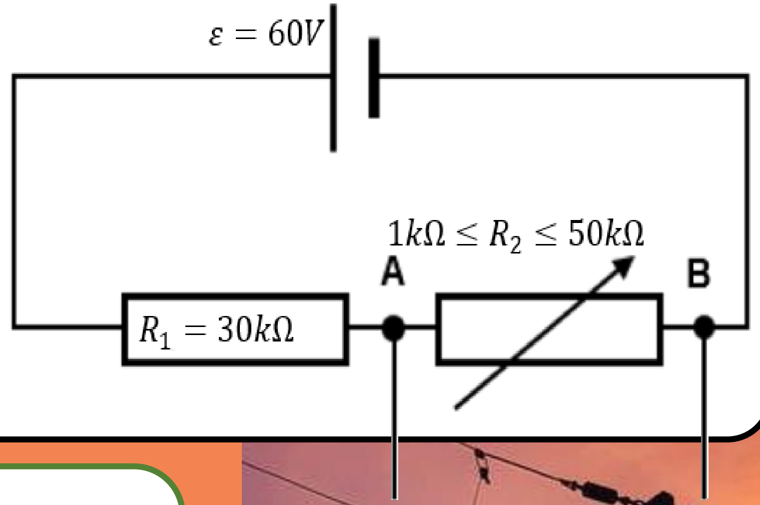
- Commonly used as fire detection alarm system or air conditioner temperature regulating system

Note:

The graph and responses have the same behavior as the thermistors.

Practice Example 3

The potential divider circuit shown on the rightis used for providing Variable output potential difference V_{AB}. The resistance R_2 of the variable resistor ranges from $1.0k\Omega$ to $50k\Omega$. Given by the input p.d of $60V$ and $R_1 = 30k\Omega$, find the range of out p.d. V_{AB} that can be produced.



Use minimum value of $R_2 = 1k\Omega$ for solve the min output p.d:

$$V_{AB,min} = \left(\frac{R_2}{R_1 + R_2}\right)\varepsilon = \left(\frac{1k\Omega}{30k\Omega + 1k\Omega}\right)60V = 1.94V$$

Use maximum value of $R_2 = 50k\Omega$ for solve the max output p.d:

$$V_{AB,max} = \left(\frac{50k\Omega}{30k\Omega + 50k\Omega}\right)60V = 37.5V$$

Answer: $1.94V \le V_{AB} \le 37.5V$

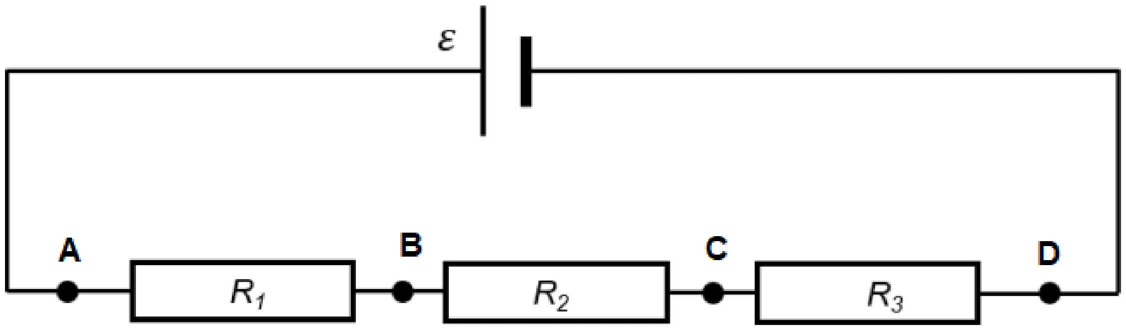
Potential and Potential Difference

POTENTIAL and POTENTIAL DIFFERENCE

• Potential at a point in a circuit depends on the chosen reference point.
  • you are free to choose the reference point and give it an arbitrary value. It affects the potential but not the p.d.
  • The ground is the reference point (zero potential, 0V) if it is included in the circuit.
  • If there is no ground, assigning the negative terminal to be the zero potential is more convenient for calculations.
• Potential difference is the same regardless of the chosen reference point.

PROBLEM SOLVING TIPS
1. Identify the reference point (ground or negative terminal). 2. Draw the direction of conventional current. 3. Calculate p.d. across each circuit component using potential divider formula. 4. Label (+ or -) potential values on points starting from the reference point. (Current flows point of higher to lower potential.)

Practice Example 4



A series circuit comprises of an emf source of $\varepsilon = 7V$ and three resistors $R_1 = 7\ \Omega, R_2 = 4\ \Omega, R_3 = 10\ \Omega$. If the potential at point C is $3.33V$, what are the potentials at points **A**, **B**, and **D**?

Step 1: Let the reference point for the zero potential be at the negative terminal of the cell.
Step 2: counterclockwise (+ to - terminal)
Step 3: $R_{tot} = R_1 + R_2 + R_3 = 21\Omega$

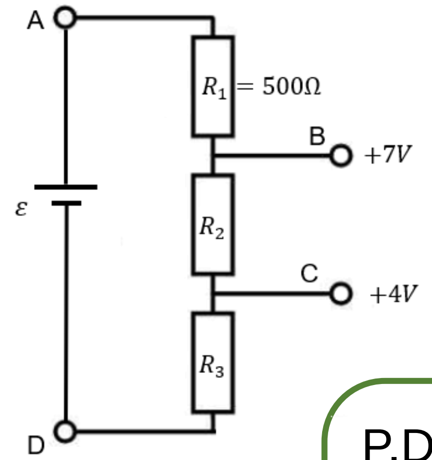
$$V_{AB} = \frac{R_1}{R_{tot}}\varepsilon = 2.33V$$
$$V_{BC} = \frac{R_2}{R_{tot}}\varepsilon = 1.33V$$
$$V_{CD} = \frac{R_3}{R_{tot}}\varepsilon = 3.33V$$

Step 4: Since current flows from $A \rightarrow D$, the potential must also drop in this order.

$V_A = 7V$, $V_B = 4.66V$, $V_C = 3.33V$ (given) $V_D = 0V$

Note that the difference between two points must agree with the results in step 3

Practice Example 5



Three resistors ($R_1 = 500\ \Omega$, R_2, R_3) and a battery with an emf of $\varepsilon = 15.0\ V$ are connected in series as shown on the left figure. If point **D** has a potential of 0V, and points **B** and **C** has +7V and +4V, respectively, calculate the resistance of R_2 and R_3.

P.D. across pts D and C: $V_{CD} = V_C - V_D = 4V - 0V = 4V$

P.D. across pts D and B: $V_{BD} = V_B - V_D = 7V - 0V = 7V$

We can have two output channels for the potential divider shown above: output p.d. across (i) R_3 and (ii) R_2 and R_3.

Output voltage across R_3: $V_{out} = V_{CD} = 4V$

$$V_{out} = \frac{R_3}{R_{tot}}\varepsilon \quad \rightarrow \quad 4V = \frac{15R_3}{500 + R_2 + R_3} \quad \rightarrow \quad \underline{11R_3 = 4R_2 + 2000}$$

Output voltage across R_2 and R_3: $V_{out} = V_{BD} = 7V$

$$V_{out} = \frac{R_2 + R_3}{R_{tot}}\varepsilon \quad \rightarrow \quad 7V = \frac{15(R_2 + R_3)}{500 + R_2 + R_3} \quad \rightarrow \quad \underline{8R_3 = -8R_2 + 3500}$$

Solving R_2 and R_3 using the system of two equations (underlined)

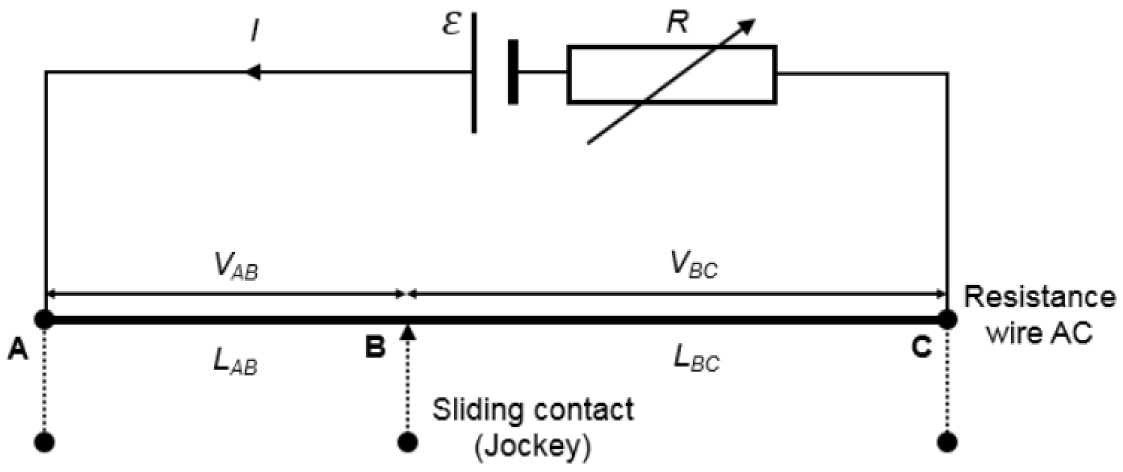
$$\Longrightarrow 0 = (32 + 88)R_2 - 22{,}500 \quad \Longrightarrow R_2 = 187.5\Omega$$

$$\Longrightarrow R_3 = \frac{1}{11}(4R_2 + 2000) \quad \Longrightarrow \quad R_3 = 250\Omega$$

To check $V_3 = IR_3$ and $V_2 = IR_2$ where $I = \frac{\varepsilon}{R_{tot}} = 0.016$

Potentiometer

- Used for more accurate measurement of p.d. (voltmeter has finite resistance)
- Consists of a wire of uniform resistance R_{AB} and length L, connected in series with an emf source ε and a variable resistor R



- The sliding contact moves point **C** and thus changing the p.d. across points **A** & **B**, and **B** & **C.**

$$\frac{V_{AB}}{V_{AC}} = \frac{R_{AB}}{R_{AC}} = \frac{L_{AB}}{L_{AC}}$$

V_{xy} = voltage across points x and y
R_{xy} = resistance from point x to y
L_{xy} = length from point x to y

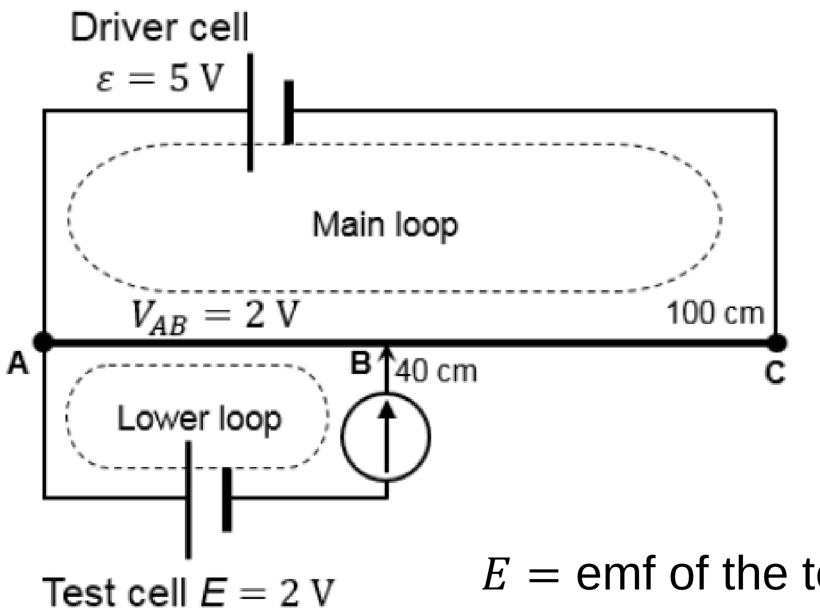
Potential Gradient

$$k = \frac{V_{AC}}{L_{AC}}$$

Derivation
Using potential divided rule $$\frac{V_{AB}}{V_{AC}} = \frac{R_{AB}}{R_{AC}} \quad (1)$$ Recall $R = \frac{\rho L}{A}$: $$R_{AB} = \rho\frac{L_{AB}}{A}, \qquad R_{AC} = \rho\frac{L_{AC}}{A}$$ Elimate ρ and A from these two equations $$\frac{R_{AB}}{R_{AC}} = \frac{L_{AB}}{L_{AC}} \quad (2)$$ Comparing equations (1) and (2), we obtain the formula above

Balance Point

- The contact point where the test cell nullifies the current flowing in the lower circuit. (galvanometer shows no deflection)
- This is where $V_{AB} = E$



Sample Circuit

E = emf of the test cell

Balance Length

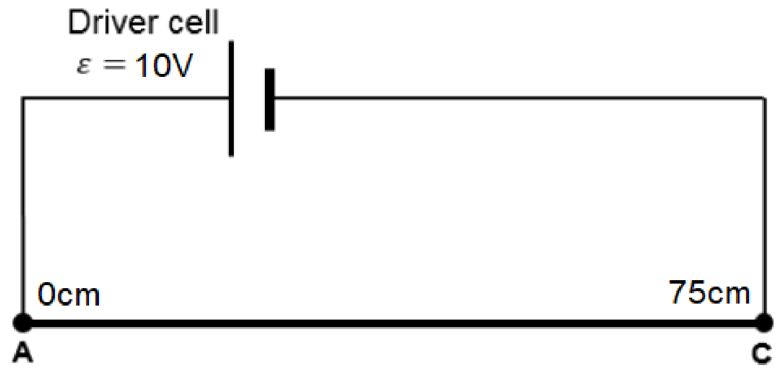
$$L_{AB} = \frac{V_{AB}}{V_{AC}} L_{AC}$$

Note:

- Though at balance point, no current flows in the lower circuit, there is still a constant current flowing in the main loop

Practice Example 6

On the right is circuit diagram of a potentiometer. The driver cell has an emf of $10V$ and the resistance wire has length of $75cm$.



a.) What is the potential difference across points **A** and **C?**

Assuming that the driver cell has negligible internal resistance $V_{AC} = \varepsilon = 10.0V$

b.) Solve the p.d. between the points in the

i. 0-cm and 50-cm mark

ii. 30-cm and 75-cm mark

Define: A=0cm mark, B=50cm mark, X=25cm mark

i. $V_{AB} = \frac{L_{AB}}{L_{AC}} V_{AC} = \left(\frac{50cm}{75cm}\right) 10V = 6.66V$

ii. $V_{XC} = \frac{L_{XC}}{L_{AC}} V_{AC} = \left(\frac{75cm - 30cm}{75cm}\right) 10V = 6.0V$

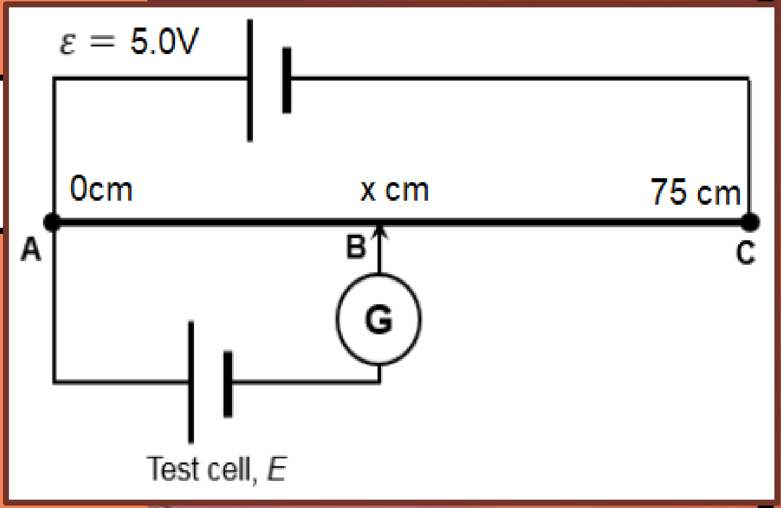
c.) Where should the sliding contact (point **B**) be located such that the potential difference between **A** and **B** is 3.0V?

$$V_{AB} = \frac{L_{AB}}{L_{AC}} V_{AC} \rightarrow L_{AB} = \frac{V_{AB} L_{AC}}{V_{AC}} = \frac{(3.0V)75cm}{10V} = 22.5cm$$

Ans: at the 22.5-cm mark

Practice Example 7

A potentiometer is set-up as shown on the left.



a.) Find the balance point if the test cell has an emf of $E = 2.0V$.

At balance point $V_{AB} = E = 2.0V$

$$\frac{V_{AB}}{V_{AC}} = \frac{L_{AB}}{L_{AC}} \rightarrow L_{AB} = \frac{V_{AB}}{V_{AC}} L_{AC} = \frac{2.0V}{5.0V} 75cm = 30.0cm$$

b.) How would your answer to part (a) be different if the test cell has an internal resistance r_T?

Answer stays the same. At balance point, the current in the lower loop, thereby also through the test cell, is zero. This means that the terminal voltage across the test cell ($V_T = E - Ir$) is the same as E. Hence, solution for part (a) should hold up.

c.) How would your answer to part (a) be different if the test cell has an internal resistance r_D?

Following $V_T = \varepsilon - Ir_D$, the terminal voltage of the driving cell yields to slightly smaller value. This results to smaller V_{AC}. $L_{AB} = \frac{V_{AB}}{V_{AC}} L_{AC}$. With V_{AB} and L_{AC} staying fixed, the balance point would **increase**.

OVERMUGGED
By Tutors, For Students

For more notes & learning materials, visit: www.overmugged.com

IG handle: **@overmugged**

Join our telegram channel: **@overmuggedAlevels**

‘A’ levels crash course program

Professionally designed crash course to help you get a **condensed revision** before your ‘A’ Levels!

Each H2 subject will have **3 crash course modules** which will cover their entire H2 syllabus.

The **4 hour module** focuses on going through **key concepts** and **identifying commonly tested questions!**

The crash courses modules will begin in **June 2021 and last till Oct 2021**.

Pre-register now on our website and secure your slots!

Need help?

EMMANUEL ER
(Private Tutor)

97338620
(WhatsApp)

@emmannyyy
(Telegram)