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Topic 12: Linear Law (4049)

THE ABOUT

CHAPTER ANALYSIS

- Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight-line graph

MASTERY

- Relatively straight forward chapter
- 1 **key** concept

EXAM

- Concepts usually tested as a stand-alone topic
- Most time-consuming chapter, especially if there is graph-sketching involved

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, an 10m question, 1 question in one of the papers

KEY CONCEPT

Straight line graphs
Common Transformations

Steps for Linearisation

Follow this step-by-step breakdown for linearisation

1. Transform the non-linear equation into a straight-line function of the form $Y = mX + c$, where X and Y are expressions in x or y or both, m is the gradient of the straight line and c is the intercept on the Y-axis
2. Use the experimental values of x and y to compute the corresponding values of X and Y
3. Use the values of X and Y to plot the graph of $Y = mX + c$ with the values of X on the horizontal axis and the values of Y on the vertical axis. Draw a straight-line through the plotted points or as close to them as possible
4. From the graph, obtain the values of m and c
5. Express the required unknown constants in terms of m and/or c and determine their values

Things to note:

- Do not mix up X as x and Y as y
- Renames axis X and Y cannot consists of any **unknown constants**
- At times, there may be more than 1 way to linearise an equation

Sometimes, we are given the transformed straight line instead which

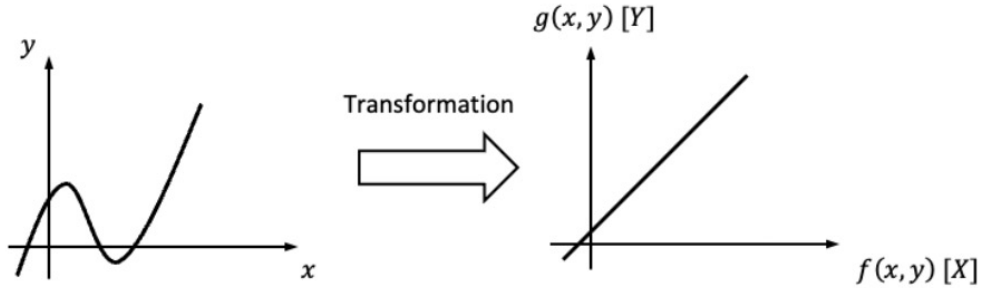
- pass through two given points
- has a known gradient and which passes through a given point

Unknown constants can be determined using the above information without the need to draw a line of best fit

Linearisation

All non-linear relationships will be transformed into the form

$$\boldsymbol{Y = mX + c}$$



The new vertical axis is names $\boldsymbol{Y}$, the new horizontal axis is named $\boldsymbol{X}$

- $\boldsymbol{g(x, y)}$ and $\boldsymbol{f(x, y)}$ are functions involving variables $\boldsymbol{x}$ and/or $\boldsymbol{y}$
- $\boldsymbol{m}$ is the gradient of the new linear line
- $\boldsymbol{c}$ is the new y-intercept of the new linear line

Common Transformations

1.

$$y = \frac{a}{x} + b$$

To linearise the following:

$$y = a\left(\frac{1}{x}\right) + b$$

Plot $\boldsymbol{y}$ against $\boldsymbol{\frac{1}{x}}$, a is the gradient, b is the y-intercept

2.

$$y = a\sqrt{x} + \frac{b}{\sqrt{x}}$$

To linearise the following:

$$y\sqrt{x} = ax + b$$

Plot $\boldsymbol{y\sqrt{x}}$ against $\boldsymbol{x}$, a is the gradient, b is the y-intercept

Multiply by $\sqrt{x}$ throughout

3.

$$xy = \frac{a}{x} + bx$$

To linearise the following:

$$y = \frac{a}{x^2} + b$$

$$y = a\left(\frac{1}{x^2}\right) + b$$

Plot $\boldsymbol{y}$ against $\boldsymbol{\frac{1}{x^2}}$, a is the gradient, b is the y-intercept

Divide by x throughout

4.

$$y = ab^x$$

To linearise the following:

$$\lg y = \lg\left[ab^x\right]$$

$$\lg y = (\lg b)\, x + \lg a$$

Plot $\boldsymbol{\lg y}$ against $\boldsymbol{x}$, $\lg b$ is the gradient, $\lg a$ is the y-intercept

Take common/natural logarithms on both sides

Common Transformations

5.

$$y = ax^b$$

To linearise the following:

$$\lg y = \lg \left[ax^b\right]$$
$$\lg y = b\,(\lg x) + \lg a$$

Plot $\mathbf{\lg y}$ against $\mathbf{\lg x}$, b is the gradient, $\lg a$ is the y-intercept

Take common/natural logarithms on both sides

6.

$$y = \frac{a}{x - b}$$

To linearise the following:

$$\frac{1}{y} = \frac{x - b}{a}$$
$$\frac{1}{y} = \frac{x}{a} - \frac{b}{a}$$
$$\frac{1}{y} = \left(\frac{1}{a}\right)x - \frac{b}{a}$$

Plot $\mathbf{\frac{1}{y}}$ against $\boldsymbol{x}$, $\frac{1}{a}$ is the gradient, $\frac{b}{a}$ is the y-intercept

Take reciprocals on both sides

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