GCSE Preliminary Examination Mock Paper 2022 SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

CANDIDATE
NAME

CENTRE $\square$

## ELEMENTARY MATHEMATICS

4048/02
August 2022
2 hours 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graph Paper (1 sheet).

## READ THESE INSTRUCTIONS FIRST

Write your name and centre in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 siginificant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is $\mathbf{1 0 0}$.

## Grade Tables: For Examiner's Use

| Total Score | Deductions |  |  | Grade |
| :--- | :---: | :---: | :---: | :---: |
|  | Rounding | Units | Presentation |  |
|  |  |  |  |  |

## Setter: Kaiwen :)

This question paper consists of $\underline{\mathbf{2 7}}$ printed pages including the cover page

## Grade Tables: For Examiner's Use

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 9 |  |
| 3 | 7 |  |
| 4 | 8 |  |
| 5 | 11 |  |
| 6 | 6 |  |


| Question | Points | Score |
| :---: | :---: | :---: |
| 7 | 7 |  |
| 8 | 11 |  |
| 9 | 10 |  |
| 10 | 11 |  |
| 11 | 10 |  |
| Total: | 100 |  |

## Examiner's Comments

## List of Mathematical Formulae

## Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

## Mensuration

Curved surface area of a cone $=\pi r l$

$$
\text { Surface area of a sphere }=4 \pi r^{2}
$$

$$
\begin{gathered}
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h \\
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
\end{gathered}
$$

Arc length $=r \theta$, where $\theta$ is in radians
Sector area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians
Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

## Answer all questions: $\mathbf{1 0 0}$ marks

1. (a) The following dot diagram shows the number of people in each group who booked a tour package. There are 20 groups

(i) Find the median number of people in a group

Answer $\qquad$
(ii) Find the mean number of people in the group

Answer
(iii) Explain whether the median or mean is a more appropriate average to use to analyse the information in this distribution
Answer
(iv) Find the standard deviation of the number of people in the group

Answer $\qquad$
(v) The standard deviation of the number of people in the group who booked another tour package was 3.52 . Use this information to comment on one difference between the two distribution
Answer
(b) There are 5 balls numbered 1 to 5 placed in a box. A game involves a player drawing two balls, one after another without replacement, from the box. A player wins a prize if the product of the two balls drawn is an odd number
(i) Draw a possibility diagram to show the possible outcomes

Answer
(ii) Find, as a fraction in its simplest form, the probability that (a) a player wins a prize

> Answer
(b) only one out of two players wins a prize

Answer $\qquad$
(c) at least one out of two players wins a prize
2. (a) A string of beads on a table is partially covered by a piece of cloth as shown


There are 2 white beads between every 2 black beads. Altogether, there are 14 black beads. John guessed that there are $\mathbf{2 8}$ white beads. Do you agree? Justify your answer with calculations Answer
(b) Find the integers such that

$$
2 x+1<9<3 x+1
$$

$\qquad$
(c) Factorise the following completely

$$
a^{2}+9 b^{2}-6 a b-2 a+6 b
$$

Answer
(d) It is given that

$$
3 b=4 a \quad 2 c=5 a
$$

(i) Find $a: b: c$

Answer $\qquad$ [1]
(ii) If $a+b+c=10$, find the value of $b$
$\qquad$
3. A soccer club offers annual memberships for both adults and juniors. The adult annual membership fee is $\$ 150$. Junior members need to pay $80 \%$ of the adult annual membership fee
(a) Calculate the discount each junior member receives

Answer
If an adult member does not pay the membership fee by the due date, the club will charge a penalty of $5 \%$ per month until the fee is paid. Simon paid the $\$ 150$ membership fee exactly 2 months after the due date
(b) Calculate the penalty that Simon will be charged

Answer

The soccer club received a statement of the transactions in its saving account for the month of January 2017

| Date | Details | Deposit | Withdrawal | Balance |
| :---: | :---: | :---: | :---: | :---: |
| 01 Jan 2017 | Brought forward |  |  | $\$ 63950.00$ |
| 09 Jan 2017 | Math Fees | $\$ 750$ |  | $\$ 64700.00$ |
| 15 Jan 2017 | Withdrawal |  | $x$ | $\$ 42700.00$ |
| 23 Jan 2017 | Membership Fees | $\$ 3800.00$ |  | $\$ 46500.00$ |
| 31 Jan 2017 | Interest | $\$ 124.54$ |  | $\$ 46624.54$ |

(c) (i) Calculate the withdrawal amount $\$ x$ on 15 Jan 2017

Answer $\qquad$
(ii) Interest on the account is calculated on the minimum balance for the month and added to the account on the last day of the month. What is the annual interest rate of this account, corrected to one decimal place

Answer $\qquad$
(d) The soccer club plans to invest $\$ 120000$ in an account which pays compound interest at the rate of $2 \%$ per annum, compounded monthly. Find the total amount that can be withdrawn at the end of 4 years
$\qquad$
4. In January 2013, the price of rice in Singapore was $\$ x$ per kilograms. A food catering company spent $\$ 450$ on rice each month
(a) Write down an expression, in terms of $x$, for the amount of rice in kilograms that this food catering company ordered in January 2013

## Answer

In January 2016, the price of each kilograms of rice had increased by 15 cents. The company continued to spend $\$ 450$ on rice each month
(b) Write down an expression, in terms of $x$, for the amount of rice in kilograms ordered in January 2016

Answer $\qquad$ [1]
(c) The difference in the amount of rice ordered between January 2013 and January 2016 is 30 kg . Write down an equation in $x$ to represent this information, and show that it reduces to

$$
20 x^{2}+3 x-45=0
$$

Answer
(d) Solve the equation in part (c), giving your solutions correct to 2 decimal places

Answer
(e) Hence, calculate the average amount of rice ordered in January 2016. Give your answer correct to the nearest kilogram
5. (a) $A, B, C, D$ and $E$ lie on a circle with centre $O$. The line $P Q$ is tangent to the circle at $A$. $A D$ is the diameter of the circle, $\angle C E D=x^{\circ}$ and $\angle A D E=y^{\circ}$


Find, in terms of $x$ and $y$,
(i) $\angle C A D$

> Answer
$\qquad$
(ii) $\angle C O D$

Answer $\qquad$
(iii) $\angle E A O$

Answer $\qquad$
(iv) $\angle E D C$

Answer $\qquad$ [1]
(v) $\angle E A Q$
$\qquad$
(b) In the diagram, $R$ is the point $(12,-6)$ and $P R$ is a straight line parallel to the $x$-axis. Points $Q$ and $K$ lie on the $y$-axis and $L$ is the midpoint of the straight line $Q R$.


Given that $O K=2 O Q$, find
(i) the coordinates of $K$ and $Q$

Answer
(ii) the value of $a$ given that the coordinates of $L$ is $(6, a)$
(iii) the area of $\triangle K Q L$
6. The first four terms in a sequence of numbers are given below

$$
\begin{gathered}
T_{1}=1 \times 3 \times 5=15 \\
T_{2}=2 \times 4 \times 6=48 \\
T_{3}=3 \times 5 \times 7=105 \\
T_{4}=4 \times 6 \times 8=192
\end{gathered}
$$

(a) Write down the 5 th term, $T_{5}$, of the sequence in a similar form

Answer $\qquad$
(b) Find an expression, in terms of $n$, for the $n$th term, $T_{n}$ of the sequence

Answer
(c) Evaluate $T_{23}$

Answer $\qquad$ [1]
(d) $T_{k}$ and $T_{k+2}$ are two terms in the sequence. Find and simplify an expression, in terms of $k$, for

$$
T_{k+2}-T_{k}
$$

Answer $\qquad$
(e) Explain why the value of $T_{k+2}-T_{k}$ must be even for all values of $k$ Answer
7. In the diagram $A B C D$ is a square and $E B G H$ is a parallelogram


The lines $B C, E G$ and $H J$ intersect at the point $F . E$ and $F$ are midpoints of $A D$ and $B C$ respectively and $4 D H=3 D C$
(a) Show that
(i) $\triangle E D H$ and $\triangle B F G$ are congruent Answer
(ii) $\triangle C H F$ and $\triangle A E B$ are similar Answer
(b) Find the numerical value of (i)

$$
\frac{D H}{E G}
$$

Answer
(ii)

Area of $\triangle B G F$
Area of $\triangle B E F$

Answer $\qquad$
(iii)
$\frac{\text { Area of } E B G H}{\text { Area of } A B C D}$

Answer
8. The variables $x$ and $y$ are connected by the equation

$$
y=2 x+\frac{50}{x}-30, \quad x \neq 0
$$

Some corresponding values of $x$ and $y$ are given in the following table, corrected to 2 decimal places

| $x$ | 1 | 1.5 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 22.00 | $p$ | -7.33 | -10.00 | -8.86 | -6.44 | $q$ | -0.15 | 3.33 | 5.13 |

(a) Find the value of $p$ and of $q$

## Answer

$\qquad$ [1]
(b) Using a scale of 1 cm to 1 unit on the horizontal $x$-axis and 2 cm to 5 units on the vertical $y$-axis, draw the graph (on the sheet of graph paper provided) of $y=2 x+\frac{50}{x}-30$ for $1 \leq x \leq 16$
(c) By drawing a tangent, find the gradient of the curve at the point $x=10$
(d) (i) On the same axes, draw the line

$$
y=3-2 x
$$

(ii) From the graph, state the $x$-coordinate of the points where this line intersects the curve

Answer
(iii) The values of $x$ are the solutions of the equation

$$
A x^{2}-33 x+B=0
$$

Find the value of $A$ and of $B$

Answer
9. In the diagram below, $O P Q$ is a triangle such that $\overrightarrow{O P}=\mathbf{p}$ and $\overrightarrow{O Q}=\mathbf{q} . Q$ is the mid-point of $O S$ and $R$ lies on $O P$ produced such that $3 P R=2 O P$. The point $T$ on $P Q$ such that $P T: T Q=3: 5$

(a) Express in terms of $\mathbf{p}$ and/or $\mathbf{q}$
(i) $\overrightarrow{O S}$

Answer $\qquad$
(ii) $\overrightarrow{O R}$

Answer $\qquad$
(iii) $\overrightarrow{O T}$

Answer
(b) Given that $\overrightarrow{Q U}=\frac{10}{9} \mathbf{p}-\frac{1}{3} \mathbf{q}$, show that $O, T$ and $U$ are collinear points Answer
(c) Find
(i)

$$
\frac{R U}{U S}
$$

Answer
(ii)
$\frac{\text { Area of } \triangle O T Q}{\text { Area of } \triangle O U S}$

Answer $\qquad$
(iii) area of $\triangle O U S$ if the area of $\triangle O T P$ is $54 \mathrm{~cm}^{2}$
$\qquad$
10. Given that $A, B, C$ and $D$ are on a horizontal ground. $A B=7 \mathrm{~m}, B C=5 \mathrm{~m}$ and $A C=10 \mathrm{~m}$

(a) Show that $\angle B A C=27.7^{\circ}$, correct to 1 decimal place]

Answer
(b) Calculate the area of $\triangle A B C$
$\qquad$
(c) Calculate the shortest distance from $B$ to $A C$

## Answer

$\qquad$
(d) Given that $B D=12 \mathrm{~m}$, find the $\angle B D C$

Answer
(e) A camera is placed at the top of the vertical metal pole at $B$. A cameraman estimates that the difference of the angles of elevation of the camera from $A$ and from $D$ is at most $10^{\circ}$. The metal pole is 4.5 m tall. Is his estimate accurate? Justify your answer with calculations clearly
$\qquad$
11. A chain, like the one shown below, is commonly used to drop anchors into the sea


A single unit chain can be modelled by the figure below in which two cylindrical metal pieces are bent and joined together at the edges


Some data on the Stud Link Anchor Chain Dimensions are shown in Annex A to help you answer the following questions
(a) Calculate the surface area of the single unit chain in the form of $k \pi$ inches ${ }^{2}$, where $k$ is an integer, if the diameter of each cylinder is 1 inch . It is assumed that the surface area of the figure does not change

Answer $\qquad$
(b) Calculate the volume of the single unit chain, in $\mathrm{mm}^{3}$, if the diameter of each cylinder is 1 inch
(c) Some anchor chains are sold in sets of 15 fathom shots. The weight per 15 fathom shots in the table is given in kilograms. A fishing boat goes out to sea with the intention of anchoring at a depth of 600 metres. The maximum allowable load for the chain is 45 tonnes. Note that 1 tonne is equivalent to 1000 kg . If a 1.5 inches chain size is selected, determine, showing your calculation, whether the chain size choice is recommended. If otherwise, recommend which chain size should be used
Annex A - Data for Question 11

| Stud Link Anchor Chain Dimensions |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Diameter | Dimensions (inches) |  |  | Weight | Grade 2 |  | Grade 3 |  | Oil Rig Quality |  | Link per shot |
| Chain Size | A | B | C | - | X | Y | X | Y | X | Y | - |
| 5/8 | 33/4 | $21 / 4$ | 133/4 | 365 | 23,745 | 33,220 | 33,220 | 47,465 |  |  | 432 |
| 3/4 | $41 / 2$ | 2 5/8 | $161 / 2$ | 480 | 34,000 | 47,600 | 47,600 | 68,000 |  |  | 357 |
| 13/16 | 47/8 | $27 / 8$ | $177 / 8$ | 570 | 39,800 | 55,700 | 55,700 | 79,500 |  |  | 329 |
| 7/8 | $51 / 4$ | $31 / 8$ | 19 1/4 | 660 | 46,000 | 64,400 | 64,400 | 91,800 |  |  | 305 |
| 15/16 | 5 5/8 | $35 / 16$ | $205 / 8$ | 760 | 52,600 | 73,700 | 73,700 | 105,000 |  |  | 285 |
| 1 | 6 | 3 9/16 | 22 | 860 | 59,700 | 83,600 | 83,600 | 119,500 | 84,000 | 129,000 | 267 |
| $11 / 16$ | $63 / 8$ | $33 / 4$ | 23 3/8 | 970 | 67,200 | 94,100 | 94,100 | 135,000 |  |  | 251 |
| $11 / 8$ | $63 / 4$ | 4 | $243 / 4$ | 1,080 | 75,000 | 105,000 | 105,000 | 150,000 | 106,000 | 161,000 | 237 |
| $113 / 16$ | $71 / 8$ | $41 / 4$ | $261 / 8$ | 1,220 | 83,400 | 116,500 | 116,500 | 167,000 |  |  | 225 |
| $11 / 4$ | $71 / 2$ | $41 / 2$ | 27 1/2 | 1,350 | 92,200 | 129,000 | 129,000 | 184,000 | 130,000 | 198,000 | 213 |
| $15 / 16$ | 77/8 | $43 / 4$ | $287 / 8$ | 1,490 | 1,500 | 142,000 | 142,000 | 203,000 |  |  | 203 |
| $13 / 8$ | $81 / 4$ | $415 / 16$ | $301 / 4$ | 1,630 | 111,000 | 155,000 | 155,000 | 222,000 | 157,000 | 235,000 | 195 |
| 17/16 | $85 / 8$ | $53 / 16$ | 315/8 | 1,780 | 120,500 | 169,000 | 169,000 | 241,000 |  |  | 187 |
| $11 / 2$ | 9 | $53 / 8$ | 33 | 1,940 | 131,000 | 183,500 | 183,500 | 262,000 | 185,000 | 280,000 | 179 |
| 19/16 | 93/8 | 5 5/8 | 343/8 | 2,090 | 142,000 | 198,500 | 198,500 | 284,000 |  |  | 171 |
| $15 / 8$ | 93/4 | $57 / 8$ | 353/4 | 2,240 | 153,000 | 214,000 | 214,000 | 306,000 | 216,000 | 325,000 | 165 |
| 111/16 | 10 1/8 | $61 / 16$ | $371 / 8$ | 2,410 | 166,500 | 229,000 | 229,000 | 327,000 |  |  | 159 |
| 13/4 | 10 1/2 | 65/16 | $381 / 2$ | 2,590 | 176,000 | 247,000 | 247,000 | 352,000 | 249,000 | 380,000 | 153 |
| $113 / 16$ | $107 / 8$ | $61 / 2$ | $397 / 8$ | 2,790 | 188,500 | 264,000 | 264,000 | 377,000 |  |  | 147 |
| 17/8 | $111 / 4$ | $63 / 4$ | 41 1/4 | 2,980 | 201,000 | 281,000 | 281,000 | 402,000 | 285,000 | 432,000 | 143 |
| $115 / 16$ | 115/8 | 7 | $425 / 8$ | 3,180 | 214,000 | 299,000 | 299,000 | 427,000 |  |  | 139 |
| 2 | 12 | $73 / 16$ | 44 | 3,360 | 227,000 | 318,000 | 318,000 | 454,000 | 322,000 | 488,000 | 133 |

- X denotes the Proof Test, Y denotes the Break Test.

Question Source

| Question | Credit | Remarks |
| :---: | :---: | :---: |
| 1 | S4 P2/GDSS 2017 PRELIM Qn 9 | Modified |
| 2 | S4 P2/YISS 2017 PRELIM Qn 1 \& 2 | Modified |
| 3 | S4 P2/TKSS 2017 PRELIM Qn 1 | - |
| 4 | S4 P2/RSS 2017 PRELIM Qn 9 | - |
| $5(\mathrm{a})$ | S4 P2/PRSS 2017 PRELIM Qn 2(b) | Modified |
| $5(\mathrm{~b})$ | S4 P2/BGSSS 2017 PRELIM Qn 7(b) | - |
| 6 | S4 P2/NSS 2017 PRELIM Qn 3 | - |
| 7 | S4 P2/NBSS 2017 PRELIM Qn 2 | - |
| 8 | S4 P2/JYSS 2017 PRELIM Qn 8 | - |
| 9 | S4 P2/MSHS 2017 PRELIM Qn 9 | - |
| 10 | S4 P2/BMSS 2017 PRELIM Qn 8 | - |
| 11 | S4 P2/BBSS 2021 MYE Qn 10 | Modified |

