Mid-Year Examination Mock Paper 2022
SECONDARY 3 EXPRESS / NORMAL ACADEMIC

CANDIDATE
NAME

CENTRE $\square$

## ADDITIONAL MATHEMATICS

4049/01

Mock Paper

April/May 2022
1 hour 30 minutes
Candidates answer on the Question Paper.
No Additional Materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your name and centre in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 siginificant figures, or decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60 .

## Additional Instructions:

- Question 7 is a bonus question. Attempt only if you have completed the paper :)
- This is a RESTRICTED OPEN BOOK examination.

Candidates are allowed $\mathbf{1}$ handwritten A6 double-sided help-sheet

## Setter: Kaiwen :)

This question paper consists of 16 printed pages including the cover page

Grade Tables: For Examiner's Use

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 10 | 10 | 10 | 10 | 10 | 10 | 60 |
| Score: |  |  |  |  |  |  |  |

Bonus Question

| Question: | 7 | Total |
| :--- | :---: | :---: |
| Bonus Points: | 3 | 3 |
| Score: |  |  |

Total Score

| Total Score | Deductions | Grade |
| :---: | :---: | :---: |
|  |  |  |

## Examiner's Comments:

## List of Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Binomial Expansion

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and

$$
\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1) \ldots(n-r+1)}{r!}
$$

## 2. TRIGONOMETRY

## Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A \\
\sin (A \pm B)=\sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B)=\cos A \cos B \mp \sin A \sin B \\
\tan (A \pm B)=\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\
\sin 2 A=2 \sin A \cos A \\
\cos 2 A=\cos ^{2} A-\sin ^{2} A=2 \cos ^{2} A-1=1-2 \sin ^{2} A \\
\tan 2 A=\frac{2 \tan A}{1-\tan ^{2} A}
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1. (a) Solve the simultaneous equations

$$
\begin{gathered}
2 x+y+2=0 \\
\frac{1}{x}+\frac{2}{y}=\frac{1}{2}
\end{gathered}
$$

(b) (i) Find the range of values of $k$ for which the curve lies entirely above or below the $x$-axis

$$
y=(k-2) x^{2}+2 k x+(k+3)
$$

Answer
(ii) Justify whether the curve lies entirely above or below the $x$ axis
2. The function is given as the following, where $a$ and $b$ are constants

$$
f(x)=x^{3}+a x^{2}+b x+3
$$

This function is exactly divisible by $(x+1)$ and leaves a remainder of 48 when divided by $(x-3)$
(a) Find the value of $a$ and of $b$
$\qquad$
(b) Determine, showing all necessary working, the number of real roots of the equation

$$
f(x)=0
$$

Answer $\qquad$ [5]
3. (a) Express the following in partial fractions

$$
\frac{3\left(x^{2}-2 x+9\right)}{(x-3)\left(x^{2}+9\right)}
$$

Answer
(b) The volume of a square pyramid is $(2+5 \sqrt{5}) \mathrm{cm}^{3}$. If the length of the perpendicular height of the pyramid is $(3+2 \sqrt{5}) \mathrm{cm}$, find, without using a calculator, the area of the base of the pyramid in the form $(a+b \sqrt{5}) \mathrm{cm}^{2}$, where $a$ and $b$ are integers
4. (a) Find the coefficient of $x^{2}$ in the binomial expansion of

$$
\left(x-\frac{1}{3 x}\right)^{8}
$$

(b) Find the coefficient of $x^{2}$ in the expansion of

$$
(1+3 x)^{7}(2-x)^{4}
$$

Answer
5. (a) A prism with volume $3\left(x^{2}-5\right) \mathrm{cm}^{3}$ has a base area of $(x-1) \mathrm{cm}^{2}$. Calculate the range of values of $x$ for which the height of the prism is greater than 10 mm

Answer $\qquad$ [5]
(b) Find the value of $6^{x}$ given that

$$
\frac{8^{x}}{5^{x}}=\frac{5^{3-x}}{27^{x}}
$$

$\qquad$
6. (a) A quadratic equation is defined by

$$
(x-a)(b-x)=m
$$

Given that the roots of the equation are equal, show that the value of $m$ is

$$
\left(\frac{a-b}{2}\right)^{2}
$$

(b) (i) Show that

$$
(1-\sqrt{a})^{5}-(1+\sqrt{a})^{5}=-10 \sqrt{a}-20 a \sqrt{a}-2 a^{2} \sqrt{a}
$$

(ii) Hence, deduce the exact value of

$$
(1-\sqrt{3})^{5}-(1+\sqrt{3})^{5}
$$

## 7. BONUS QUESTION

Define $\{x\}=x-\lfloor x\rfloor$, where the floor function $\lfloor x\rfloor$ denotes the largest integer not exceeding $x$. If $-8 \leq x \leq 8$, show that the number of real solutions to the following equation is $\mathbf{1 1 3}$

$$
\{x\}+\left\{x^{2}\right\}=1
$$

