Name:	Level/Subject: **4049 Sec 4 A-Math**
Material: **March Practice Questions 2022**	Centre: **Overmugged**

Instructions

- Answer all questions
- If working is needed for any question it must be shown with the answer
- Omission of essential working will result in loss of marks
- You are expected to use a scientific calculator to evaluate explicit numerical expressions
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures
- Give answers in degrees to one decimal place
- For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π
- A copy of the formula list is provided for you on the third page

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Question Source

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level A-Math Examination. All questions compiled here are from **2009 - 2021 School Mid-Year / Prelim Papers**. Questions are categorised into the various topics and range in varying difficulties. If questions are sourced from respective sources, credit will be given when appropriate.

How to read:

[S4 ABCSS P1/2011 PRELIM Qn 1]

Secondary 4, ABC Secondary School, Paper 1, 2011, Prelim, Question 1

Prepared by: **Kaiwen :)**

This question paper consists of 35 printed pages including the cover page

Contents

List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Quadratic Equations & Inequalities

1. The equation of a curve is given, where k is a constant

$$y = 2x^2 - 6x + k$$

   (a) In the case when $k = -20$, find the set of values of x for which $y < 0$ [2]

   (b) In the case when $k = 10$, show that the line $y + 2x = 8$ is a tangent to the curve [3]

   Credit: **S4 ACS(B) P1/2017 PRELIM Qn 1**

2. The solution to the inequality $-ax^2 + bx - 1 > 0$, where a and b are constants is $\frac{1}{4} < x < 1$

   (a) Find the value of a and of b [3]

   (b) Using the values of a and b found in part (i), find the set of values of x which the curve lies completely below the line [3]

$$y = -ax^2 + bx - 1$$
$$y = 1 - 4x$$

   Credit: **S4 BDSS P1/2017 PRELIM Qn 6**

3. (a) Find the range of values of p which satisfy the inequality [4]

$$px^2 + 8x + p > 6$$

   (b) Show that the line will intersect the curve at two distinct points for all real values of q, where $q \neq -1$ [3]

$$y + qx = q$$
$$y = (q + 1)x^2 + qx - 1$$

   Credit: **S4 NCHS P1/2017 PRELIM Qn 1**

4. (a) Given that $px^2 + qx + 2q$ is always negative, what conditions must apply to the constants p and q [4]

   (b) Give an example of values of p and q which satisfy the conditions found in part (i) [1]

   Credit: **S4 TKSS P1/2017 PRELIM Qn 5**

5. The equation of a curve is $y = 2x^2 + 5x + 8$. The line $y = mx + c$, where c is a constant, does not intersect the curve

   (a) Show that [3]

$$m^2 - 10m - 39 + 8c < 0$$

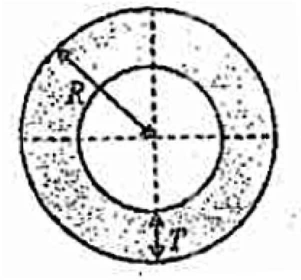
   (b) Hence, find the value of c for which $-5 < m < 15$ is the solution set for [3]

$$m^2 - 10m - 39 + 8c < 0$$

   Credit: **S4 WGSS P1/2017 PRELIM Qn 11(b)**

2 Surds

1. A hollow copper pipe with an external radius, $R = \left(4\sqrt{3} - 1\right)$ cm has a thickness, $T = \sqrt{3}$ cm. The volume of copper needed to make the pipe is $\left(521\sqrt{3} - 108\right) \pi$ cm^3



Find

(a) the cross sectional area of the pipe, in the form $\pi\left(a + b\sqrt{3}\right)$, where a and b are integers [2]

(b) the length of the pipe in the form $\left(c + d\sqrt{3}\right)$, where c and d are integers [3]

Credit: **S4 GESS P1/2017 MYE Qn 1**

2. A right cylinder has a base radius, r cm, where

$$r = \frac{3}{\sqrt{6}} + \sqrt{3}$$

(a) Find the base area of the cylinder, expression your answer in the form $\frac{\pi(a + b\sqrt{c})}{2}$, where a, b and c are integers [3]

(b) Given that the curved surface area of the cylinder is $\pi\left(20\sqrt{2} + 10\right)$. Find the height of the cylinder in the form $p\sqrt{6} + q\sqrt{3}$ [4]

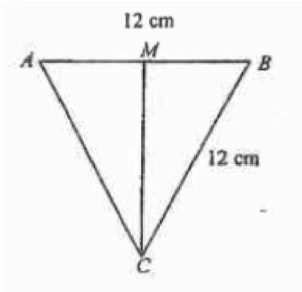
Credit: **S4 HYSS P2/2017 PRELIM Qn 4**

3. Find the value of a and of b, without using a calculator, given that [5]

$$\sqrt{a + b\sqrt{3}} = \frac{2\sqrt{3}}{3 - \sqrt{3}}$$

Credit: **S4 NBSS P1/2017 PRELIM Qn 3**

4. (a) The diagram shows a conical cup with slant height and diameter being 12 cm each [4]



There is a tiny spider at C. Given that the spider climbs at a constant speed of $\frac{6-3\sqrt{3}}{4}$ cm/s, find the time, in seconds, taken by the spider to climb up along CM, giving your answer in the form $a\sqrt{3}+b$, where a and b are integers. You may assume that the spider is of negligible size

(b) Find the value of k, where k is a rational function, given that [3]

$$125^k = \sqrt[3]{25\sqrt{5}}$$

Credit: **S4 SGSS P2/2017 PRELIM Qn 3**

5. (a) Given that m is a positive integer, find the value of m where [3]

$$\frac{15^{2k} \times 9^{4k} \times 5^{6k}}{3^{2k}} = m^{8k}$$

(b) Without using a calculator, find the value of k such that [3]

$$\left(\frac{4}{\sqrt{3}} + \frac{2\sqrt{15}}{3} - \frac{8}{\sqrt{12}}\right) \times \sqrt{6} = 2\sqrt{k}$$

(c) The lengths of the diagonals PR and QS of a rhombus $PQRS$ are $\left(4+2\sqrt{3}\right)$ cm and $\left(6+\frac{4}{\sqrt{3}}\right)$ cm respectively. Leaving your answers in surd form, find

(i) the value of PQ^2 [3]

(ii) the area of the triangle PQR [3]

Credit: **S4 WGSS P2/2017 PRELIM Qn 9**

3 Polynomials

1. The function $f(x) = x^4 + 6x^3 + 2ax^2 + bx - 3a$, where a and b are constants. Given that $x^2 + 2x - 3$ is a factor of $f(x)$,
   (a) find the value of a and of b [5]
   (b) hence, showing all necessary working, find the number of real roots of the equation $f(x) = 0$ [3]

   Credit: **S4 HIHS P2/2017 MYE Qn 4**

2. Solve the following equation, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers [5]
$$x^3 - 4x^2 - 8x + 8 = 0$$

   Credit: **S4 NBSS P2/2017 PRELIM Qn 4**

3. Given that a and b are constants, there is a function
$$f(x) = 3x^3 + ax^2 + bx + 2$$
   $(x - 1)$ is a factor of $f(x)$. The remainder when $f(x)$ is divided by $(x - 2)$ is $2\frac{1}{2}$ times the remainder when $f(x)$ is divided by $(x + 1)$
   (a) Show that $a = 2$ and $b = -7$ [4]
   (b) Without using a calculator, solve $f(x) = 0$ [3]
   (c) Hence, solve for $0 \leq y \leq 360°$ [4]
$$3\sin^2 y - 2\sec y - 2\cos y + 4 = 0$$

   Credit: **S4 XMSS P2/2017 PRELIM Qn 8**

4. A function is given by, where n is a positive integer [4]
$$f(x) = 2\left(7^{n+2}\right) + 7^n + 3\left(7^{n+1}\right)$$
   Billy says that the function is divisible by 10. Explain whether his comment is correct, justifying your answer with clear working

   Credit: **S4 HYSS P1/2017 PRELIM Qn 3**

5. (a) By using long division, divide $2x^4 + 5x^3 - 8x^2 - 8x + 3$ by $x^2 + 3x - 1$ [2]
   (b) Factorise the following completely [2]
$$2x^4 + 5x^3 - 8x^2 - 8x + 3$$
   (c) Hence, find the exact solutions to the equation [4]
$$32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0$$

   Credit: **S4 ZHSS P2/2018 PRELIM Qn 2**

4 Partial Fractions

1. It is given that

$$P(x) = 3x^3 - 9x^2 - 18x + 24$$
$$Q(x) = x^2 - 9$$

   (a) Express the following in partial fractions [5]

$$\frac{P(x)}{Q(x)}$$

   (b) (i) Solve for $P(x) = 0$ [5]

   (ii) Hence, solve the following equation, expressing your answers in powers of 2 [3]

$$3\left(\log_2 \sqrt{y}\right)^3 - 9\left(\log_2 \sqrt{y}\right)^2 - 18\left(\log_2 \sqrt{y}\right) + 24 = 0$$

   Credit: **S4 AISS P1/2017 MYE Qn 10**

2. Express the following in partial fractions [5]

$$\frac{4}{\left(x^2+4\right)(x-2)}$$

   Credit: **S4 YTSS P1/2017 PRELIM Qn 1**

3. Express the following in partial fractions [5]

$$\frac{2x^3 - 3x - 1}{(x+3)(x-1)}$$

   Credit: **S4 AHS P2/2018 PRELIM Qn 3(a)**

4. Express the following in partial fractions [5]

$$\frac{8x^2 - 2x + 19}{(1-x)(4+x^2)}$$

   Credit: **S4 TKSS P1/2018 PRELIM Qn 1**

5. (a) Factorise completely the cubic polynomial [3]

$$2x^3 - 11x^2 + 12x + 9$$

   (b) Hence, express the following in partial fractions [5]

$$\frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9}$$

   Credit: **S4 NCHS P1/2018 PRELIM Qn 2**

5 Binomial Theorem

1. (a) Find the term independent of x in the expansion of [3]

$$\left(x^2 - \frac{1}{2x^3}\right)^{10}$$

(b) Write down and simplify, the first 3 terms of the expansion in ascending powers of x

(i) (a) $(2 - 3x)^7$ [2]

(b) $\left(1 + \frac{x}{3}\right)^7$ [2]

(ii) Hence, find the coefficient of x^2 in the expansion of [3]

$$\left(2 - \frac{7}{3}x - x^2\right)^7$$

Credit: **S4 AISS P1/2017 MYE Qn 8**

2. (a) Find the values of a and b, given that, in ascending powers of x, [4]

$$(1 + ax + bx^2)^8 = 1 - 40x + 748x^2 + ...$$

(b) Evaluate the term independent of x in the binomial expansion of [2]

$$\left(x^2 - \frac{1}{2x^6}\right)^{16}$$

(c) In the binomial expansion of the following, where k is a positive constant

$$\left(x + \frac{k}{x}\right)^9$$

the coefficient of x and x^3 are equal

(i) Find the value of k [2]

(ii) Use the value of k found in part (a) to find the coefficient of x^3 in the expansion of [3]

$$(1 - 3x^2)\left(x + \frac{k}{x}\right)^9$$

Credit: **S4 GESS P2/2017 MYE Qn 3**

3. (a) (i) Expand and simplify $(1+a)^8$, in ascending powers of a, up to the term containing a^3 [2]

(ii) Given that $a = x + x^2$, write down the expansion of the following, up to the term containing x^3 [3]

$$\left(1+x+x^2\right)^8$$

(iii) Using your expansion and a suitable value for x, find the value of 1.0101^8, giving your answer correct to 6 decimal places [2]

(b) (i) Write down the general term in the binomial expansion [1]

$$\left(3x - \frac{2}{x^2}\right)^{12}$$

(ii) Write down the power of x in this general term [1]

(iii) Hence, explain why there is no term in x^5 in the binomial expansion of [2]

$$\left(3x - \frac{2}{x^2}\right)^{12}$$

Credit: **S4 TKSS P2/2017 PRELIM Qn 4**

4. (a) In the expansion of $(3x-1)(1-kx)^7$ where k is a non-zero constants, there is no term in x^2. Find the value of k [4]

(b) In the binomial expansion of $\left(\frac{2}{x^3} - x^2\right)^{12}$, in ascending powers of x, find the term in which the power of x first becomes positive [4]

Credit: **S4 SCGS P2/2018 PRELIM Qn 1**

5. (a) Find the coefficient of x^{10} in the binomial expansion [2]

$$\left(3-2x^2\right)^8$$

(b) In the following binomial expansion, in ascending powers of x, where m is a positive integer, the difference between the coefficients of x^2 and x is 462. Find the value of m [4]

$$(1+3x)^m$$

Credit: **S4 HIHS P1/2017 MYE Qn 7**

6 Exponential & Logarithms

1. A particular species of fish living in a fish farm is being studied. After t years, its population P is given by the following, where k is a constant

$$P = 300\left(2 + 5e^{-kt}\right)$$

(a) Find the initial population of the fish in the farm [1]

The population of the fish in the farm after 3 years is predicted to be 2400

(b) Find the value of k [2]

The fish farm owner has to replenish the supply of fish in the farm when the population drops below 1000

(c) Using the k value obtained in part (b), determine, with working, whether the fish farm owner needs to replenish the fish supply after 5 years [2]

Credit: **S4 CHIJ STC P2/2017 MYE Qn 1**

2. A structured deposit pays a compound interest of $r\%$ per annum. In n years, the principal amount P_0 will become P_n where

$$P_n = P_0\left(1 + \frac{r}{100}\right)^n$$

Many invests \$20000 and receives \$22497.28 in 3 years

(a) Find the value of r [2]

(b) Find the number of years Mandy has to invest if she wishes to double the principal amount [2]

Credit: **S4 CWSS P1/2017 PRELIM Qn 3**

3. (a) Simplify [2]

$$\log_3 2 \times \log_4 3 \times \log_5 4 \times ... \times \log_{n+1} n$$

(b) Using the substitution $u = 6^x$, solve the equation [4]

$$6^{x+1} - 6^{1-x} = 5$$

Credit: **S4 XMSS P1/2017 PRELIM Qn 5**

4. (a) Solve [5]

$$2\log_2(1-x) - \log_2 x - 2 = \log_2 2x + 1$$

(b) Express y as a power of x given that [4]

$$\frac{(\log_x y)^3}{\log_y x} - 20 = 61$$

Credit: **S4 YISS P2/2017 PRELIM Qn 5**

5. Solve the equation

(a) $3\log_3 x - \log_x 3 = 2$ [5]

(b) $2\log_2(1-2x) - \log_2(6-5x) = 0$ [4]

Credit: **S4 YTSS P1/2018 PRELIM Qn 8**

7 Trigonometry

1. (a) (i) Show that [2]

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

(ii) Hence, without the use of calculators, deduce the value of [3]

$$\sin\left(\frac{7\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$$

(b) (i) Prove the identity [3]

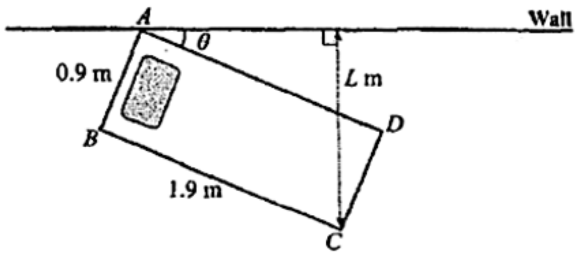
$$\frac{\sec^2 x + 2\tan x}{1 + 2\sin x\cos x} = \sec^2 x$$

(ii) Hence, solve for $0 < x < 2\pi$, the equation. Leave your answers in terms of π [4]

$$\frac{\sec^2\left(x-\frac{\pi}{3}\right) + 2\tan\left(x-\frac{\pi}{3}\right)}{1 + 2\sin\left(x-\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)} = \frac{4}{3}$$

Credit: **S4 BSS P2/2017 MYE Qn 11**

2. The diagram shows a rectangular single bed with wheels, $ABCD$, which is hinged to the wall at A.



It is given that the dimensions of the bed is 1.9 m by 0.9 m and L m is the perpendicular distance from the wall to C. The bed can be rolled such that the angle between the wall and the side, AD of the bed is θ and that $0° < \theta < 90°$

(a) Show that the length, L m, can be expressed as [3]

$$L = 1.9\sin\theta + 0.9\cos\theta$$

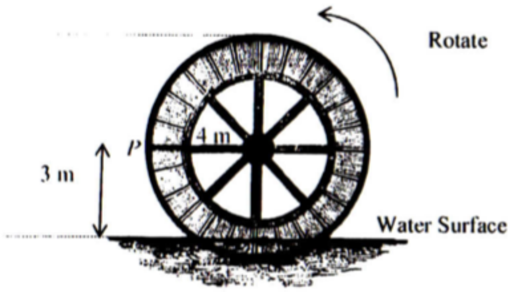
(b) Express L in the form $R\sin(\theta+\alpha)$ where $R > 0$ and α is an acute angle [3]

(c) Hence, find the maximum value of L and the corresponding value of θ [3]

(d) Find the value of θ when $L = 1.3$ m [2]

Credit: **S4 GMS(S) P1/2017 MYE Qn 11**

3. The diagram shows a water wheel which rotates at 3 revolutions per minute in an anticlockwise direction



At the start of the revolution, a point P on the rim of the wheel is at the height of 3 m above the surface of the water. The radius of the water wheel is 4 m. The height, h m, of point P above the water surface is given as, where t is the time in seconds

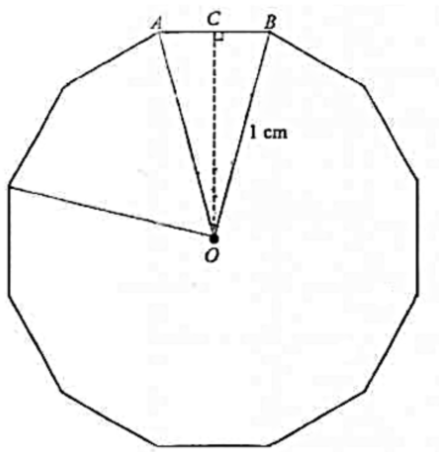
$$h = a \sin\left(\frac{\pi}{b}t\right) + c$$

(a) State the values of a, b and c [3]

(b) Find the time t, where point P first emerge from the water [3]

Credit: **S4 ACS(B) P2/2017 PRELIM Qn 2**

4. **Diagram is not drawn to scale**



The diagram shows a regular 12-sided polygon with centre O. AB is one side of the polygon, C is the midpoint of AB and $OB = 1$ cm

(a) Show that [2]

$$AB = 2\sin 15°$$

(b) (i) Express $\cos 30°$ in terms of $\sin 15°$ [1]

(ii) Show that [2]

$$\sin 15° = \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

Credit: **S4 CHIJ P1/2017 PRELIM Qn 6**

5. (a) State the values between which the principal value of $\tan^{-1} x$ must lie [1]

(b) Given that $\tan A = -p$ where A is a reflex angle, without the use of a calculator, obtain an expression, in terms of p, for

(i) $\sin A$ [1]

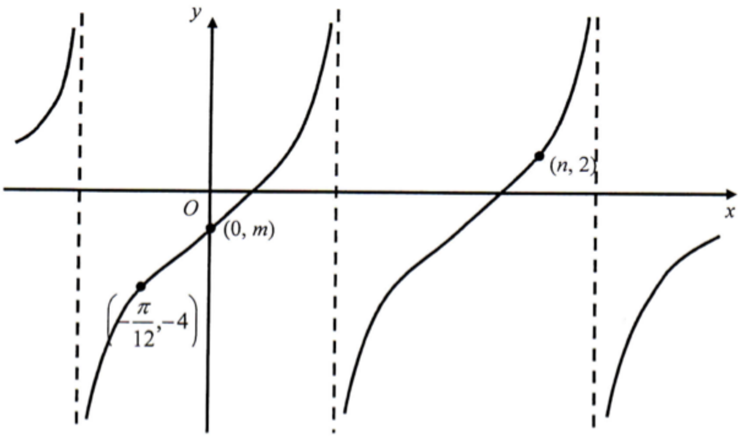
(ii) $\sec A$ [1]

(iii) $\cot(-A)$ [1]

(iv) $\tan(90 - A)^\circ$ [1]

(c) The diagram shows part of the graph [3]

$$y = m + 3\tan 3x$$



The graph passes through the point $\left(-\frac{\pi}{12}, -4\right)$, $(0, m)$ and $(n, 2)$. Find the value of m and of n

Credit: **S4 FMS(S) P1/2017 PRELIM Qn 7**

6. (a) The acute angles A and B are such that $\sin(A+B) = \frac{56}{65}$ and $\cos A \sin B = \frac{4}{13}$. Without using the calculator, find the value of

(i) $\sin A \cos B$ [1]

(ii) $\dfrac{\tan A}{\tan B}$ [2]

(iii) $\cos(A+B)$, given that $(A+B)$ is obtuse [2]

(b) (i) Express $3\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ [4]

(ii) Hence, solve for $0 < y < \pi$, the equation [4]

$$3\sin 2y + \cos 2y = 2$$

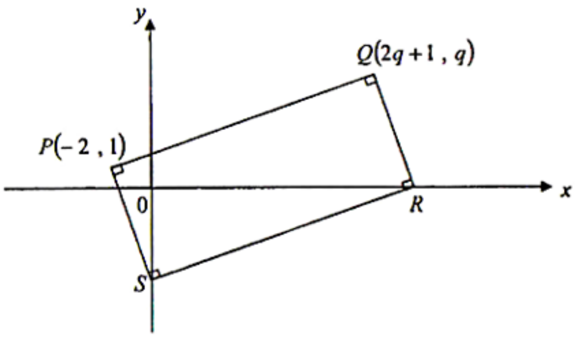
(iii) Find the greatest value of the following, leaving your answer in exact form [2]

$$\frac{1}{3\sin\theta + \cos\theta + 5}$$

Credit: **S4 SKSS P2/2017 PRELIM Qn 7 & 8**

8 Coordinate Geometry

1. The diagram shows a rectangle $PQRS$ in which $P(-2, 1)$ and $Q(2q + 1, q)$.



S is a point on the y-axis and R is a point on the x-axis. The length of PS is $2\sqrt{10}$ units. Find

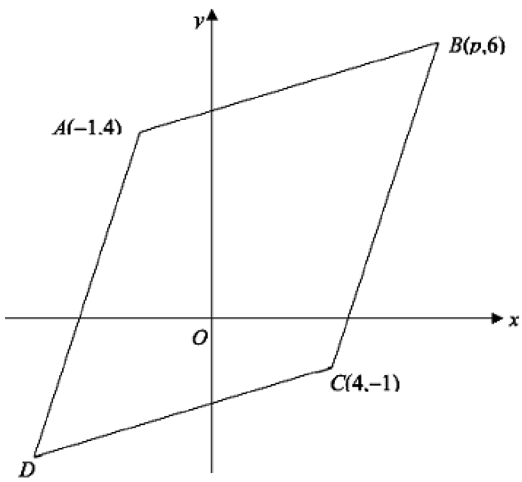
(a) the coordinates of S [3]

(b) the value of q [3]

(c) the area of rectangle $PQRS$ [3]

Credit: **S4 BSS P1/2017 MYE Qn 11**

2. **Solutions to this question by accurate drawing will not be accepted** The diagram shows a parallelogram with vertices $A(-1, 4)$, $B(p, 6)$, $C(4, -1)$ and D



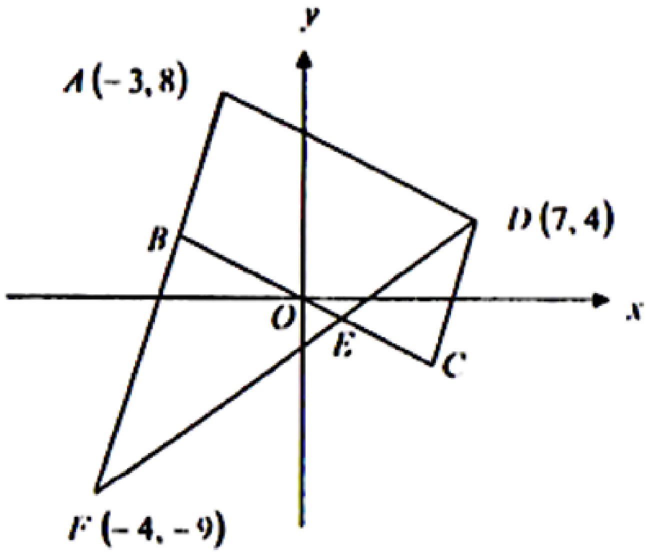
(a) Given that AC is perpendicular to BD, show that $p = 6$ [4]

(b) Find the coordinates of D [2]

(c) Find the area of the parallelogram $ABCD$ [2]

Credit: **S4 AHS P2/2018 PRELIM Qn 5**

3. **Solutions to this question by accurate drawing will not be accepted**
The diagram shows a parallelogram, $ABCD$.



ABF and DEF are straight lines. The line BC intersects DF at E. The point B divides AF such that $AB : BF = 1 : 2$. Points A, D and F have coordinates $(-3, 8)$, $(7, 4)$ and $(-4, -9)$ respectively

(a) (i) Find the equation of the perpendicular bisector of AD and show that it passes through F [4]

(ii) Hence, state one property about triangle ADF [1]

(b) Find the coordinates of B [3]

(c) Given that the area of triangle ADF is 87 units^2, find the area of the parallelogram $ABCD$ [2]

Credit: **S4 MFSS P1/2017 PRELIM Qn 12**

4. A triangle ABC is such that point $A(6, 6)$ and the point C is above point A and lie on the y-axis. $\angle ABC = 90°$ and $AB = BC = \sqrt{20}$ units. The equation of AB is $y + 2x = 18$

(a) Find the coordinates of C and hence find the equation of BC [5]

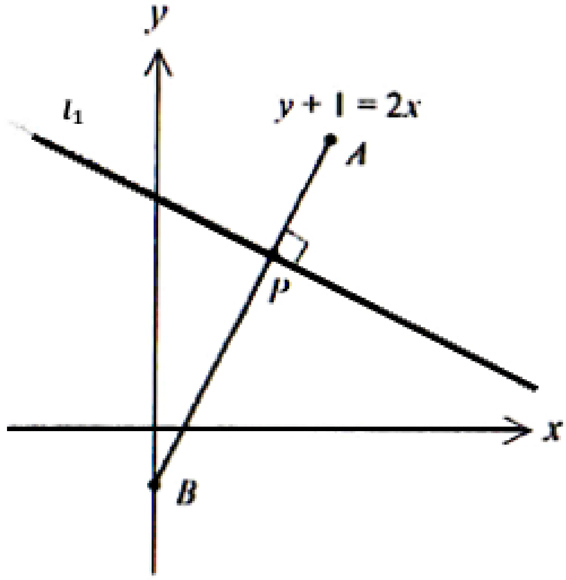
(b) State the coordinates of M, the midpoint of AC [1]

(c) Show that the coordinates of B is $(4, 10)$ [2]

(d) Calculate the area of quadrilateral $OMBC$ [3]

Credit: **S4 YTSS P1/2017 PRELIM Qn 11**

5. In the diagram, which is not drawn to scale, point P lies on the line AB



The point P is $(2, 3)$ and the equation of line AB is $y + 1 = 2x$

(a) Find the equation of the line l_1, that is perpendicular to AB and passes through point P [2]

(b) The point $C(4, 2)$. Show that point C lies on the line l_1 from part (i) [1]

(c) The point D is such that $ACBD$ is a kite. Find the coordinates of D [2]

(d) Given the length of AP and CP are equal, find the coordinates of A [4]

(e) Find the area of the kite $ACBD$ [2]

Credit: **S4 YHSS P1/2017 PRELIM Qn 11**

9 Further Coordinate Geometry

1. The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle C at the points $(-1, 8)$ and $(3, 6)$ respectively
   (a) Show that the equation of C is [5]
   $$x^2 + y^2 + 2x - 6y - 15 = 0$$
   (b) Explain whether or not the x-axis is tangent to C [3]
   (c) The points Q and R also lie on the circle, and the length of the chord QR is 2. Calculate the shortest distance from the centre of C to the chord QR [2]

   Credit: **S4 CWSS P2/2017 PRELIM Qn 7**

2. The line $x = 17$ is a tangent to a circle and the points $A(1, 9)$ and $B(1, -7)$ are on the circumference of the circle
   (a) Show that the radius of the circle is 10 units [4]
   (b) State the coordinates of the centre of the circle [1]
   (c) Write down the equation of the circle [2]
   (d) The circle is reflected along the line $y = -1$, show that the point $(3, 10)$ does not lie on the reflected circle [3]

   Credit: **S4 BDSS P2/2017 PRELIM Qn 12**

3. The equation of a circle C_1 is
   $$3x^2 - 30x + 75 - 12y + 3y^2 = 0$$
   (a) Find the radius and the coordinates of the centre of C_1 [3]
   (b) Show that the circle C_1 touches the x-axis [2]

   A second circle, C_2, has the same centre as the circle C_1 and a diameter AB. Given that the coordinates of A are $(1, 6)$, find
   (c) the equation of the circle C_2 [2]
   (d) the equation of the tangent to C_2 at B [3]

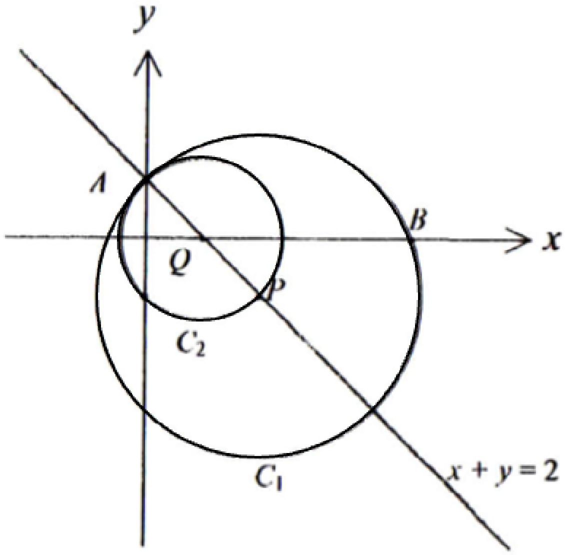
   A point, P, which lies on the circle C_2, has the same distance from the x-axis as the point A
   (e) Find the equation of PB [2]

   Credit: **S4 FMS(S) P2/2017 PRELIM Qn 9**

4. The line $x = 3$ and $y = -5$ are tangents to a circle C_1
   (a) Given that the centre of the circle C_1 lies on the negative x-axis, find the equation of C_1 [3]
   (b) Circle C_2 is a reflection of circle C_1 along the line $x + y = 0$, find the equation of C_2 [3]

   Credit: **S4 OPSS P2/2017 PRELIM Qn 7**

5. The diagram shows two circles C_1 and C_2.



Circle C_1 has its centre at P and circle C_2 has its centre at Q. Point P lies on the circumference of circle C_2 and point Q lies on the x-axis. The line $x + y = 2$ passes through the centres of both circles and intersects the circumference of both circles at point $A(0, 2)$.

Find, in terms of $x^2 + y^2 + 2gx + 2fy + c = 0$
   (a) the equation of C_2 [2]
   (b) the equation of C_1 [3]

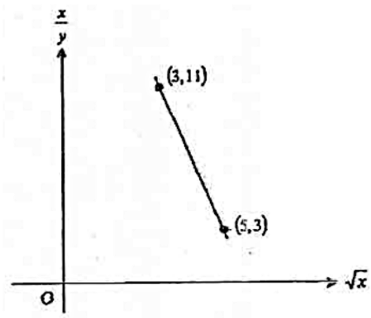
$B(k, 0)$ is a point on the circle C_1, where $k > 0$
   (c) Find the value of k, express your answer in terms of $a + b\sqrt{7}$ where a and b are integers [3]
   (d) Show that the gradient of the tangent to the circle C_1 at point B is $-\sqrt{7}$ and that this tangent intersects the y-axis at $4\sqrt{7} + 14$ [3]

   Credit: **S4 YHSS P2/2017 PRELIM Qn 12**

10 Linear Law

1. The diagram shows part of a straight line graph drawn to represent the equation, where a and b are constants [4]

$$y = \frac{x}{b\sqrt{x} - a}$$



Given that the line passes through $(3, 11)$ and $(5, 3)$, find the values of a and of b

Credit: **S4 CHIJ STC P1/2017 MYE Qn 3**

2. (a) The variables x and y are related such that when $\lg y$ is plotted against x^2, a straight line passing through $(2, 12)$ and $(3, 8)$ is obtained. Express y in terms of x [4]

(b) The table shows experimental values of two variables, x and y which are related by the equation, where a and b are constants

$$\sqrt{y} = a\left(x^2 + b\right)$$

x	1	1.8	3.5	4.3	5.5
y	16	115	1425	3189	8418

(a) Using graph paper, plot $\sqrt{y}$ against x^2 and draw a straight line graph [2]

(b) Use the graph to estimate the value of a and of b [3]

(c) Estimate the value of x when $y = 36$ [1]

Credit: **S4 HIHS P1/2017 MYE Qn 11**

3. The value, $\$V$, of a mobile phone usually decreases with time. An analyst claims that this decrease is exponential and so can be modelled by an equation of the form

$$V = V_0 e^{kt}$$

where V_0 and k are constants and t is the time in months after the release of the mobile phone. The table below gives some values of V and t of a mobile phone

t months	1	4	7	9
$\$V$	955	825	730	652

(a) Plot a suitable straight line graph and explain how it shows that the model is value for $t = 1$ to $t = 9$. The vertical axis should start at 6.4 and have a scale of 2 cm to 0.1 [3]

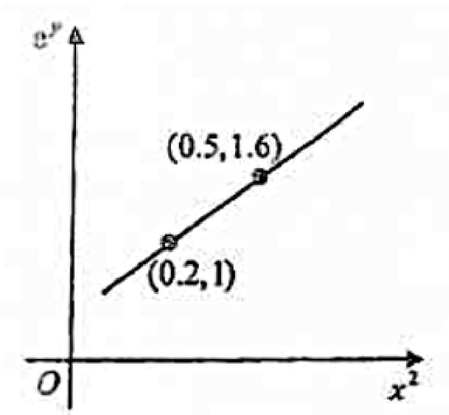
(b) Estimate the value of V_0 and explain what this term represents [2]

(c) Estimate the value of k [2]

(d) Assuming that the model is still appropriate, estimate the value of the mobile phone after 15 months [2]

Credit: **S4 MFSS P2/2017 PRELIM Qn 1**

4. Variables x and y are such that, when e^y is plotted against x^2, a straight line passing through the points $(0.2, 1)$ and $(0.5, 1.6)$ is obtained



(a) Find the value of e^y when $x = 0$ [2]

(b) Express y in terms of x [1]

Credit: **S4 SGSS P1/2017 PRELIM Qn 5**

5. **Answer the whole of this part of the question on a sheet of graph paper**
The table shows the values of two variables, x and y, obtained from an experiment. The variables x and y are related by the equation, where h and k are constants

$$y = \frac{h}{kx} + \frac{1}{kx^2}$$

x	1	2	3	4	5	6
y	2.601	0.551	0.194	0.089	0.040	0.017

(a) Based on the values given in the above table, construct a table for the values of x^2y [1]

(b) Plot x^2y against x, using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.5 unit on the x^2y axis. Hence, draw the line of best fit [3]

(c) Using the graph of (ii) to find the value of

(i) y when $x = 2.5$ [1]

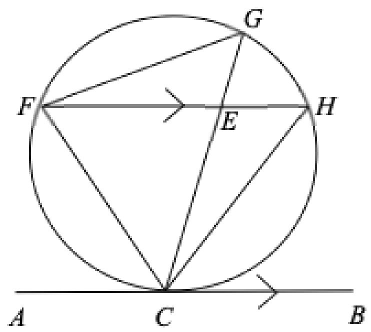
(ii) k [1]

(iii) h [1]

Credit: **S4 YISS P1/2017 PRELIM Qn 4**

11 Proofs of Plane Geometry

1. The diagram shows a point C on a circle and line ACB is a tangent to the circle



Points F, G and H lie on the circle such that FH is parallel to AB. The lines GC and FH intersect at E

(a) (i) Prove that $\triangle ECF$ and $\triangle FCG$ are similar [2]

(ii) Hence, show that [2]

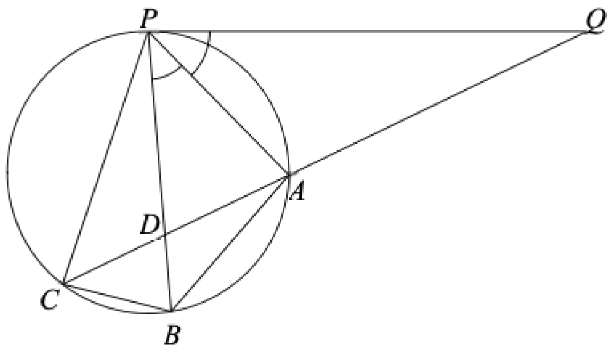
$$EC \times CG = (CF)^2$$

(b) By using similar triangles, show that [5]

$$FE \times EH = CF^2 - EC^2$$

Credit: **S4 BPGHS P2/2018 PRELIM Qn 10**

2. The diagram shows a point P on a circle and PQ is a tangent to the circle



Points A, B and C lie on the circle such that PA bisects $\angle QPB$ and QAC is a straight line. The line QC and PB intersect at D. Prove that

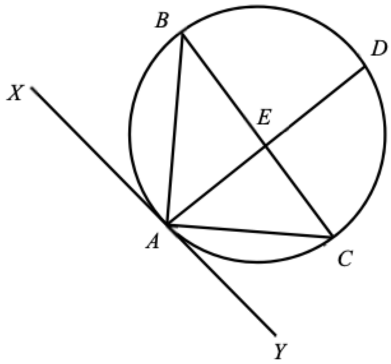
(a) $AP = AB$ [4]

(b) CD bisects $\angle PCB$ [4]

(c) $\triangle CDP$ is similar to $\triangle CBA$ [2]

Credit: **S4 CHIJ KC P2/2018 PRELIM Qn 7**

3. The diagram shows that AD and BC are straight lines, AC bisects $\angle DAY$ and AB bisects $\angle DAX$



Show that

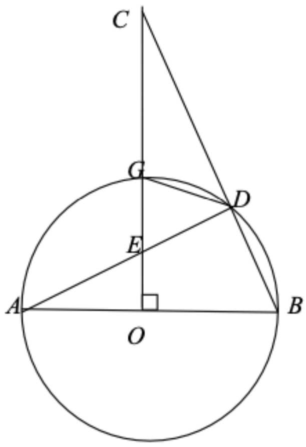
(a) $AC^2 = EC \times BC$ [3]

(b) BC is a diameter of the circle [3]

(c) AD and BC are perpendicular to each other [3]

Credit: **S4 CGS P2/2018 PRELIM Qn 10**

4. AB is a diameter of the circle with centre O



C is a point on OG produced and CB intersects the circle at D. OG is perpendicular to AB and OG intersects the chord AD at E

(a) Prove that [4]

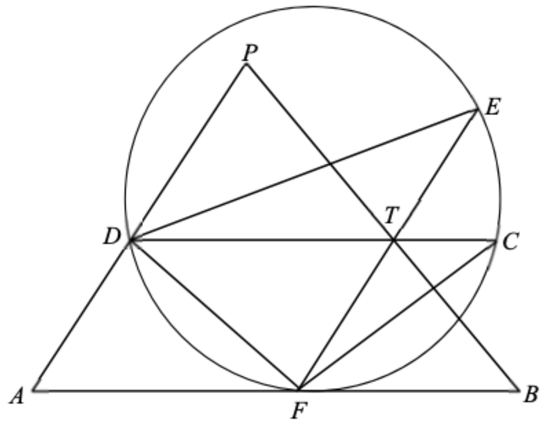
$$AE \times ED = OE \times EC$$

(b) Explain why C is at an equal distance from A and B [2]

(c) Explain why a circle with BC as a diameter passes through O [2]

Credit: **S4 SST P1/2018 PRELIM Qn 8**

5. The diagram shows a circle passing through points D, E, C and F, where $FC = FD$.



The point D lies on AP such that $AD = DP$. DC and EF cut PB at T such that $PT = TB$

(a) Show that AB is a tangent to the circle at point F [3]

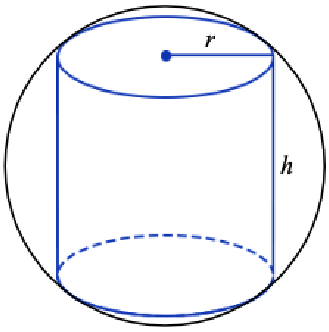
(b) By showing that $\triangle DFT$ and $\triangle EFD$ are similar, show that [4]

$$DF^2 - FT^2 = FT \times ET$$

Credit: **S4 SCGS P1/2018 PRELIM Qn 12**

12 Differentiation

1. The diagram shows a cylinder of height h cm and base radius r cm inscribed in a sphere of radius 35 cm



(a) Show that the height of the cylinder, h cm, is given by [2]

$$h = 2\sqrt{1225 - r^2}$$

(b) Given that r can vary, find the maximum volume of the cylinder [4]

Credit: **S4 CGSS P1/2018 PRELIM Qn 5**

2. It is given that, for $0 < x < \frac{\pi}{2}$

$$y = x - \ln(\sec x + \tan x)$$

(a) Show that [3]

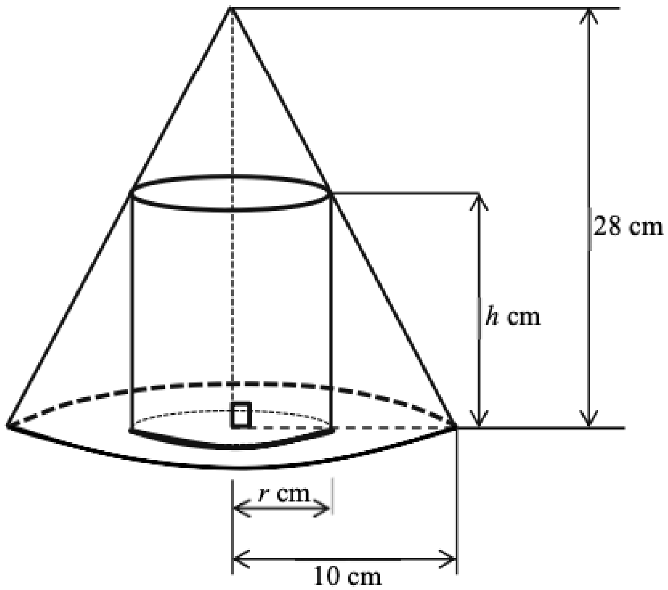
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

(b) Hence, express $\frac{dy}{dx}$ in the form $a + b \sec x$, where a and b are integers [3]

(c) Deduce that y is a decreasing function [2]

Credit: **S4 CCHS(M) P2/2018 PRELIM Qn 2**

3. The diagram shows a cylinder of height h cm and base radius r cm inscribed in a cone of height 28 cm and base radius 10 cm.



Show that

(a) (i) the height, h cm, of the cylinder is given by [1]

$$h = 28 - \frac{14}{5}r$$

(ii) the volume, V cm^3, of the cylinder is given by [1]

$$V = 14\pi r^2\left(2 - \frac{r}{5}\right)$$

(b) (i) Given that r can vary, find the maximum volume of the cylinder [5]

(ii) Show that, in this case, the cylinder occupies $\frac{4}{9}$ of the volume of the cone [2]

Credit: **S4 CWSS P1/2018 PRELIM Qn 6**

4. Find the value of the constant k for which [4]

$$y = x^2e^{1-2x}$$

is a solution of the equation

$$\frac{d^2y}{dx^2} - \frac{2y}{x^2} = k\left(\frac{dy}{dx} + y\right)$$

Credit: **S4 SST P2/2018 PRELIM Qn 1**

5. Diagram I shows a right angled $\triangle ABC$, with hypotenuse AB of length 4 m. This triangle is revolved around BC to generate a right circular cone as shown in Diagram II

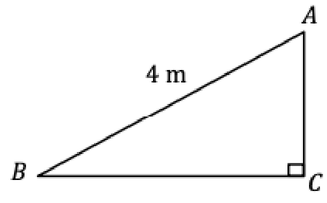


Diagram I

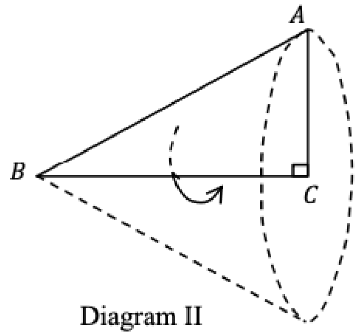


Diagram II

(a) Find the **exact** height that gives the maximum volume of the cone [6]

(b) Show that this maximum volume is obtained when [2]

$$BC : CA = 1 : \sqrt{2}$$

Credit: **S4 NCHS P1/2018 PRELIM Qn 10**

13 Integration

1. (a) Show that [3]

$$\frac{2}{\tan\theta + \cot\theta} = \sin 2\theta$$

(b) Hence, find the value of p, giving your answer in terms of π, where $0 < p < \frac{\pi}{4}$, for which [4]

$$\int_0^p \frac{4}{\tan 2x + \cot 2x}\ dx = \frac{1}{4}$$

Credit: **S4 CHIJ SNGS P1/2018 PRELIM Qn 6**

2. A curve is such that, where a is a constant

$$\frac{d^2y}{dx^2} = ax - 2$$

The curve has a minimum gradient at $x = \frac{1}{3}$

(a) Show that $a = 6$ [1]

The tangent to the curve at (1,4) is $y = 2x + 2$

(b) Find the equation of the curve [6]

Credit: **S4 CCHS(M) P1/2018 PRELIM Qn 1**

3. It is given that, where c is a constant of integration

$$\int f'(x)\ dx = \frac{x}{2} - \frac{\sin kx}{8} + c \qquad \int_0^{\frac{\pi}{8}} f'(x)\ dx = \frac{\pi}{16} - \frac{1}{8}$$

(a) Show that $k = 4$ [2]

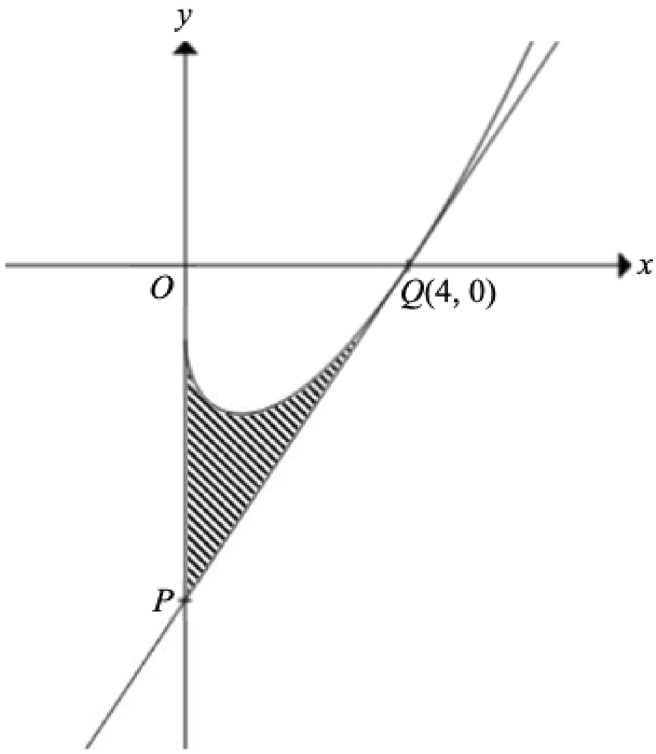
(b) Hence, find $f'(x)$, expressing your answer in $\sin^2 px$, where p is a constant [2]

(c) Find the equation of the curve $y = f(x)$, given that the point $\left(\frac{\pi}{4}, 0\right)$ lies on the curve [2]

Credit: **S4 CGS P1/2018 PRELIM Qn 5**

4. The diagram below shows part of a curve $y = f(x)$. The curve is such that

$$f'(x) = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$



The curve passes through the point $Q(4, 0)$. The tangent at Q meets the y-axis at the point P

(a) Find $f(x)$ [3]

(b) Show that the y-coordinate of P is -6 [3]

(c) Find the area of the shaded region [4]

Credit: **S4 SCGS P2/2018 PRELIM Qn 11**

5. (a) Find k, given that [2]

$$\sin(A + B) + \sin(A - B) = k \sin A \cos B$$

(b) Hence, find the exact value of [4]

$$\int_0^{\frac{\pi}{4}} \sin 2x \cos x \; dx$$

Credit: **S4 YTSS P2/2018 PRELIM Qn 5**

14 Differentiation & Integration

1. (a) Differentiate the following with respect to x and simplify your answer as a single fraction [2]

$$(x-5)\sqrt{2x-1}$$

(b) Hence, evaluate the following, leaving your answer in exact form [4]

$$\int_1^2 \frac{3x-9}{\sqrt{2x-1}}\,dx$$

Credit: **S4 BPGHS P1/2018 PRELIM Qn 9**

2. (a) Show that [2]

$$\frac{d}{dx}(\sin x\cos x) = 2\cos^2 x - 1$$

(b) Hence, without using a calculator, find the value of each of the constants a and b for which [4]

$$\int_0^{\frac{\pi}{4}} \cos^2 x\ dx = a + b\pi$$

Credit: **S4 CGSS P2/2018 PRELIM Qn 2**

3. The curve $y = f(x)$ passes through the point $(0, 3)$ and is such that

$$f'(x) = \left(e^x + \frac{1}{e^x}\right)^2$$

(a) Find the equation of the curve [4]

(b) Find the value of x for which $f''(x) = 3$ [4]

Credit: **S4 CHIJ SNGS P2/2018 PRELIM Qn 9**

4. Given that

$$y = e^x \sin x$$

   (a) show that [4]

$$2\left(\frac{dy}{dx}\right) - \frac{d^2y}{dx^2} = 2y$$

   (b) Hence, or otherwise, find the value of [4]

$$\int_0^{\frac{\pi}{3}} e^x \sin x \, dx$$

Credit: **S4 CWSS P2/2018 PRELIM Qn 3**

5. (a) Given that [3]

$$y = x^2\sqrt{2x+1}$$

   show that

$$\frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}}$$

   (b) Hence

   (i) find the coordinates of the stationary points on the curve $y = x^2\sqrt{2x+1}$ and determine the nature of these stationary points [5]

   (ii) evaluate [4]

$$\int_1^5 \frac{5x^2+2x-3}{\sqrt{2x+1}} \, dx$$

Credit: **S4 MGS P2/2018 PRELIM Qn 11**

15 Kinematics

1. A particle moves in a straight line so that its velocity, v m/s, is given by the following, where t is the time in seconds, after leaving a fixed point O

$$v = 2 - \frac{18}{(t+2)^2}$$

Its displacement from O is 9 m when it is at instantaneous rest. Find

(a) the value of t when it is at instantaneous rest [2]

(b) the distance travelled during the first 4 seconds [4]

At $t = 7$, the particle starts with a new velocity, V m/s, is given by

$$V = -h\left(t^2 - 7t\right) + k$$

(c) Find the value of k [1]

(d) Given that the deceleration is 0.9 m/s^2 when $t = 8$, find the value of h [2]

Credit: **S4 CHIJ KC P1/2018 PRELIM Qn 12**

2. A particle moves in a straight line passes through a fixed point X with velocity 5 m/s. Its acceleration is given by, where t is the time in seconds after passing X

$$a = 4 - 2t$$

(a) Calculate the value of t when the particle is instantaneously at rest [4]

(b) Find the total distance travelled by the particle in the first 6 seconds [4]

Credit: **S4 MGS P1/2018 PRELIM Qn 8**

3. A particle, P, travels along a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by, where k is a constant

$$v = 12e^{kt} + 18$$

(a) Find the initial velocity of the particle [1]

Two seconds later, its velocity is 40 m/s

(b) Show that $k = 0.3031$, correct to 4 significant figures [3]

(c) Sketch the graph of $v = 12e^{kt} + 18$, for $0 \leq t \leq 4$ [3]

(d) Explain why the distance travelled by P during the first 4 seconds does not exceed 180 metres [2]

(e) Find the maximum acceleration of P during the interval $0 \leq t \leq 4$ [2]

Credit: **S4 TKSS P2/2018 PRELIM Qn 10**

4. A particle moves pass a point A is a straight line with displacement of -4 m from a fixed point O. Its acceleration, a m/s^2, is given by, where t seconds is the time elapsed after passing through point A

$$a = \frac{t}{2}$$

Given that the initial velocity is -1 m/s, find,

(a) the velocity when $t = 2$ [3]

(b) the distance travelled by the particle in the first 5 seconds [5]

Credit: **S4 YTSS P2/2018 PRELIM Qn 13**

5. A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, v m/s, of the car after he applied the brakes is given by, where t the time after he applied the brakes is measured in seconds

$$v = 40e^{-\frac{1}{3}t} - 15$$

(a) Calculate the initial acceleration of the car [2]

(b) Calculate the time taken to stop the car [2]

(c) Obtain an expression, in terms of t, for the displacement of the car, t seconds after the brakes has been applies [3]

(d) Calculate the braking distance [1]

Credit: **S4 ZHSS P2/2018 PRELIM Qn 9**

END OF PRACTICE QUESTIONS