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Topic 2:
Equations and Inequalities (4049)

THE ABOUT

CHAPTER ANALYSIS

- Conditions for a quadratic equation to have:
  - Two real roots
  - Two equal roots
  - No real roots

  and relating conditions for a given line to:
  - Intersect a given curve
  - Be a tangent to a given curve
  - Not intersect a given curve

- Solving quadratic inequalities, and representing the solution on the number line

MASTERY

- Relatively straight forward chapter
- 3 **key** concepts

EXAM

- Concepts usually tested separately
- Usually tested as an application to other topics
  - ➢ Topics that involve Graphs

WEIGHTAGE

- High overall weightage
- Tested consistently every year
- Typically, 5m - 7m questions, 1 question per paper

KEY CONCEPT

Quadratic Equations and its Roots

Nature of Roots

Quadratic Equations and its Roots

A quadratic function that is equated to a number is a quadratic equation. The solutions to the equation are known as **roots**

1. Factorisation

Factorise the quadratic equation

$$\boxed{ax^2 + bx + c = 0} \Rightarrow \boxed{a(x-\alpha)(x-\beta) = 0}$$

Solving the equation gives $x = \alpha$ or $x = \beta$

2. Quadratic Formula

The roots of the equation $ax^2 + bx + c = 0$ can be obtained with

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad a \neq 0}$$

IMPORTANT

Students are not allowed to assume the discriminant signature as this is a show question. Students are supposed to reach the discriminant signature on their own, hence, proving the claim of the question

Example

Show that the following has real and distinct roots for all real values of x

$$(p+1)x^2+(4p+3)x+2p-1=0$$

[S4 CWSS P1/2009 PRELIM Qn 9(b)]

Solution:

To show that the function has real and distinct roots

$$\text{WTS: } b^2-4ac>0$$

$$\begin{aligned} D &= (4p+3)^2-4(p+1)(2p-1) \\ &= 16p^2+24p+9-4\left(2p^2+p-1\right) \\ &= 16p^2+24p+9-8p^2-4p+4 \\ &= 8p^2+20p+13 \\ &= 8\left(p^2+\frac{5}{2}p+\frac{13}{8}\right) \\ &= 8\left[\left(p+\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2+\frac{13}{8}\right] \\ &= 8\left(p+\frac{5}{4}\right)^2+\frac{1}{2} \end{aligned}$$

Since $\left(p+\frac{5}{4}\right)^2>0$,

$$8\left(p+\frac{5}{4}\right)^2>0$$

$$8\left(p+\frac{5}{4}\right)^2+\frac{1}{2}>0$$

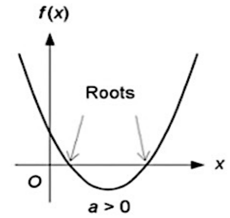
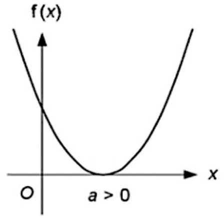
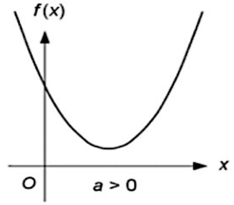
Hence, the function has real and distinct roots **(shown)**

□

Nature of Roots

The expression (b^2-4ac) is known as the **discriminant/determinant** of the quadratic equation

The nature of the roots can be discriminated by examining the discriminant

Nature of Roots	Discriminant	Graphical Representation
2 real and distinct roots	$b^2-4ac>0$	
2 equal roots	$b^2-4ac=0$	
No real roots	$b^2-4ac<0$	

*There is also a possibility for "2 real roots". In this special case, $b^2-4ac\geq 0$

IMPORTANT

Students are not allowed to assume the discriminant signature as this is a show question. Students are supposed to reach the discriminant signature on their own, hence, proving the claim of the question

Example

Show that the line meets the curve at 2 distinct points for all real values of k

$$y = 5 - k$$

$$y = x^2 - kx$$

[S3 SQSS P1/2011 MYE Qn 10(b) (MODIFIED)]

Solution:

$$y = 5 - k \text{ ......(1)}$$
$$y = x^2 - kx \text{ ......(2)}$$

Take Equation (1) = Equation (2)

$$5 - k = x^2 - kx$$
$$x^2 - kx + (k - 5) = 0$$

To show that the line meets the curve

$$\text{WTS: } b^2 - 4ac > 0$$

$$\begin{aligned} D &= (-k)^2 - 4(1)(k - 5) \\ &= k^2 - 4k + 20 \\ &= (k - 2)^2 - (2)^2 + 20 \\ &= (k - 2)^2 + 16 \end{aligned}$$

Since k can take any real value,

$$(k - 2)^2 > 0$$
$$(k - 2)^2 + 16 > 0$$

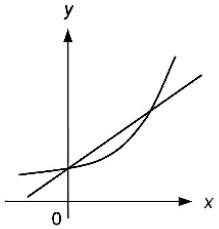
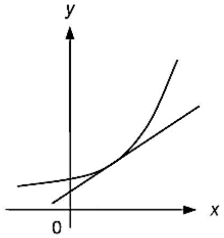
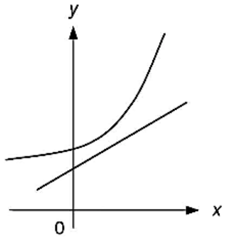
Hence, the line will meet the curve at 2 distinct points for all real values of k **(shown)**

□

Line intersecting Curves

Solve the equations simultaneously by substituting the equation of the line into the equation of the curve to eliminate one of the variables

The number of intersection points can be identified by examining the discriminant

Intersection	Discriminant	Graphical Representation
Intersect at 2 distinct points	$b^2 - 4ac > 0$	
Tangential	$b^2 - 4ac = 0$	
Do not intersect	$b^2 - 4ac < 0$	

KEY CONCEPT

Quadratic Inequality
Critical Regions

Quadratic Inequality Properties

Let a, b, c and d be any 4 real numbers

Property / Conditions	Result
$a - b > 0$	$a > b$
$a - b < 0$	$a < b$
$a - b = 0$	$a = b$
$a > b$ and $b > c$	$a > c$
$a > b$	$a + c > b + c$ or $a - c > b - c$
$a < b$	$a + c < b + c$ or $a - c < b - c$
$a > b$ and $c > 0$	$ac > bc$ or $\frac{a}{c} > \frac{b}{c}$
$a > b$ and $c < 0$	$ac < bc$ or $\frac{a}{c} < \frac{b}{c}$

When multiplying / dividing by a negative number, remember to reverse the direction of the inequality

IMPORTANT

Note that for this method to work, we only deal with happy face curves where the coeffiicient of x^2 is **greater than 0**

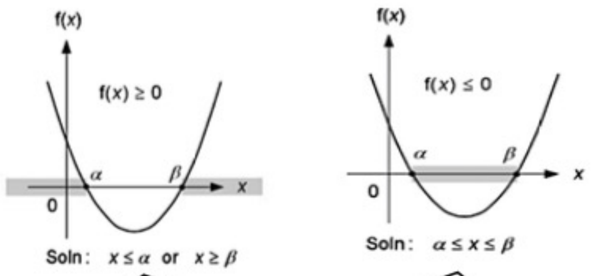
$$a > 0$$

In the event the coefficient of the quadratic term is not positive, factorise out the negative sign before continuing on with the quadratic curve and inequality

$$-x^2 + bx + c < 0$$
$$x^2 - bx - c > 0$$

$$-x^2 + bx + c > 0$$
$$x^2 - bx - c < 0$$

Note the flip in the inequality symbol. Since we are dividing by a negative value, the inequality symbol changes direction



If the inequality symbol of the quadratic inequation is pointing to the right, then the critical region will be outside of the curve

If the inequality symbol of the quadratic inequation is pointing to the left, then the critical region will be on the inside of the curve

Expressing Quadratic Inequalities

For Quadratic inequalities, you must sketch out the graph to see where are the critical regions related to the inequality

Rewrite the inequality into the form

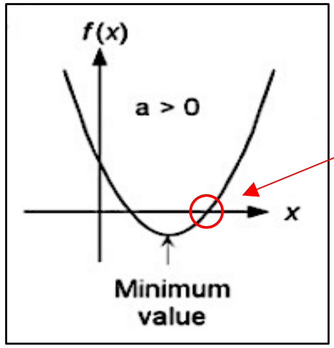
$$\boxed{f(x) = 0}$$

Factorise the equation so that it will get into the form

$$\boxed{a(x-\alpha)(x-\beta) = 0}$$

Solve the equation to get the roots of the equation $x = \alpha$ or $x = \beta$

To figure out the critical regions in relation to the inequality, draw the number line and sketch the curve. Observe the direction of the inequality symbol



Imagine you are standing at that position, facing the top of the paper

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