Name: ____________________ Material: A-Math Mock Paper 2021

OVERMUGGED MOCK PAPER 2021
SECONDARY 4 EXPRESS
SECONDARY 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS **4049/01**

Specimen Paper **MARKING SCHEME**

Date: **1 September 2021** **Duration: 2 hours 15 minutes**

Candidates answer on separate writing paper

READ THESE INSTRUCTIONS FIRST

Write your name on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

At the end of the exam, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question
The total number of marks for this paper is 90.

Setter: Ong Kai Wen

This question paper consists of 18 printed pages including the cover page

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1. In view of a contagious virus, the government of a particular country has imposed a 'Stay-Home-Notice' on her people to reduce the number of human-to-human transmission cases. It is estimated that the percentage of the population, P, complying to the Stay-Home-Notice is given by the following equation, where t is the number of days after the imposition

$$P = 100(1 - e^{-0.15t})$$

(a) Find the percentage of population complying to the 'Stay-Home-Notice' after 5 days of the imposition

[1]

Solution

$$P_5 = 100(1 - e^{-0.15(5)})$$
$$= 52.76334 \ldots$$
$$= 52.8\% \text{ (3.s.f.)}$$

(b) Find the number of complete days after the imposition that it will take for at least 90% of the population to comply

[2]

Solution

$$90 = 100(1 - e^{-0.15t})$$
$$1 - e^{-0.15t} = 0.9$$
$$e^{-0.15t} = 0.1$$
$$-0.15t = \ln(0.1)$$
$$t = \frac{\ln(0.1)}{-0.15}$$
$$= 15.35056 \ldots$$
$$\approx 16 \text{ completed days}$$

(c) Is It possible for the percentage of this country's population complying to the 'Stay-Home-Notice' to reach 100%. Explain your answer

[1]

Solution

As t gets very large, $e^{-0.15t}$ approaches 0 but will never reach 0. Hence, it is not possible for the percentage to reach 100%

[S4 ZHSS P1/2020 PRELIM Qn 4]

[**Total: 4 marks**]

2. It is given that the following line meets the curve, where k is a constant.

$$y = 2x - 3k + 3$$
$$y = x^2 - kx + k^2$$

Find the smallest and largest possible of k

[5]

Solution

$y = 2x - 3k + 3 \ldots\ldots\ldots (1)$

$y = x^2 - kx + k^2 \ldots\ldots\ldots (2)$

$(1) = (2),$

$2x - 3k + 3 = x^2 - kx + k^2$

$x^2 + (-k - 2)x + (k^2 + 3k - 3) = 0$

Since the line meets the curve, $b^2 - 4ac \geq 0$

$(-k - 2)^2 - 4(1)(k^2 + 3k - 3) \geq 0$

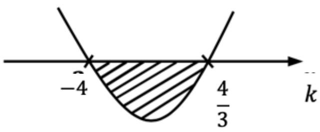
$k^2 - 4k + 4 - 4k^2 - 12k + 12 \geq 0$

$-3k^2 - 8k + 16 \geq 0$

$3k^2 + 8k - 16 \leq 0$

$(k + 4)(3k - 4) \leq 0$

$-4 \leq k \leq \frac{4}{3}$



The largest value of k is $\frac{4}{3}$ and the smallest value of k is -4

[S4 YISS P1/2020 PRELIM Qn 2]

[**Total: 5 marks**]

3. Evaluate the following

$$\int \frac{x^2+7}{(x-3)(x+1)^2}\,dx$$

[Hint: Split the fraction using Partial Fractions]

[8]

Solution

$$\frac{x^2+7}{(x-3)(x+1)^2} = \frac{A}{(x-3)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$x^2+7 = A(x+1)^2 + B(x-3)(x+1) + C(x-3)$$

Let $x=-1$,

$(-1)^2+7 = C(-1-3)$

$-4C = 8$

$C=-2$

Let $x=3$,

$(3)^2+7 = A(3+1)^2$

$16A = 16$

$A = 1$

Let $x=0$

$(0)^2+7 = 1(0+1)^2 + B(0-3)(0+1) - 2(0-3)$

$-3B = 0$

$B = 0$

$$\frac{x^2+7}{(x-3)(x+1)^2} = \frac{1}{(x-3)} - \frac{2}{(x+1)^2}$$

$$\therefore \int \frac{x^2+7}{(x-3)(x+1)^2}\,dx = \int \frac{1}{x-3} - \frac{2}{(x+1)^2}\,dx$$

$$= \ln|x-3| + \frac{2}{x+1} + c$$

[S4 YSS P1/2020 PRELIM Qn 7 (Modified)]

[**Total: 8 marks**]

4. Find the value of each of the constants $\boldsymbol{A}$ and $\boldsymbol{B}$ given the following equations

$$y = \frac{1}{2}(e^{2x} + 5e^{-2x})$$
$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = Ae^{2x} + Be^{-2x}$$

[4]

Solution

$$y = \frac{1}{2}(e^{2x} + 5e^{-2x})$$
$$= \frac{1}{2}e^{2x} + \frac{5}{2}e^{-2x}$$

$$\frac{dy}{dx} = e^{2x} - 5e^{-2x}$$
$$\frac{d^2y}{dx^2} = 2e^{2x} + 10e^{-2x}$$

$$\therefore \frac{d^2y}{dx^2} - \frac{dy}{dx} = (2e^{2x} + 10e^{-2x}) - (e^{2x} - 5e^{-2x})$$
$$= e^{2x} + 15e^{-2x}$$

$A = 1, \qquad B = 15$

[S4 TSS P1/2020 PRELIM Qn 3]

[**Total**: **4 marks**]

5. Given that $\log_2 a = h$ and $\log_2 b = k$, express the following in terms of h and k

$$\log_2 \sqrt{\frac{32a}{b^3}}$$

[4]

Solution

$$\begin{aligned}
\log_2 \sqrt{\frac{32a}{b^3}} &= \frac{1}{2}\log_2 \frac{32a}{b^3} \\
&= \frac{1}{2}\log_2 32a - \frac{1}{2}\log_2 b^3 \\
&= \frac{1}{2}\log_2 32 + \frac{1}{2}\log_2 a - \frac{3}{2}\log_2 b \\
&= \frac{5}{2} + \frac{1}{2}h - \frac{3}{2}k \\
&= \frac{1}{2}(5 + h - 3k)
\end{aligned}$$

[S4 SMSS P1/2020 PRELIM Qn 4]

[**Total**: **4 marks**]

6. **(a)** A trigonometric function is defined by the following for $\mathbf{0^\circ \le x \le 360^\circ}$

$$f(x) = 2 - 3\cos x$$

(i) State the amplitude of the graph of $y = f(x)$ [1]

Solution

$\textbf{Amplitude} = 3$

(ii) State the period of the graph of $y = f(x)$ [1]

Solution

$\textbf{Period} = 360^\circ$

(b) Another trigonometric function, where $a < 0$, has a maximum value of 5, a minimum value of -1 and a period of 540°

$$g(x) = a\sin\left(\frac{x}{b}\right) + c$$

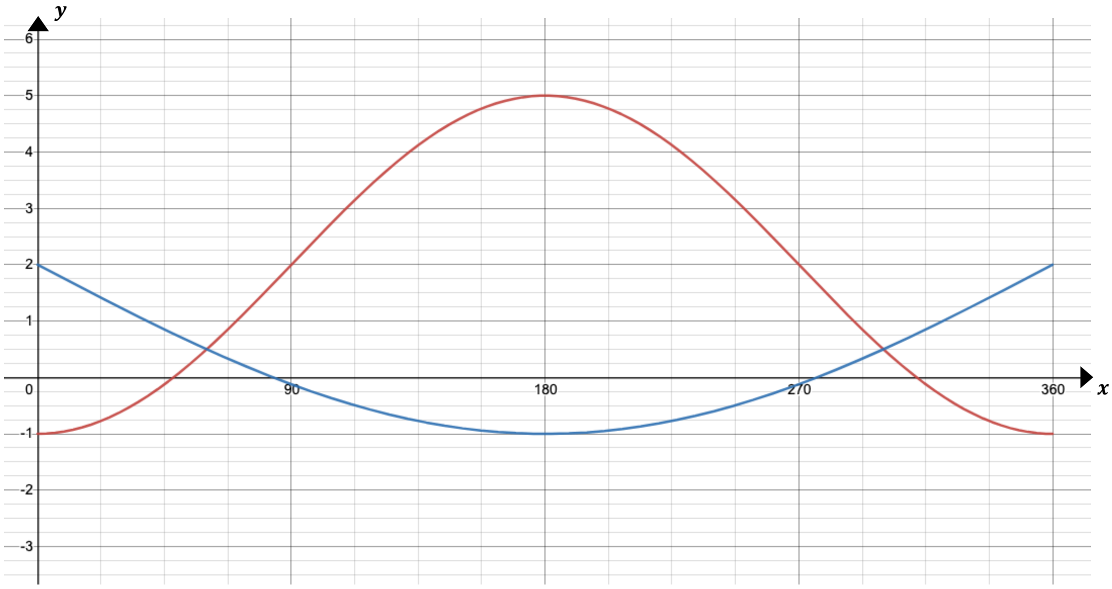
Find the values of a, b and c [3]

Solution

$a = -3, \quad b = 2, \quad c = 2$

(c) On the same diagram, sketch the graphs of $y = f(x)$ and $y = g(x)$ for $0° \le x \le 360°$ [4]

Solution



(d) State the number of solutions of

$$a\sin\left(\frac{x}{b}\right) + 3\cos x + c = 2$$

[1]

Solution

Number of solutions = 2

[S4 JSS P1/2020 PRELIM Qn 12]

[Total: 10 marks]

7. The equation of a curve is $y = x^3 + kx^2 - 3x + 1$, where k is a constant

(a) Find, in terms of k, the gradient of tangent to the curve at the point $x = 1$

[2]

Solution

$y = x^3 + kx^2 - 3x + 1$

$\frac{dy}{dx} = 3x^2 + 2kx - 3$

$\left.\frac{dy}{dx}\right|_{x=1} = 3(1)^2 + 2k(1) - 3$

$= 2k$

(b) Explain why the curve has two stationary points, for any real values of k

[3]

Solution

Since we are dealing with stationary points, $\frac{dy}{dx} = 0$

$3x^2 + 2kx - 3 = 0$

To explain, we shall use discriminant

$b^2 - 4ac = (2k)^2 - 4(3)(-3)$

$= 4k^2 + 36$

$= 4(k^2 + 9)$

Since k^2 is always positive, $4(k^2 + 9) > 0$. Since the discriminant is greater than 0, we know that there are 2 solutions to the equation, hence 2 stationary points

(c) If $k = -4$, find the range of values of x for which y is a decreasing function

[2]

Solution

Since y is a decreasing function, $\frac{dy}{dx} < 0$

$3x^2 - 8x - 3 < 0$

$(3x + 1)(x - 3) < 0$

$-\frac{1}{3} < x < 3$

[S4 SST P1/2020 PRELIM Qn 5]

[**Total: 7 marks**]

8. In the following expansion, where m is a positive constant, the term independent of x is 61236

$$\left(x^2 + \frac{m}{x}\right)^9$$

(a) Show that $m = 3$

[4]

Solution

We first find the general $(r+1)$ term

$$(r+1) \text{ term} = \binom{9}{r}(x^2)^{9-r}\left(\frac{m}{x}\right)^r$$
$$= \binom{9}{r} m^r (x^{18-3r})$$

For the independent term of x,

$x^{18-3r} = x^0$

$18 - 3r = 0$

$\text{r} = 6$

$\binom{9}{6} m^6 = 61236$

$m = 3 \text{ (shown)}$

(b) Hence or otherwise, find the coefficient of x^6 in the expansion of

$$(2 - 5x^3)\left(x^2 + \frac{3}{x}\right)^9$$

[4]

Solution

To find the coefficient of x^6, we need the x^3 and x^6 terms

When $r = 5, \binom{9}{5}(x^2)^{9-5}\left(\frac{3}{x}\right)^5 = 30618x^3$

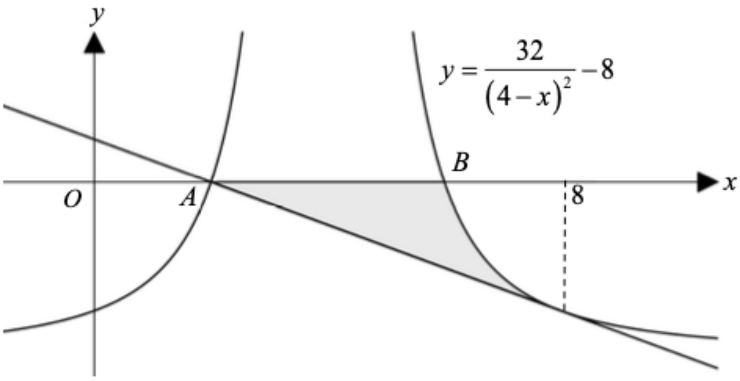
When $r = 4, \binom{9}{5}(x^2)^{9-4}\left(\frac{3}{x}\right)^4 = 10206x^6$

Coefficient of $x^6 = 2(10206x^6) - (5x^3)(30618x^3)$
$= -132678x^3$

[S4 PCSS P1/2020 PRELIM Qn 6]

[**Total: 8 marks**]

9.



The diagram shows part of the curve, intersecting the x-axis at A and B

$$y = \frac{32}{(4-x)^2} - 8$$

The tangent to the curve at $x = 8$ meets the x-axis at A

(a) Find the equation of the tangent

[4]

Solution

$$y = \frac{32}{(4-x)^2} - 8$$

When $x = 8$,

$$y = \frac{32}{(4-8)^2} - 8$$
$$= -6$$

$$\frac{dy}{dx} = \frac{64}{(4-x)^3}$$
$$\left.\frac{dy}{dx}\right|_{x=8} = \frac{64}{(4-8)^2}$$
$$= -1$$

Equation of tangent:

$$y - (-6) = -1(x - 8)$$
$$y = -x + 2$$

(b) Find the area of the shaded region

[6]

Solution

We first find the coordinates of A and B

At A & B, $y = 0$

$$0 = \frac{32}{(4-x)^2} - 8$$

$$x = 2 \quad \text{or} \quad x = 6$$

$$\therefore A(2, 0) \text{ and } B(6, 0)$$

Area of shaded region = Area of triangle − Area under the curve

$$= \frac{1}{2}(8-2)(6) - \left| \int_6^8 \frac{32}{(4-x)^2} - 8 \, dx \right|$$

$$= 18 - \left| \left[\frac{32}{(4-x)} - 8x \right]_6^8 \right|$$

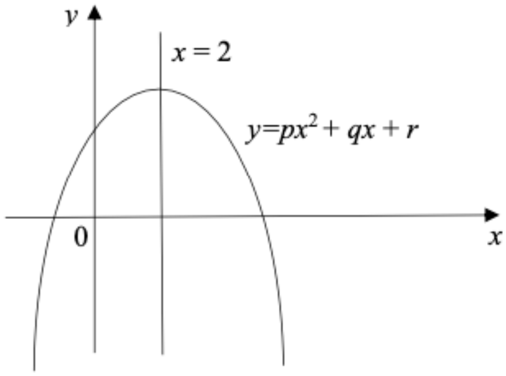
$$= 18 - \left| \left(\frac{32}{(4-8)} - 8(8) \right) - \left(\frac{32}{(4-6)} - 8(6) \right) \right|$$

$$= 10 \text{ units}^2$$

[S4 NHHS P1/2020 PRELIM Qn 10]

[Total: 10 marks]

10.



The diagram above shows part of the graph of

$$y = px^2 + qx + r$$

The equation of the line of symmetry of the curve is $x = 2$. Determine the conditions for each of the following expressions, justifying your answer

(a)

$$q^2 - 4pr$$

[2]

Solution

Since the curve cuts the $x -$ axis at two real and distinct points, $q^2 - 4pr > 0$

(b)

$$\frac{dy}{dx} \text{ when } x = 2$$

[2]

Solution

Since the line of symmetry is $x = 2$, the point where $x = 2$ is a stationary point

$$\frac{dy}{dx} = 0$$

(c)

$\boldsymbol{q}$

[**2**]

Solution

$$\frac{dy}{dx} = 2px + q$$

When $x = 2$, $\frac{dy}{dx} = 0$

$2p(2) + q = 0$

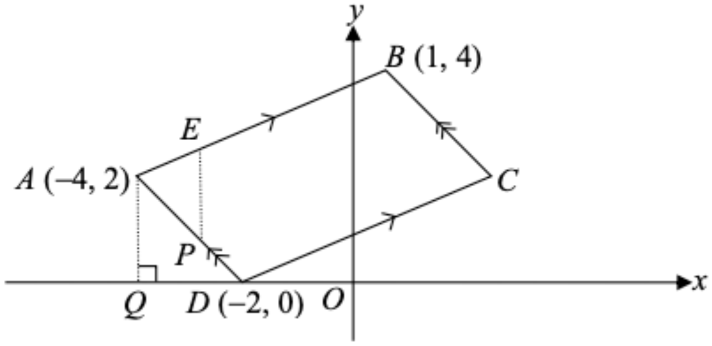
$q = -4p$

Since the curve has a maximum point, $p < 0$. This means that $q > 0$

[S4 MGS P1/2020 PRELIM Qn 10]

[**Total: 6 marks**]

11. Solutions to this question by accurate drawing will not be accepted

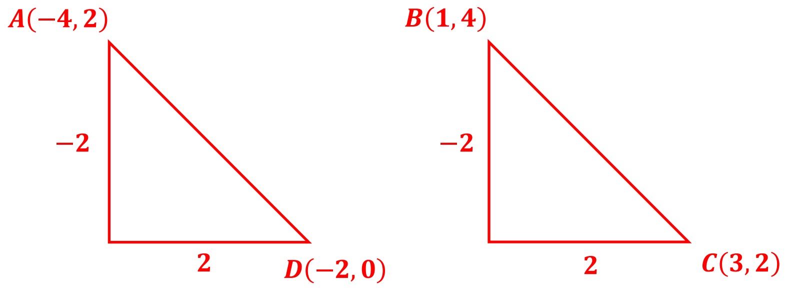


$ABCD$ is a parallelogram where A is $(-4, 2)$, B is $(1, 4)$ and D is $(-2, 0)$. The point E lies on AB such that $AE : EB = 2 : 5$. Lines are drawn, parallel to the y-axis, from A to meet the x-axis at Q and from E to meet AD at P

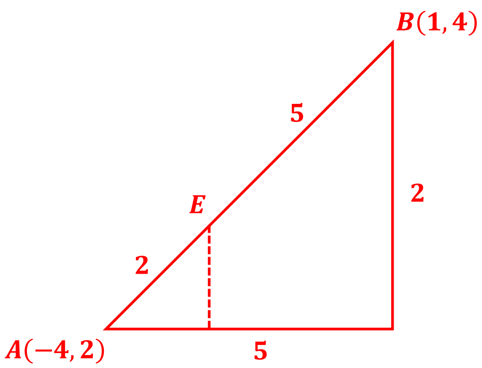
(a) Calculate the coordinates of C and of E

[4]

Solution



By observations, $C(3, 2)$



By observations,

$$E = \left(\left[1 - \frac{5}{7}(5)\right], \left[4 - \frac{5}{7}(2)\right]\right)$$

$$= \left(-2\frac{4}{7}, 2\frac{4}{7}\right)$$

(b) Find the equation of AD and calculate the coordinates of P

[3]

Solution

$$m_{AD} = \frac{2-0}{-4+2}$$
$$= -1$$

$$\therefore y = -(x-(-2))$$
$$y = -x-2$$

At $P, x = -2\frac{4}{7}$

$$\therefore y = -\left(-2\frac{4}{7}\right) - 2$$
$$= \frac{4}{7}$$

$$P\left(-2\frac{4}{7}, \frac{4}{7}\right)$$

(c) Explain why $AEPQ$ is a parallelogram and calculate its area

[3]

Solution

$$m_{PQ} = \frac{\left(\frac{4}{7}\right)}{\left(-2\frac{4}{7}-(-4)\right)} = \frac{2}{5}$$

$$m_{AE} = \frac{\left(2\frac{4}{7}-2\right)}{\left(-2\frac{4}{7}-(-4)\right)} = \frac{2}{5}$$

Since $m_{AE} = m_{PQ}$, the lines are parallel, $AEPQ$ is a parallelogram

$$\text{Area of } AEPQ = AQ \times \text{height}$$
$$= 2\left(4 - 2\frac{4}{7}\right)$$
$$= 2\frac{6}{7} \text{ units}^2$$

[S4 MSS P1/2020 PRELIM Qn 12]

[**Total**: **10 marks**]

12. The acute angles A and B are such that

$$\tan(A+B) = 8$$
$$\tan A = \frac{1}{5}$$

Without using a calculator, find the exact value of

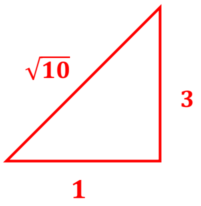
$$\cos B$$

[5]

Solution

$$\tan(A+B) = 8$$
$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 8$$
$$\frac{\frac{1}{5} + \tan B}{1 - \left(\frac{1}{5}\right)\tan B} = 8$$
$$\frac{1}{5} + \tan B = 8 - \frac{8}{5}\tan B$$
$$\frac{13}{5}\tan B = \frac{39}{5}$$
$$\tan B = 3$$

	$\sin y$	$\cos y$	$\tan y$
x	$\frac{3}{\sqrt{10}}$	$\frac{1}{\sqrt{10}}$	3



$$\cos B = \frac{1}{\sqrt{10}}$$

[S4 JWSS P1/2020 PRELIM Qn 2]

[Total: 5 marks]

13. A company manufactures aluminium beverage cans and the design of the can is modelled using a right cylindrical can of radius r cm and height h cm. Each can needs to hold at least $355\ \text{cm}^3$ of liquid. The company would like to minimise the area of the aluminium sheet used for each can. Using suitable calculations, advise the company of the dimensions of the can.

[Hint: State the appropriate height and radius of the can]

[9]

Solution

$V_{\text{can}}: \pi r^2 h = 355 \ldots\ldots\ldots (1)$

$SA_{\text{can}} = 2\pi r^2 + 2\pi r h$

$\frac{dA}{dr} = 4\pi r + 2\pi h$

Since the surface area is minimum, $\frac{dA}{dr} = 0$

$4\pi r + 2\pi h = 0$

$r = -\frac{1}{2}h \ldots\ldots\ldots (2)$

Substitute (2) into (1),

$\pi\left(-\frac{1}{2}h\right)^2 h = 355$

$h = \sqrt[3]{\frac{355}{\pi\left(-\frac{1}{2}\right)^2}}$

$= 7.67443\ldots$

$= 7.67\ \text{cm}\ (3.\text{s.f.})$

Substitute $h = \sqrt[3]{\frac{355}{\pi\left(-\frac{1}{2}\right)^2}}$ **into Equation (1),**

$\pi r^2 h = 355$

$r = \sqrt{\frac{355}{\pi\left(\sqrt[3]{\frac{355}{\pi\left(-\frac{1}{2}\right)^2}}\right)}}$

$= 3.837215\ldots$

$= 3.84\ \text{cm}\ (3.\text{s.f.})\ (\text{rej} - \text{ve})$

[S4 NGHS P1/2020 PRELIM Qn 12]

[**Total: 9 marks**]

End of Paper ☺