Name:	Level/Subject: **4049 Sec 4 A-Math**
Material: **February Practice Questions 2022**	Centre: **Overmugged**

Instructions

- Answer all questions
- If working is needed for any question it must be shown with the answer
- Omission of essential working will result in loss of marks
- You are expected to use a scientific calculator to evaluate explicit numerical expressions
- If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures
- Give answers in degrees to one decimal place
- For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π
- A copy of the formula list is provided for you on the next page

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Question Source

All questions are sourced and selected based on the known abilities of students sitting for the 'O' Level A-Math Examination. All questions compiled here are from **2009 - 2021 School Mid-Year / Prelim Papers**. Questions are categorised into the various topics and range in varying difficulties. If questions are sourced from respective sources, credit will be given when appropriate.

How to read:

[S4 ABCSS P1/2011 PRELIM Qn 1]

Secondary 4, ABC Secondary School, Paper 1, 2011, Prelim, Question 1

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This question paper consists of 32 printed pages including the cover page

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List of Mathematical Formulae

1. ALGEBRA

Quadratic Equation
For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + ... + \binom{n}{r} a^{n-r}b^r + ... + b^n$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Quadratic Equations & Inequalities

1. (a) Find the range of values of p for which the expression is never negative [4]

$$x^2 - 2px + 2p^2 - \frac{1}{4}(5p + 6)$$

(b) (i) Express $-x^2 + 12x - 37$ in the form $k(x + a)^2 + b$ [2]

Hence,

(ii) Sketch the curve [2]

$$y = -x^2 + 12x - 37$$

(iii) State the range of y [1]

Credit: **S4 CHIJ SNGS P1/2009 PRELIM Qn 12**

2. A quadratic equation is defined by [5]

$$(x - a)(b - x) = m$$

Given that the roots of the equation are equal, show that the value of m is

$$\left(\frac{a - b}{2}\right)^2$$

Credit: **S4 CGS P1/2011 PRELIM Qn 6**

3. (a) Find the range of values of p for all real values of x [5]

$$px^2 + 4x + p > 3$$

(b) Find the range of values of k for which the line does not intersect the curve [5]

$$5y = k - x$$

$$5x^2 + 5xy + 4 = 0$$

Credit: **S4 CHIJ SNGS P1/2018 PRELIM Qn 8**

4. Given that curve $y = x^2$ lies above the line $y = px - q^2$ for $-2 < p < 2$. Find the value(s) of constant q [4]

Credit: **S4 CCHS(M) P1/2014 PRELIM Qn 2**

2 Surds

1. (a) Show that [3]
$$\left(1-\sqrt{a}\right)^5-\left(1+\sqrt{a}\right)^5=-10\sqrt{a}-20a\sqrt{a}-2a^2\sqrt{a}$$

   (b) Hence, deduce the exact value of [2]
$$\left(1-\sqrt{3}\right)^5-\left(1+\sqrt{3}\right)^5$$

   Credit: **S4 CGSS P1/2009 PRELIM Qn 10(a)**

2. Two geometrically similar cones are such that the ratio of their volumes is $1:2\sqrt{2}$. If the height of the smaller cone is $\dfrac{3+2\sqrt{2}}{\left(1-\sqrt{2}\right)^2}$ cm, find the **exact** height of the larger cone in the simplest form [4]

   Credit: **S4 CGSS P1/2011 PRELIM Qn 1**

3. (a) Express the following in the form $a+b\sqrt{5}$, where a and b are integers [4]
$$\left(\frac{4}{2+\sqrt{5}}-3-2\sqrt{5}\right)^2$$

   (b) By factorising $ab-4b+a-4$, solve the following [4]
$$6^x-4\left(3^x\right)+2^x-4=0$$

   Credit: **S4 HIHS P1/2014 PRELIM Qn 5 (MODIFIED)**

4. A prism has a square base of side of $\left(2+\sqrt{3}\right)$ m and its volume is $\left(11+6\sqrt{3}\right)$ m^3. Find, without using a calculator, the height of the prism in the form $\left(a-b\sqrt{3}\right)$ m, where a and b are integers [4]

   Credit: **S4 NHHS P1/2014 PRELIM Qn 2**

3 Polynomials

1. Given that $x^2 + x - 2$ is a factor of

$$f(x) = 3x^3 + ax^2 - bx - 10$$

(a) Find the values of a and b [4]

(b) Hence, factorise $f(x)$ completely [2]

(c) Find the remainder when $f(x)$ is divided by $(2x - 1)$ [2]

Credit: **S4 BNSS P1/2009 PRELIM Qn 6**

2. (a) The cubic polynomial $f(x)$ is such that the coefficient of x^3 IS 2 and the roots of the equation $f(x) = 0$ are -1, 3 and k. Given that $f(x)$ has a remainder of 20 when divided by $(x - 4)$,

(i) Show that $k = 2$ [2]

(ii) Hence, or otherwise, find the remainder when $f(x)$ is divided by $2x - 1$ [2]

(b) When the polynomial $x^{10} - px^3 + q$ is divided by $x^2 - 1$, the remainder is $4x + 3$. Find the values of p and q [4]

Credit: **S4 CCHS(M) P2/2011 PRELIM Qn 4**

3. The function, where a and b are constants, is exactly divisible by $(x - 2)$

$$f(x) = x^3 + ax^2 + bx + 4$$

Given that $f(x)$ leaves a remainder of -3 when divided by $(x + 1)$,

(a) find the value of a and of b [4]

(b) express $f(x)$ in the following form, where d is an integer [3]

$$f(x) = (x - 2)\left(x - 1 - \sqrt{d}\right)\left(x - 1 + \sqrt{d}\right)$$

Credit: **S4 CHIJ SNGS P2/2014 PRELIM Qn 4**

4. The polynomial, where a and b are constants, leaves a remainder of 10 when divided by $(2x-1)$

$$P(x) = 2x^3 + ax^2 + bx + 8$$

Given that $(x + 2)$ is a factor of $P(x)$,

(a) find the value of a and of b [5]

(b) Explain why the equation $P(x) = 0$ has only 1 real root. Hence, find this root [4]

Credit: **S4 CHS P2/2015 PRELIM Qn 11**

4 Partial Fractions

1. (a) Express the following in partial fractions [4]

$$\frac{13x-6}{2x^2+3x-9}$$

(b) Hence, or otherwise, evaluate the following integral [4]

$$\int \frac{17x-3}{2x^2+3x-9}\,dx$$

Credit: **S4 SSS P2/2009 PRELIM Qn 8(a)**

2. Express the following in partial fractions [5]

$$\frac{x^4-5x^3+6x^2-18}{x^3-3x^2}$$

Credit: **S4 VS P1/2009 PRELIM Qn 4**

3. Express the following in partial fractions [5]

$$\frac{x-4}{(2x-1)(x+1)^2}$$

Credit: **S4 SPSS P1/2012 PRELIM Qn 2**

4. (a) Express the following in partial fractions [6]

$$\frac{2x^2-3x+1}{9x^3-6x^2+x}$$

(b) Hence, find [3]

$$\int \frac{2x^2-3x+1}{9x^3-6x^2+x}\,dx$$

Credit: **S4 CGS P2/2014 PRELIM Qn 5**

5 Binomial Theorem

1. (a) Find, in ascending powers of x, the first three terms in the expansion of [2]

$$(1+px)^6$$

(b) Given that the first two non-zero terms in the expansion of the following are 1 and $-\frac{7}{3}x^2$ [3]

$$(1+px)^6(1+qx)$$

Find the possible values of p and q

Credit: **S4 AMKSS P1/2009 PRELIM Qn 10**

2. (a) Given that the coefficients of x^3 and x^6 in the expansion of $\left(x^2-\frac{3}{x}\right)^9$ are p and q respectively, find the value of $\frac{p}{q}$ [4]

(b) (i) Find the first 3 terms in the expansion of the following, in ascending powers of x, [2]

$$\left(2+\frac{x}{2}\right)^5$$

(ii) Given that the coefficient of x^2 in the expansion of the following is -12, find the values of the constant of k [3]

$$(1-kx)^2\left(2+\frac{x}{2}\right)^5$$

Credit: **S4 SCGS P1/2010 PRELIM Qn 6**

3. (a) Find the first five terms of the binomial expansion in ascending powers of x [1]

$$\left(1-\frac{x}{2}\right)^9$$

Hence,

(b) find the value of a given that the coefficient of x^2 in the expansion of the following is $54\frac{3}{8}$ [2]

$$\left(4-\frac{1}{x}+\frac{a}{x^2}\right)\left(1-\frac{x}{2}\right)^9$$

(c) Expand the following, up to and including the term in x^3 [3]

$$\left(1-\frac{1}{2}x-x^2\right)^9$$

Credit: **S4 ANDSS P1/2012 PRELIM Qn 5**

4. (a) Write down the first three terms in the expansion, in ascending powers of x, of the following, where n is a positive integer greater than 2 [2]

$$\left(1-\frac{x}{3}\right)^n$$

(b) Find, in terms of n and p, the first three terms in the expansion, in ascending powers of x, where p is a constant, of [2]

$$\left(2+px+\frac{5}{2}x^2\right)\left(1-\frac{x}{3}\right)^n$$

Given that in the following expansion, in ascending powers of x, the first three terms are

$$\left(2+px+\frac{5}{2}x^2\right)\left(1-\frac{x}{3}\right)^n = 2+\frac{31p}{3}x+\frac{25}{3}x^2+...$$

(c) find the value of n and of p [3]

(d) Hence, find the coefficient of x^3 in the expansion of [3]

$$\left(2+px+\frac{5}{2}x^2\right)\left(1-\frac{x}{3}\right)^n$$

Credit: **S4 PLMGS P2/2014 PRELIM Qn 7**

6 Exponential & Logarithms

1. A man buys an apartment during the economic crisis. The value of the apartment decreases with time so that its value, $\$V$, after t months, where k is a constant is given by

$$V = 45000e^{-kt}$$

The value of the apartment is expected to be \$36300 after 11 months

(a) Find the value of the apartment when the man bought it [1]

(b) Calculate the value, to the nearest \$100, of the apartment after 9 months [3]

The man decided to sell off the apartment when it reached $\frac{2}{3}$ of its original value

(c) Calculate the length of time, to the nearest month, before the apartment is sold off [2]

Credit: **S4 ASS P2/2009 PRELIM Qn 1**

2. (a) Sketch the graph of the following, for $x > -3$,

$$y = \ln(x + 3)$$

Indicate the x-intercept and y-intercept clearly

(b) Insert on your sketch the straight line graph required to illustrate how a graphical solution of the following may be obtain. Show all appropriate working

$$ex = e^{4-3x} - 3e$$

Credit: **S4 SCSS P1/2009 PRELIM Qn 6**

3. (a) Solve the equation, giving your answer correct to 2 decimal places [4]

$$5^{x+2} - 25^{x+\frac{1}{2}} = 2\left(5^{x+1}\right)$$

(b) Solve the simultaneous equations [6]

$$64^x \div 8^y = 32$$

$$27^{2x}\left(\frac{1}{\sqrt{3}}\right)^{y+1} = 9\sqrt{3}$$

Credit: **S4 CHIJ TP P2/2011 PRELIM Qn 4**

4. (a) Express p in terms of q [3]

$$\log_x \frac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x (p-q)$$

(b) Given that $a = \log_2 3$ and $b = \log_2 7$, express the following in terms of a and b [4]

$$\log_2 21 + \log_4 \frac{16}{7}$$

(c) **Without using the calculator**, simplify the following [3]

$$\frac{(\sqrt[10]{x}+1)\left(x^{\frac{21}{10}} - x^2\right)}{\sqrt[5]{x}-1}$$

Credit: **S4 AHS P2/2014 PRELIM Qn 6**

7 Trigonometry

1. (a) Prove that [1]

$$\tan(\theta - 45^\circ) = \frac{\tan\theta - 1}{1 + \tan\theta}$$

(b) Hence, show that [3]

$$\cot 15^\circ = 2 + \sqrt{3}$$

Credit: **S4 SGSS P1/2009 PRELIM Qn 4**

2. (a) (i) Show that [2]

$$1 + 4\sin^2 x = 3 - 2\cos 2x$$

(ii) State the period and amplitude of [1]

$$y = 1 + 4\sin^2 x$$

(b) Sketch the graph of the following, for $-\pi \leq x \leq \pi$ [3]

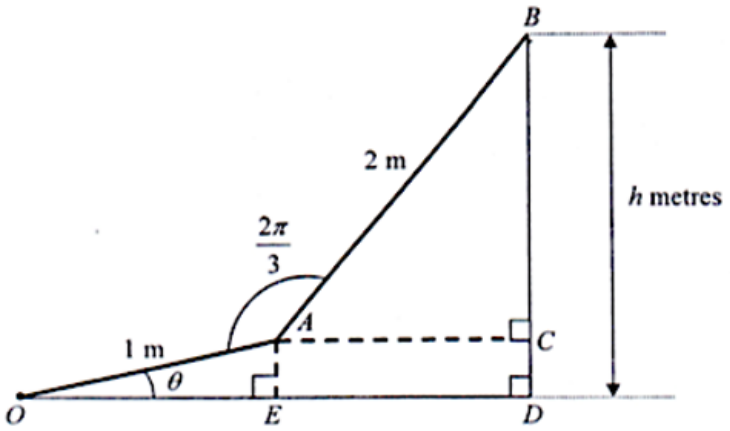
$$y = 1 + 4\sin^2 x$$

(c) On the same axes, draw a suitable straight line to find the number of solutions to the equation. State the number of solutions [3]

$$\pi \cos 2x = x$$

Credit: **S4 ANDSS P2/2011 PRELIM Qn 5**

3. The diagram shows the bent rod OAB which is formed from a piece of three-metre long steel bar. The bent rod lies in a vertical plane with $OA = 1$ m, $AB = 2$ m and angle OAB is fixed at $\frac{2\pi}{3}$ radians. The bent rod is hinged at O and it is free to rotate in the vertical plane.



In the diagram, a typical position of the bent rod is shown where OA makes an angle of θ radians with the horizontal and $0 < \theta < \frac{\pi}{2}$. The vertical height BD of B above the level of O is h metres. Angle ACB is a right angle

(a) Show that

(i) $\angle BAC = \theta + \frac{\pi}{3}$ [1]

(ii) $h = \sin\theta + 2\sin\left(\theta + \frac{\pi}{3}\right)$ [2]

(b) Hence prove that [2]

$$h = 2\sin\theta + \sqrt{3}\cos\theta$$

(c) Express $2\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ [3]

(d) As θ varies from 0 to $\frac{\pi}{2}$, find the maximum value of h [1]

(e) Find the possible value(s) of θ for which $h = 2.5$ [3]

Credit: **S4 CGSS P2/2010 PRELIM Qn 10**

4. (a) Prove the identity [3]

$$\tan^4 x = \sec^2 x \tan^2 x - \sec^2 x + 1$$

(b) Solve, for $0^\circ \leq x \leq 360^\circ$ [5]

$$\cos^2 x + 3 \sin x \cos x + 1 = 0$$

(c) Given, for $0^\circ < \theta < 90^\circ$, that

$$\cos \theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(i) find the value of $\sin \theta$ in the form [2]

$$\frac{\sqrt{a-\sqrt{b}}}{a}$$

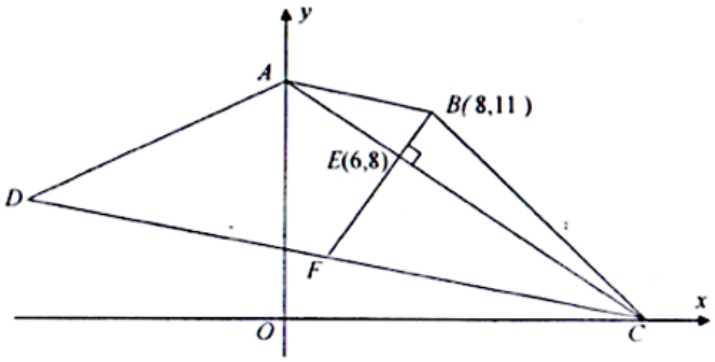
(ii) hence, prove that [3]

$$\tan \theta = \sqrt{7-4\sqrt{3}}$$

Credit: **S4 DSS P2/2014 PRELIM Qn 8**

8 Coordinate Geometry

1. The diagram, which is not drawn to scale, shows a trapezium $ABCD$ in which AB is parallel to DC. The points A and C lie on the y-axis and x-axis respectively. [3]



The point $B(8, 11)$ and $E(6, 8)$ is a point on AC such that BE is perpendicular to AC. BE produced meets DC at F and $BF = 3BE$

(a) Find

(i) the equation of AC [3]

(ii) the coordinates of A [1]

(iii) the coordinates of F [3]

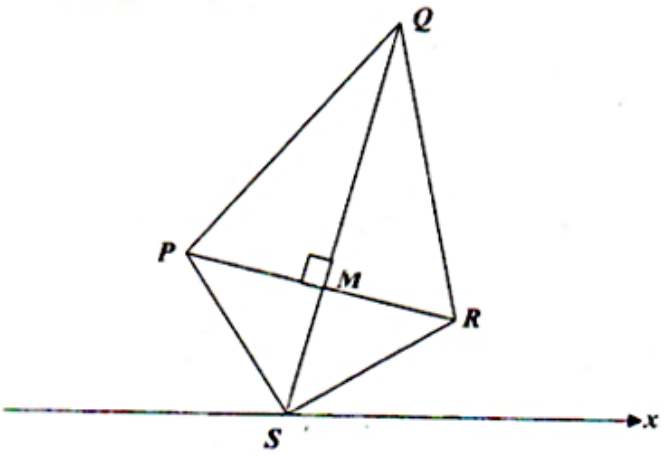
(b) Given that point P is $(4, 5)$, show that $\triangle ABP$ is isosceles. What is the special name given to quadrilateral $ABCP$ [3]

(c) Given further that $AB : DC = 1 : 4$ and F is a mid-point of DC, find the values of [1]

$$\frac{\text{Area of } \triangle ABC}{\text{Area of trapezium } ABCD}$$

Credit: **S4 BBSS P2/2009 PRELIM Qn 11**

2. The diagram shows a quadrilateral $PQRS$, where P is $(2, 4)$, M is $(5, 3)$ and S is a point on the x-axis



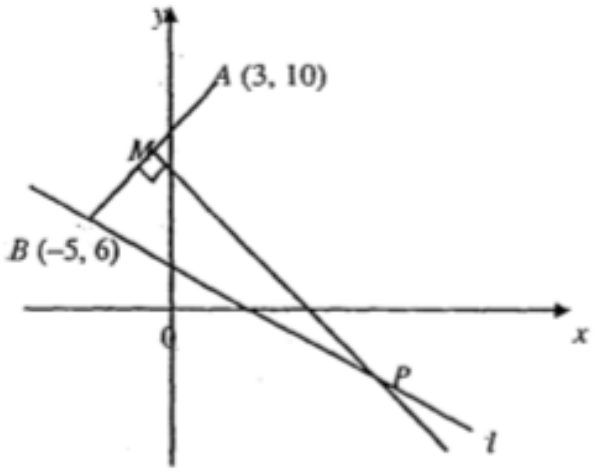
QS bisects PR at M. The equation of PQ is given by $y = x + 2$. Find

(a) the coordinates of R, S and Q [7]

(b) the area of $PQRS$ [2]

Credit: **S4 NVSS P1/2012 PRELIM Qn 10**

3. In the diagram, M is the midpoint of the line joining the points $A(3, 10)$ and $B(-5, 6)$ [6]

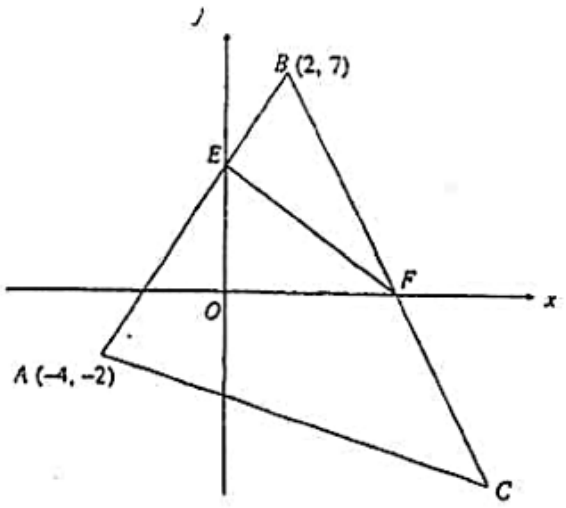


The perpendicular bisector of AB intersects the line l at the point P. Given that the line l is parallel to the line $6y + 7x = 0$, find the coordinates of P

Credit: **S4 FMS(S) P1/2014 PRELIM Qn 6**

4. **Solution to this question by accurate drawing will not be accepted**

The diagram shows a triangle ABC where $A(-4, -2)$, $B(2, 7)$ and BC is parallel to the line $2y = -4x + 1$



BC cuts the x-axis at F and AB cuts the y-axis at E

(a) Find the equation of the line BC [2]

(b) Determine whether if EF is perpendicular to AB [3]

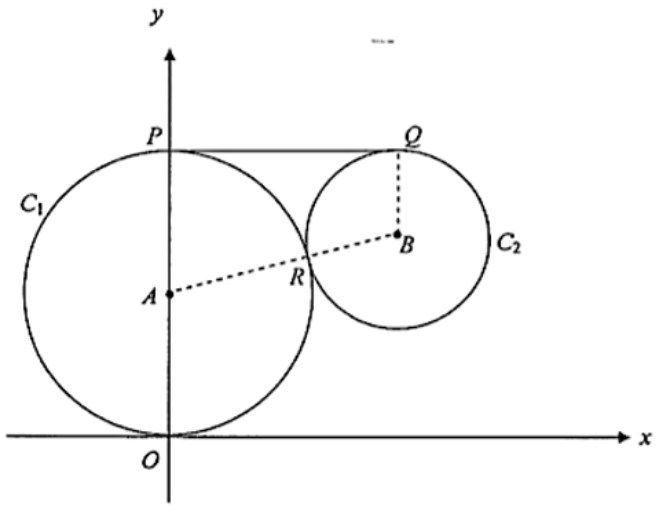
(c) Given that C is equidistant from A and E, find the coordinates of C [3]

(d) Find the area of $\triangle AEC$ [3]

Credit: **S4 ANDSS P2/2015 PRELIM Qn 11**

9 Further Coordinate Geometry

1. The diagram shows two circles C_1 and C_2 centred at A and B respectively



C_1 passes through O and P and touches C_2 at R. Q is on C_2 such that QB is parallel to the y-axis. PQ has length $2\sqrt{35}$ units and is tangent to both circles.

Given that the equation of C_1 is

$$x^2 + y^2 - 14y = 0$$

find

(a) the centre and radius of C_1 [3]

(b) the equation of C_2 [5]

(c) the equation of the perpendicular bisector of AB [4]

Credit: **S4 BSS P2/2009 PRELIM Qn 12**

2. The line $y = -4$ is a tangent to circle C, with centre $(2, -1)$

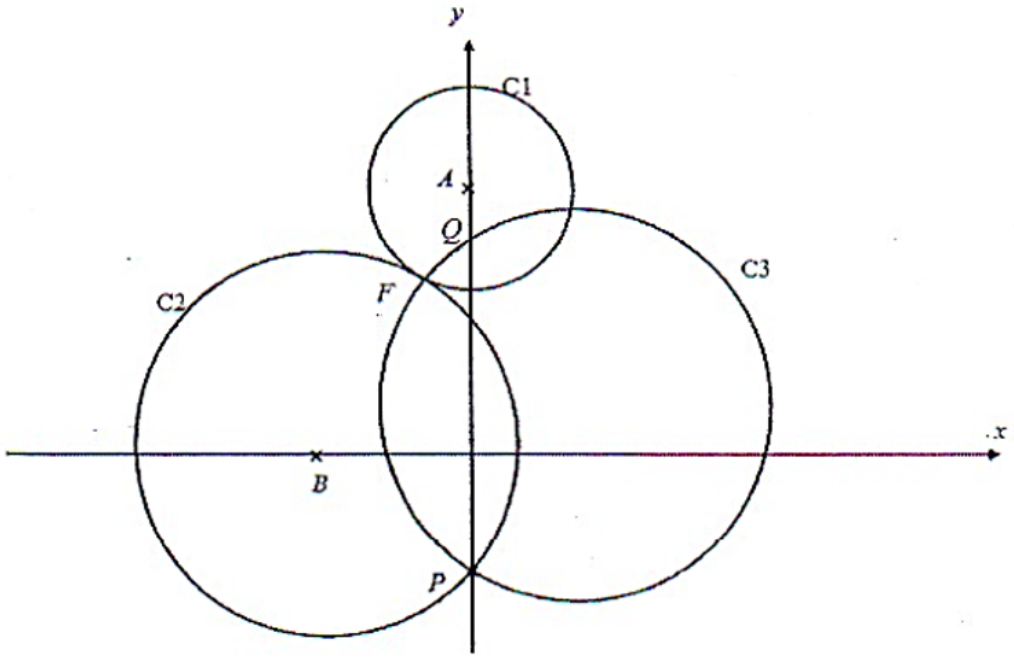
(a) Find the radius of circle C and hence, express the equation of circle C in the form $x^2 + y^2 + hx + ky + l = 0$ where h, k and l are constants [3]

(b) The line $y = 5x + 1$ cuts the circle C at the points P and Q. Find the perpendicular bisector of PQ [2]

(c) Find the centre of the circle, which is a reflection of C in the line $x = -3$ [1]

Credit: **S4 SCGS P1/2011 PRELIM Qn 8**

3. The diagram shows three circles C_1, C_2 and C_3



All three circles pass through the point $F\left(-\frac{1}{2}, 1\frac{1}{2}\right)$. Circle C_1 has its centre at A where A is on the y-axis. Circle C_2 has its centre at $B\left(-1\frac{1}{2}, 0\right)$. The line joining the points A, F and B is a straight line such that $AF : FB = 1 : 2$

(a) Find the coordinates of A [1]

(b) Find the equation of the circle C_2 [3]

Circle C_3 intersects circle C_2 at $P(0, -1)$ and cuts the y-axis at point $Q(0, 2)$

(c) Find the equation of the perpendicular bisector of PF [4]

(d) Hence, find the equation of the circle C_3 [3]

Credit: **S4 CHS P2/2011 PRELIM Qn 11**

4. A straight line passing through $(-4, 0)$ intersects the circle, C_1 and passes through the point $P(-2, 2)$. Given that C_1 has an equation of

$$x^2 + y^2 + 3x - y = 0$$

(a) find the coordinates of Q at which the line intersects the circle again [4]

(b) find the equation of the perpendicular bisector of PQ [3]

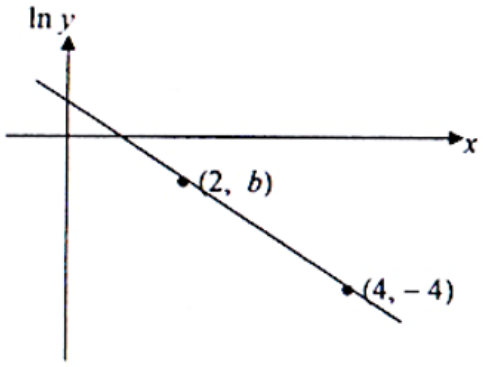
(c) find the equation of a second circle, C_2, that has the same radius, and intersects C_1 at the points P and Q [4]

Credit: **S4 MGS P1/2011 PRELIM Qn 12**

10 Linear Law

1. The variables x and y are connected by the equation, where a is a constant

$$y^2 = e^{-ax+4}$$



The diagram above shows the straight line graph, passing through the points $(2, b)$ and $(4, -4)$, obtained by plotting $\ln y$ against x

(a) Calculate the value of a and of b [5]

(b) Hence find the value of y when $x = 2$ [2]

Credit: **S4 HSS P1/2009 PRELIM Qn 13**

2. **Answer the whole of this question on a sheet of graph paper**

The table shows experimental values of two variables x and y

x	2	3	4	5	6	7.5
y	5.66	10.4	16	22.4	29.4	41.1

It is known that x and y are related by the equation, where a and b are constants

$$y = ax^{b+1}$$

(a) On graph paper, plot $\lg y$ against $\lg x$, using a scale of 2 cm to represent 0.1 unit on the $\lg x$ axis and 1 cm to represent 0.1 unit on the $\lg y$ axis. Draw a straight line graph to represent the equation [4]

$$y = ax^{b+1}$$

(b) Use your graph to estimate the value of a and of b [4]

(c) On the same diagram, draw the line representing $y = x^2$ and hence find the value of x for which [4]

$$x^{1-b} = a$$

Credit: **S4 VS P1/2010 PRELIM Qn 4**

3. The table below shows the experimental values of x and y which are known to be related by the equation, where p and q are constants

$$y = p\,(x+5)^{\frac{3}{2}} - q\sqrt{x+5}$$

x	0.5	1	1.5	2	2.5
y	24.6	28.2	31.9	35.7	39.7

(a) On graph paper, plot $\dfrac{y}{\sqrt{x+5}}$ against x and draw a straight line graph. The vertical $\dfrac{y}{\sqrt{x+5}}$-axis should start at 9 and have a scale of 2 cm to 1 [3]

(b) Use the graph to estimate the value of p and q [4]

(c) By plotting another suitable straight line on the same axes, solve graphically the equation [3]

$$p\,(x+5)^{\frac{3}{2}} = \sqrt{x+5}\,(x+10+q)$$

Credit: **S4 TKSS P1/2014 PRELIM Qn 11**

4. Variables x and y are related by the following equation, where p and q are constants

$$y = \frac{p-x}{x+q}$$

When the graph of $x(1+y)$ against y is drawn, a straight line is obtained. The line has a gradient of $-1\dfrac{1}{3}$ and passes through the point $(3, 2)$

(a) Calculate the value of p and of q [4]

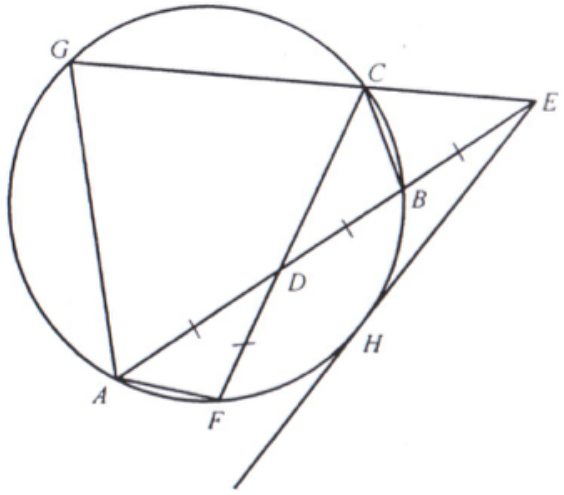
(b) Given that this line passes through $(6, k)$, find x in terms of k [2]

Credit: **S4 ACS(I) P2/2015 PRELIM Qn 3**

11 Proofs of Plane Geometry

Note: Certain parts are labelled with "**Not in Syllabus**". These questions are still doable with the 4049 syllabus, but can be solved easier with some older concepts no longer taught

1. A circle passes through 6 points A, F, H, B, C and G, as shown in the diagram.



D and B are 2 points on AE such that $AD = DB = BE = DF$. GCE and FDC are straight lines

(a) Prove that $\triangle ADF$ is congruent to $\triangle CDB$ [3]

(b) Prove that $\triangle GEA$ is similar to $\triangle CEB$ [3]

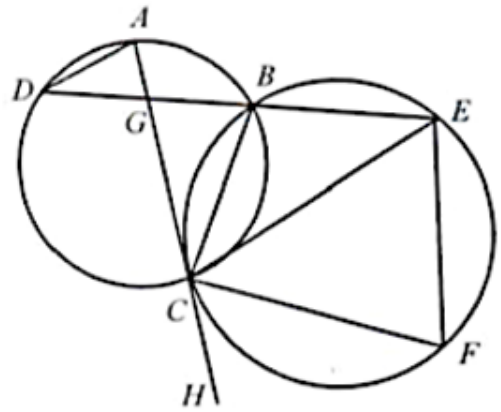
(c) Find the ratio $GA : AF$ [1]

(d) **Not in syllabus** [1]
Given that EH is a tangent to the circle at H, prove that

$$EH^2 = 3EB^2$$

Credit: **S4 BGSS P2/2009 PRELIM Qn 7**

2. In the diagram, A, B, C, D, E and F are points on the circles.



AH is a tangent to the larger circle at C. ACH and DBE are straight lies. DB meets AC at G and BC is the angle bisector of $\angle ACE$. Prove that

(a) $\triangle EGC$ is similar to $\triangle CGB$ [2]

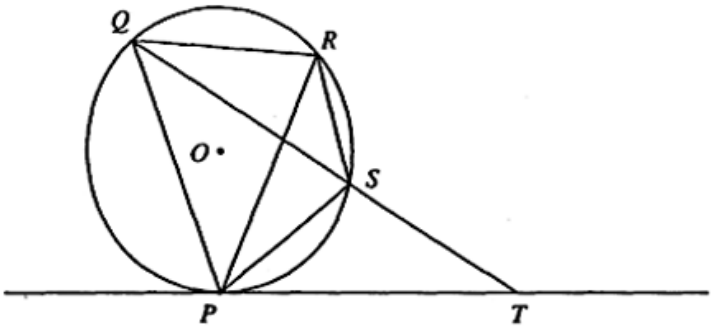
(b) $BC = BE$ [3]

(c) **Not in syllabus** [3]
$GC^2 - GB^2 = GB \times BC$

(d) **Not in syllabus** [3]
$\left(\frac{DG}{AG}\right)^2 = \frac{GE}{GB}$

Credit: **S4 VS P2/2011 PRELIM Qn 10**

3. In the figure, $RS = PS$ and the line PT is tangent to the circle



Line QT bisects angle PQR and cuts the circle at point S. Prove that

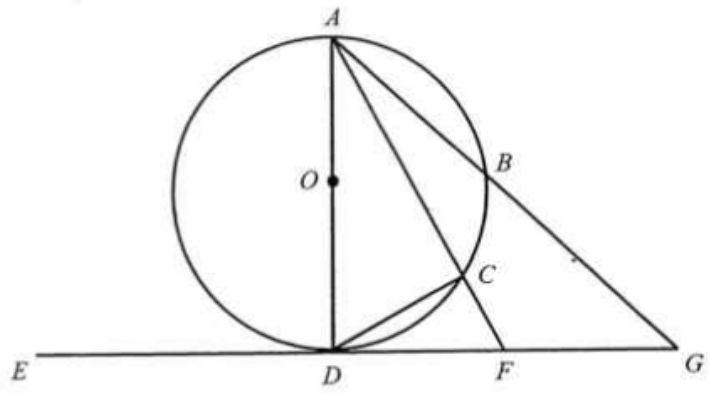
(a) $\angle TPS = \angle SPR$ [2]

(b) $\triangle SPT$ is similar to $\triangle PQT$ [2]

(c) $PT \times PQ = QT \times RS$ [4]

Credit: **S4 GMS(S) P2/2014 PRELIM Qn 5**

4. In the diagram, A, B, C and D are points on the circumference of the circle with centre O. $EDFG$ is a tangent to the circle at D



Given that $AB = BG$ and $DF = FG$, prove that

(a) ABD is an isosceles triangle [3]

(b) $DB^2 - BF^2 = \frac{1}{4}AD^2$ [2]

(c) $\triangle ADF$ is similar to $\triangle DCF$ [2]

(d) $GF^2 = AF \times CF$ [2]

Credit: **S4 PLMGS P2/2015 PRELIM Qn 10**

12 Differentiation

1. It is given that [5]

$$y = \frac{x+1}{(2x-5)^3}$$

(a) Show that [2]

$$\frac{dy}{dx} = \frac{-4x-11}{(2x-5)^4}$$

(b) Determine the values of x for which y is **not** an increasing function [2]

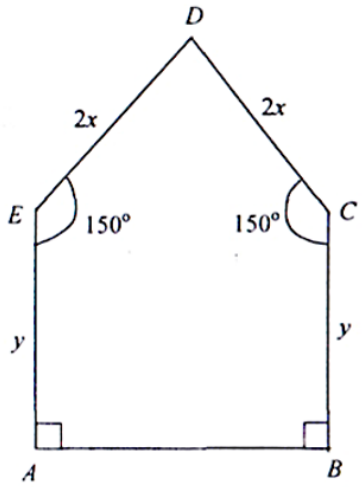
The variables x and y are such that, when $x = 3$, y is increasing at a rate of 46 units per second

(c) Find the rate of decrease of x when $x = 3$ [3]

(d) Given that the variable z is such that $z = y^3$, find the rate of change of z when $x = 3$ [3]

Credit: **S4 MFSS P2/2017 PRELIM Qn 6**

2. The diagram shows a piece of wire of length 4 m, bent to form a pentagonal frame $ABCDE$, where $CD = DE = 2x$, $BC = AE = y$ and angle AED = angle $BCD = 150°$



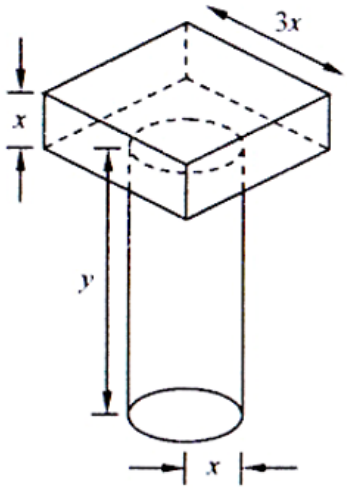
(a) Express y in terms of x and show that the area of the frame, A cm^2 is given by [4]

$$A = 4x + \left(\sqrt{3} - 6\right)x^2$$

(b) Find the exact value of x for which the area of the frame is a maximum and hence find the maximum value of A [5]

Credit: **S4 FSS P1/2009 PRELIM Qn 13**

3. The diagram shows a solid rivet which consists of a cylinder fixed to a cuboid. The cylinder has a radius of x mm and height of y mm. The cuboid has a square base of side $3x$ mm and a height of x mm



(a) Given that the total volume of the rivet is 120 mm^2, express y in terms of x [2]

(b) Show that the total surface area, A mm^2, of the rivet is given by [2]

$$A = \frac{240}{x} + 12x^2$$

Given that x can vary

(c) find the stationary value of A [3]

(d) determine whether the stationary value of A is a maximum of minimum [2]

Credit: **S4 WRSS P1/2009 PRELIM Qn 13**

4. (a) Find the value of a and of b for which [4]

$$\frac{d}{dx}\left(\frac{\sin x}{2\tan x + \cos x}\right) = \frac{a + b\sin x \tan^2 x}{(2\tan x + \cos x)^2}$$

(b) Given that [4]

$$y = (1 + x)e^{3x}$$

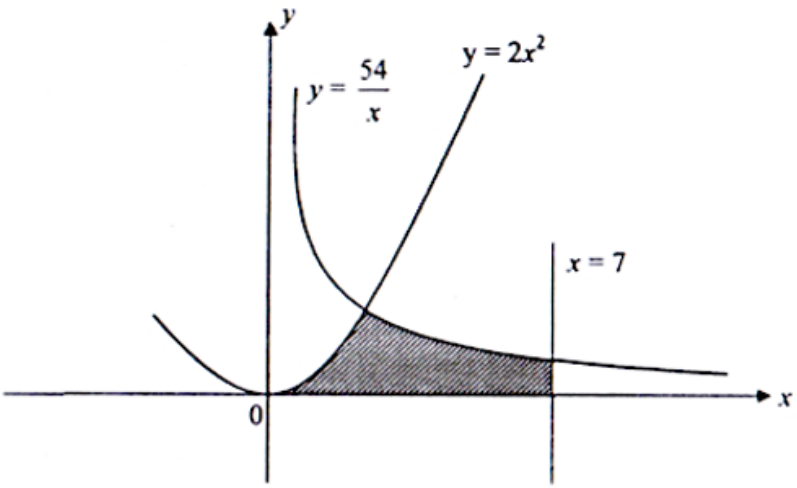
Prove that

$$6\left(\frac{dy}{dx}\right) = 9y + \frac{d^2y}{dx^2}$$

Credit: **S4 PLMGS P2/2011 PRELIM Qn 2**

13 Integration

1. The shaded region in the diagram is bounded by the line $x = 7$, curves $y = \dfrac{54}{x}$ and $y = 2x^2$ [5]



Calculate the area of the shaded region

Credit: **S4 BMSS P2/2009 PRELIM Qn 10(b)**

2. (a) Given that $\displaystyle\int_0^2 f(x)\,dx = 10$ and $\displaystyle\int_2^6 f(x)\,dx = 14$, find the value of [2]

$$\int_0^5 f(x)\,dx + \int_5^6 f(x)\,dx$$

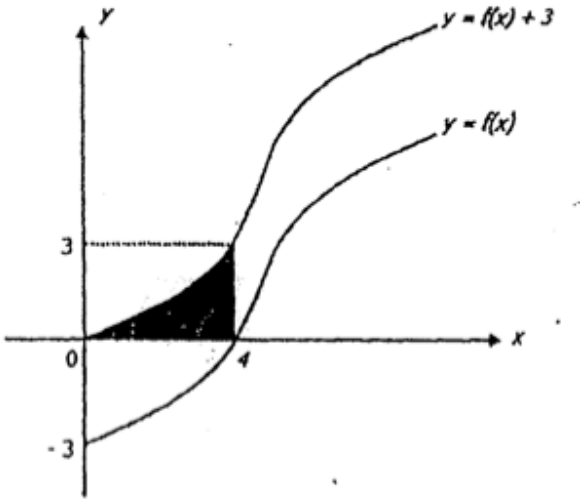
(b) Evaluate

(i) $\displaystyle\int \sqrt{2x+1}\,dx$ [1]

(ii) $\displaystyle\int \frac{2x^{\frac{1}{2}}}{x\sqrt{x}}\,dx$ [2]

Credit: **S4 BSS P1/2010 PRELIM Qn 4**

3. The diagram below shows the graphs of $y = f(x)$ and $y = f(x) + 3$.



Given that

$$\int_0^8 f(x)\ dx = 16$$

$$\int_0^4 f(x)\ dx = -7$$

Evaluate

(a) $\int_4^8 f(x)\ dx$ [1]

(b) the area of the shaded region [2]

Credit: **S4 SJI P1/2014 PRELIM Qn 3**

4. It is given that, where n is a constant,

$$f'(x) = \frac{8}{(2x+1)^n}$$

(a) Given further that $x \neq \frac{1}{2}$ and $f(1) = 0$, find an expression for $f(x)$ if

(i) $n = 1$ [3]

(ii) $n = 4$ [3]

(b) Write down the range of values of n for which $f(x)$ does not have any stationary points. Support your answer with appropriate workings [2]

Credit: **S4 NCHS P1/2014 PRELIM Qn 8**

14 Differentiation & Integration

1. The equation of a curve is

$$y = \frac{2x}{\sqrt{8x - x^2}}$$

(a) Show that [3]

$$\frac{dy}{dx} = \frac{8x}{\sqrt{(8x - x^2)^3}}$$

(b) Hence, evaluate the following, leaving your answer in the given form [4]

$$\int_2^5 \frac{2x}{\sqrt{(8x - x^2)^3}}\, dx = \frac{\sqrt{3}}{6}\left(a\sqrt{5} + b\right)$$

(c) Find the equation of normal to the curve where $x = 4$ [4]

Credit: **S4 MGS P1/2009 PRELIM Qn 9**

2. (a) Differentiate the following with respect to x [1]

$$xe^{2x}$$

(b) Find the x-coordinate of the stationary point on the curve [2]

$$y = xe^{2x}$$

(c) Using your answer from part (a), evaluate the following [3]

$$\int_0^2 4xe^{2x}\, dx$$

Credit: **S4 WSSS P1/2009 PRELIM Qn 4**

3. A curve has the equation

$$y = \frac{3x^2}{x - 1}\ ,\ x \neq 1$$

(a) Find $\dfrac{dy}{dx}$ [2]

(b) Hence, evaluate [3]

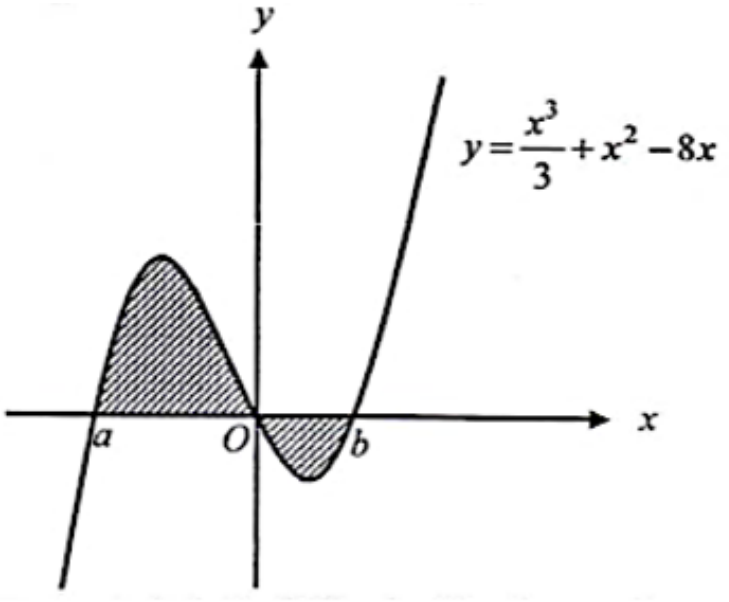
$$\int_2^4 \frac{x^2 - 2x}{3(x-1)^2}\, dx$$

(c) Hence, find the rate of change of x when $x = 3$, given that y is decreasing at a constant rate of 4 units per second [2]

Credit: **S4 CHIJ SNGS P1/2010 PRELIM Qn 7**

4. The diagram shows part of the curve

$$y = \frac{x^3}{3} + x^2 - 8x$$



The curve intersects the x-axis at $x = a$, $x = 0$ and $x = b$

(a) Find the x-coordinates of the stationary points and determine the nature of each of these points [5]

(b) The shaded region below is bounded by the curve and the x-axis. Show that the area of the shaded region is [4]

$$\left[4\left(a^2+b^2\right) - \frac{1}{12}\left(a^4+b^4\right) - \frac{1}{3}\left(a^3+b^3\right)\right] \text{ square units}$$

Credit: **S4 DHS P1/2011 PRELIM Qn 9**

5. (a) Prove that [4]

$$\frac{d}{dx}\left[\ln\left(\frac{\sin x + \cos x}{\sin x - \cos x}\right)\right] = \frac{2}{\cos 2x}$$

(b) Hence, evaluate [4]

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{1 - 2\sin^2 x}$$

Credit: **S4 BPGHS P2/2012 PRELIM Qn 8**

15 Kinematics

1. A particle moves in a straight line so that its velocity, v ms^{-1}, is given by

$$v = 3t^2 + kt$$

where t is the time in seconds, after leaving a fixed point O. If the particle passes O with acceleration -3 ms^{-2}

(a) Show that $k = -3$ [2]

(b) Calculate the values of t when the particle is instantaneously at rest [2]

(c) Calculate the average speed of the particle during the first 4 seconds of travel [5]

Credit: **S4 HKSS P2/2009 PRELIM Qn 6**

2. A particle moves in a straight line so that t s after leaving a fixed point O, its velocity, v ms^{-1} is given by

$$v = 5\left(1 - e^{1-t}\right)$$

(a) Find the value of t when the particle is instantaneously at rest [2]

(b) Find the distance travelled by the particle in the first 2 seconds [4]

(c) Find the acceleration of the particle when $t = 2.5$ [2]

(d) State the value which v approaches as t becomes very large [1]

Credit: **S4 CHS P1/2011 PRELIM Qn 14**

3. The acceleration a ms^{-2} of a particle travelling in a straight line, at the t seconds after leaving a fixed point O with a velocity of 2 ms^{-1} is given by, for $0 \leq t \leq 6\pi$ is

$$a = 2\cos\left(\frac{t}{3}\right)$$

(a) Find the inital acceleration of the particle [2]

(b) Find the values of t when the particle first comes to instantaneous rest [3]

(c) Calculate the total distance travelled by the particle in the interval $t = 0$ and $t = 15$ [3]

Credit: **S4 SPSS P1/2012 PRELIM Qn 15**

4. In the Chingay Parade procession held at the heartlands early this year, the Pioneer Generation Float was travelling on a straight road with a velocity, v ms^{-1}, given by the equation

$$v = 5t - \frac{1}{2}t^2 + 4$$

t is the time after passing a fixed point A

(a) Show that the maximum velocity is reached 5 seconds later [3]

(b) Sketch the velocity-time graph for the first 5 seconds [3]

Upon reacing its maximum velocity, the float started to decelerate uniformly at 1.5 ms^{-2}, before coming to a rest at point B to allow residents to take photographs

(c) Find the time when the float reached B [2]

(d) Find the total distance travelled from A to B [3]

Credit: **S4 AHS P2/2015 PRELIM Qn 9**

END OF PRACTICE QUESTIONS

