Topic 3: Surds (4049)

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THE ABOUT

CHAPTER ANALYSIS

- Four operations on surds, including rationalizing the denominator
- Solving equations involving surds

MASTERY

- Relatively simple chapter
- 1 **key** concept

EXAM

- Concepts usually tested as a stand-alone topic or tested with other concepts (Simultaneous Equations, Logarithms, Differentiation etc.…)
- Easy to make careless mistakes if not weary careful during the manipulations

WEIGHTAGE

- High overall weightage, scattered amongst different questions and topics
- Tested consistently every year

KEY CONCEPT

Laws of Surds

Rationalisation of Denominator (Surds)

Assumed Knowledge: Laws of Indices (E-Math)

TAKE NOTE

How to simplify surds with big numbers (non-prime) inside them

- Break down the big numbers using prime factorisation
- Use the **Multiplication Surd Law** to break up the surd

Examples

$$\sqrt{18} = \sqrt{2 \times 3^2}$$
$$= \left(\sqrt{2}\right)\left(\sqrt{3^2}\right)$$
$$= \mathbf{3\sqrt{2}}$$

$$\sqrt{360} = \sqrt{2^3 \times 3^2 \times 5}$$
$$= \left(\sqrt{2^2}\right)\left(\sqrt{2}\right)\left(\sqrt{3^2}\right)\left(\sqrt{5}\right)$$
$$= \mathbf{6\sqrt{10}}$$

A pair of conjugate surds is a pair of expressions of the form

$$\left(m\sqrt{a} \pm n\sqrt{b}\right) \qquad \left(m\sqrt{a} \mp n\sqrt{b}\right)$$

The product of a pair of conjugate surds will **ALWAYS** give a rational number

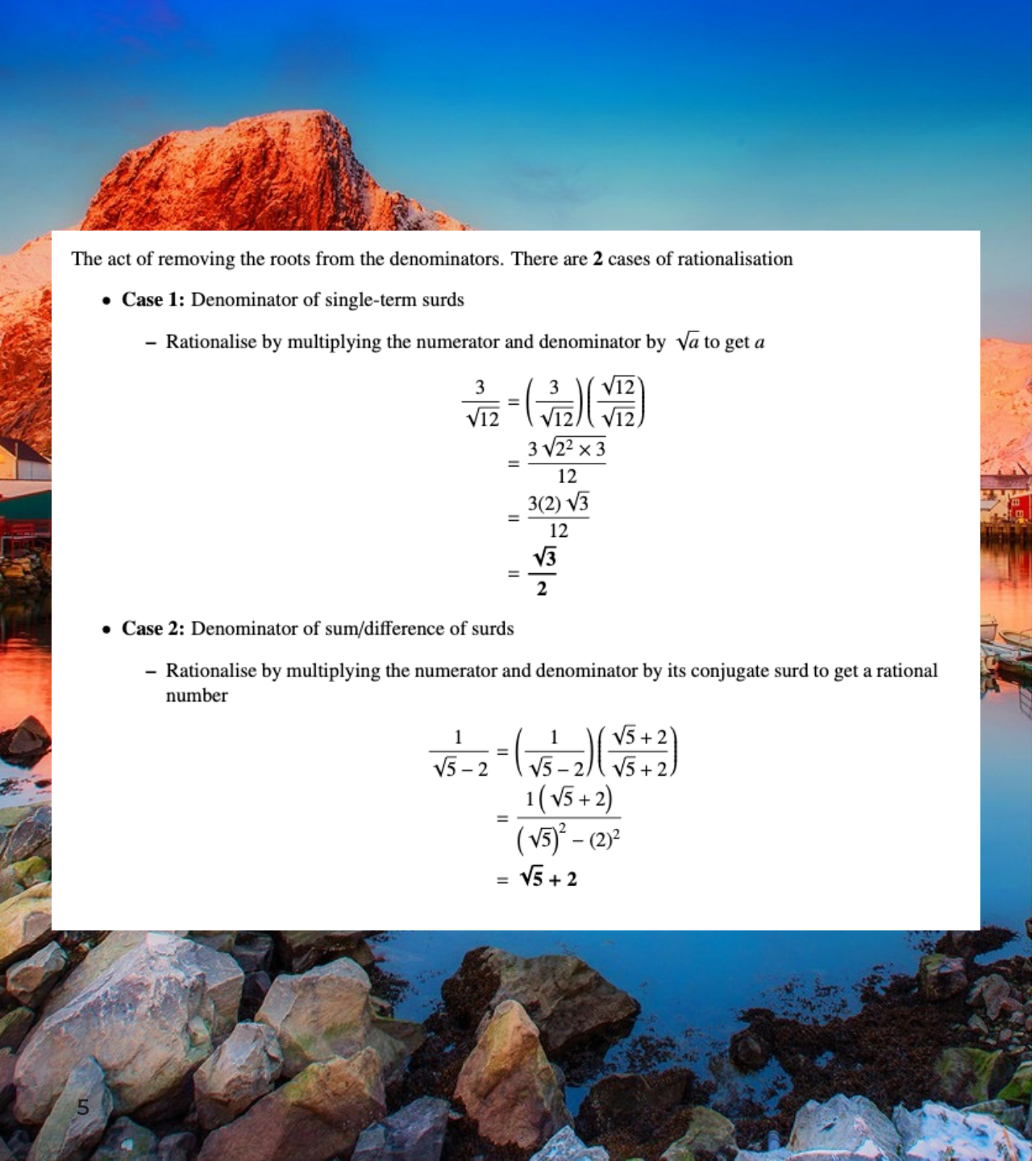
$$\left(5 + \sqrt{5}\right)\left(5 - \sqrt{5}\right) = (5)^2 - \left(\sqrt{5}\right)^2 = 25 - 5 = 20$$

This is due to the algebraic identity:

$$(a + b)(a - b) = a^2 - b^2$$

Laws of Surds

Laws of Surds	
Multiplication	$\sqrt{a}\times\sqrt{b} = a^{\frac{1}{2}}\times b^{\frac{1}{2}} = (ab)^{\frac{1}{2}} = \sqrt{ab}$
Division	$\sqrt{a} \div \sqrt{b} = \frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}} = \left(\frac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\frac{a}{b}}$
Addition	$m\sqrt{a} + n\sqrt{a} = (m + n)\sqrt{a}$
Subtraction	$m\sqrt{a} - n\sqrt{a} = (m - n)\sqrt{a}$



The act of removing the roots from the denominators. There are **2** cases of rationalisation

- **Case 1:** Denominator of single-term surds
  - Rationalise by multiplying the numerator and denominator by $\sqrt{a}$ to get a

$$\frac{3}{\sqrt{12}} = \left(\frac{3}{\sqrt{12}}\right)\left(\frac{\sqrt{12}}{\sqrt{12}}\right)$$
$$= \frac{3\sqrt{2^2 \times 3}}{12}$$
$$= \frac{3(2)\sqrt{3}}{12}$$
$$= \frac{\sqrt{3}}{2}$$

- **Case 2:** Denominator of sum/difference of surds
  - Rationalise by multiplying the numerator and denominator by its conjugate surd to get a rational number

$$\frac{1}{\sqrt{5}-2} = \left(\frac{1}{\sqrt{5}-2}\right)\left(\frac{\sqrt{5}+2}{\sqrt{5}+2}\right)$$
$$= \frac{1\left(\sqrt{5}+2\right)}{\left(\sqrt{5}\right)^2 - (2)^2}$$
$$= \sqrt{5} + 2$$

Rationalisation of Surds

The act of removing the roots from the denominators

2 cases of Rationalisation:

- **Case 1:** Denominator of single-term surd
  - Rationalise by multiplying the numerator and denominator by $\sqrt{\boldsymbol{a}}$ to get $\boldsymbol{a}$

- **Case 2:** Denominator of sum/differences of surds
  - If the denominator is of the form $\boldsymbol{m\sqrt{a} \pm n\sqrt{b}}$, rationalise it by multiplying the numerator and denominator by its conjugate surd $\boldsymbol{m\sqrt{a} \mp n\sqrt{b}}$ to get a rational number $\boldsymbol{m^2a - n^2b}$

Common Mistake

The following 2 statements are **INCORRECT**, but many students still get confused and use these 2 statements in their solutions

$$a^m + b^m = (a+b)^m \quad .....(*)$$

$$a^m - b^m = (a-b)^m$$

Proof that (*) is incorrect

By substituting values of $a = 1$, $b = 2$ and $m = 3$

$$\text{LHS} = 1^3 + 2^3 = 9$$

$$\text{RHS} = (1+2)^3 = 27$$

$$\therefore \textbf{LHS} \neq \textbf{RHS}$$

This slide is included as this is assumed knowledge from the 'O' Level E-Math syllabus

Laws of Indices

Laws of Indices	
Same Base	$a^m \times a^n = a^{m+n}$
	$\frac{a^m}{a^n} = a^{m-n}$
	$(a^m)^n = a^{mn}$
Same Power	$a^m \times b^m = (ab)^m$
	$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
Radicals	$\sqrt[n]{a} = a^{\frac{1}{n}}$
	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$
Others	$a^0 = 1$
	$a^{-n} = \frac{1}{a^n}$

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